Analytical Modeling for EVM in OFDM Transmitters Including the Effects of IIP3, I/Q imbalance, Noise, AM/AM and AM/PM Distortion

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Abstract—In this paper, we present a method for accurately calculating the Error Vector Magnitude (EVM) of OFDM transmitters based on their IQ mismatch, IIP3, noise, AM/AM and AM/PM distortion. The effects of these impairments are correlated. Thus, modeling and analyzing them in isolation results in large errors. We derive the analytical relation of the received symbol when all types of impairments are present at once and compute EVM based on this overall relation. This method helps test engineers to compute the EVM based on already measured parameters and eliminates the need to develop and set-up EVM measurements. Simulations and hardware measurements show that this calculation can be done with less than 1% error for a large range of EVM values. This error level has been shown to be within the uncertainty bounds of EVM measurements.

I. INTRODUCTION

Radio frequency (RF) transceivers require a disproportionately high effort in terms of test development time, test equipment cost, and test time. The relatively high test cost stems from the facts that these devices operate at high frequencies, requiring expensive equipment and complex load-board design and verification, and that complex specifications have to be measured, complicating test development and necessitating a long learning process for test engineers.

Most of the parameters of transceivers are correlated and these correlations have long been exploited to reduce the test cost [1]–[3]. More importantly, standard-based specifications, such as error vector magnitude (EVM) and bit error rate (BER) are tightly correlated to designer-specified system level parameters such as gain, non-linearity, and IQ imbalances [4].

For transmitters, one of the most important performance parameters is EVM, which is basically a figure of merit for modulation accuracy. In fact, BER can be obtained using the EVM result [4]. As such, it is an important parameter for production testing, yield learning, and diagnosis.

Unfortunately, measuring EVM requires a complex procedure. First, a golden receiver needs to be implemented on the load board which complicates its design and verification. The entire receiver operation needs to be duplicated on the test equipment [5] or on the load board [6]. In effect, the test engineer needs to duplicate the complete software for the receiver operation, which typically is designed by more than one person and has hardware accelerators operations, such as finding the start of the frame. Finally, the overall operation has to be verified against bench measurements. In addition to the difficulties of setting up reliable EVM measurements, EVM testing also take relatively long time. For instance for a WLAN system, EVM measurement can take 400ms while parameters with simpler test set-up take much shorter time (e.g. IQ imbalance takes 30ms and IIP3 takes 40ms to measure). As a result, despite the simple definition, EVM is one of the most challenging parameters to measure accurately and efficiently. It has been shown in the literature that EVM is directly correlated to system impairments, such as non-linearity, IQ imbalances, DC offset, and various sources of noise [7]–[11]. Ideally, these relations can be exploited to completely eliminate the EVM measurement when obtaining highly accurate results. This approach would not only reduce the test time considerably (several hundred milliseconds) but also eliminate the need to design, set-up, and verify an EVM measurement platform which takes considerable effort by test engineers [6].

In order to achieve this goal, the relation between EVM and other easy-to-measure system level parameters needs to be derived. However, this is not straightforward particularly for OFDM systems due to frequency/time domain conversions. Multiple QAM symbols are combined to generate the time domain signal that is subject to non-linearity in the power amplifier. There have been numerous attempts in the literature to analyze the effect of system impairments on EVM for a variety of reasons [5], [7]–[11]. We will explain these techniques in more detail in the next section.

DC offsets are generally removed before symbol mapping. The remaining five kinds of impairments have been shown to be the most important EVM contributions [7]–[11]. It has also been shown that the effect of noise on EVM is uncorrelated from that of IQ imbalance and non-linearity and it can be added as variance [5]. However, the contributions to EVM from IQ imbalance and non-linearity are tightly correlated as we will explain in the next section.

In this paper, we take an analytical approach for modeling the combined effect of IQ imbalance and non-linearity to accurately calculate EVM from already measured system-level parameters. This model is derived for OFDM systems without loss of generality where the contributors of IQ imbalance and non-linearity are highly correlated.

We derive an analytical model of the received symbol based on system parameters when both impairments are present. This analytical relation is used to calculate the low-frequency equivalent of transmitted signal for each OFDM symbol in each frame and it completely emulates the overall transmit-receive operation without making a single measurement. The EVM is then calculated according to whichever standard the system is designed for. Since the model is generic, there is no need to modify it with respect to the parameters of each standard. With this analytical approach, there is no need to generate statistical models that change as architectures, circuits
or standard change [5], [12].

The accuracy of the model and the associated EVM calculation method has been verified using extensive simulations and hardware measurements. To summarize, the contributions of this work are:

- An analytical model for I and Q signals at the receive end that completely emulates the entire transmit and receive operations in the presence of IQ imbalance and non-linearity.
- An algorithm to calculate EVM using the analytical model based on standard definition.
- Verification of the model and the EVM calculation method based on extensive simulations and hardware measurements.

II. RELATED WORK AND MOTIVATION OF THE PROPOSED WORK

Extensive efforts have been done to reduce the EVM test time with low degradation in measurement accuracy [13]–[15]. In [13], an optimization method for reducing the number of transmit/receive symbols is presented. A computationally efficient EVM measurement method for phase only modulation scheme is discussed in [14]. A test sequence reduction technique is presented in [15]. The more sensitive corner cases are determined and included in test vector. In all of these methods, the test time is reduced; however the test development process has to be done including golden receiver implementation, load board design, and demodulation capability implementation.

There is a body of work focusing on substituting the traditional EVM measurement with a simple measurement and estimating the EVM using statistical modeling techniques. In [12], the response of the receiver under test to a multi tone test stimulus is observed. This response is used to estimate the EVM using a multiple adaptive regression splines (MARS) learning method. An efficient multi-tone test signal is used in [16] to simplify the EVM measurement for UWB systems. In addition to multi tone test stimuli, the null carriers information is used to estimate the EVM in [17]. Test time required to gather adequate information from the device is reduced in [18]. In such statistical modeling techniques, data collection and learning process need to be repeated through change in the process, architecture, or circuit implementation.

Another research thread focuses on the effect of one or more of system impairments in single carrier systems [4], [19]–[21]. The impairment effects in these type of systems are different as they process one symbol at the time and they do not deal with inter carrier interference (ICI) problem. Thus, their derivations are not valid for multi-carrier systems.

Most of the research work on analysis of non-linearity effect in OFDM systems are based on statistical approximation [10], [11]. They assume infinite number of sub-carriers. In [11], central limit theorem is used to model the OFDM baseband signal as a complex Gaussian process with Rayleigh envelope distribution. Hence, the baseband signal amplitude expressed by using the extended Bussgang theorem with a complex coefficient of the signal and non-linear distortion noise. Gain and phase imbalance is modeled in the receiver side. Similarly, complex Gaussian assumption for OFDM signal is used in [10] to use the result of Bussgang theorem. Thus, the effect of non-linearity on the OFDM baseband signal is modeled as a scaled version of the signal plus statistically independent distortion process. The combined effect of IQ imbalance and non-linearity of EVM is modeled as affine linear functions.

In [5], the authors present an analytical technique for extraction of EVM from the system parameters. However, the two effects are still analyzed in isolation, even though they are combined in the vector domain. The accuracy of this approach would decrease with higher impairment levels as I/Q imbalance will change the peak and PAR of the signal. To obtain an accurate model, both I/Q imbalance and non-linearity has to be incorporated at the same time. One way of achieving this simultaneous modeling is to use the black-box modeling technique. For instance in [22], the authors use a behavioral simulations to establish a link between EVM and I/Q imbalance and non-linearity (includes AM/AM and AM/PM distortion) simultaneously. Here, the assumption is that RMS EVM is obtained over virtually infinite number of symbols.

In this paper, we take an analytical approach to the same modeling problem as we believe that analytical relation, if they can be obtained, provide more insight. With our technique it is possible to compute the EVM of any given OFDM frame, which would make it possible to demonstrate a frame by frame correlation with the bench and customer results. Such correlations are typically required before a test can be altered or eliminated during high volume manufacturing.

III. PROPOSED METHODOLOGY

As mentioned before, EVM is one of the most important specifications of a transmitter. EVM indicates the modulation accuracy in the system and it is basically given by the deviation of the QAM symbols from their ideal location. It is calculated according to the Eqn. (1).

\[
EVM = \frac{1}{N_f} \sum_{f=1}^{N_f} \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left( C_n(n,f) - C_{n,t}(n,f) \right)^2 / P_0}
\]  

(1)

Where \( N_f \) is the number of frames, while \( N \) is the number of sub-carriers in an OFDM symbol. \( P_0 \) is the average power of the constellation. \( C_n(n,f) \) is the transmitted QAM symbol in frame \( f \) on sub-carrier \( n \), and \( C_{n,t}(n,f) \) is its equivalent captured QAM symbol.

In this work, we study some of the most damaging impairments in transmitters, IQ mismatch, IIP3, AM/AM and AM/PM distortion. We do not focus on the effect of noise since its contribution to EVM is uncorrelated and can be added simply as variances which is discussed in prior work [5]. Different methods have been used to combine these effects. In [5], IQ imbalance and non-linearity effects are added vector wise for deterministic movements. It is not the most optimal way, as the power of the signal can be different at the output of the IQ modulator depending on the IQ mismatch in the system.
A. System Level Model

Fig. 1 shows the complete system model that is used for all the analysis. IQ mismatch is modeled in the IQ modulator path. Also non-linearity, AM/AM and AM/PM distortion is injected using a 3rd order polynomial model with complex coefficients for the power amplifier (PA). A golden direct conversion receiver is used to down convert the signal.

In order to derive our model, we have to make a few key observations. First, IQ imbalance can be due to many different factors in the circuit. Process variations in the phase shifter can inject IQ imbalance at various points. The combined effect of all these impairments is effectively a phase difference in the modulator and a gain mismatch in the I and Q arms. This part of the circuit is fairly linear, however, after we accumulate IQ imbalance at the modulator, the baseband to RF conversion complicates the modeling process. The second observation we make is that due to the linearity of the baseband operations, the overall impact of the IQ imbalance, which is normally expressed in the RF signal can be modeled before the modulator. This backwards propagation helps us express the IQ imbalance effect directly at the baseband signals. As such, the effect of IQ imbalance can be seamlessly transferred into the time domain signal in the baseband or RF domains which will be subject to the non-linearity, AM/AM and AM/PM in the power amplifier. This ability of referring the effect of IQ imbalance to the input and then applying the non-linearity and AM/PM on the baseband signals enables our modeling.

B. Phase and Gain mismatch effect

The concept of back propagating I/Q imbalance is demonstrated in Fig. 2. The $I + jQ$ signal is predistorted to reflect the IQ imbalance effect. The new $I' + jQ'$ signal is subjected to non-linear distortion to obtain the baseband equivalent of the power amplifier output, $I'' + jQ''$. Finally, this signal is compared with $I + jQ$ to model the EVM. As mentioned earlier, this result can be added with the effect of noise in variance to obtain the overall EVM for the system. Eqn. (2) shows the back-propagated signal, $C_n'$ for this purpose. Note that in this derivation, the I and Q signals are in the time domain. However, the modulator/demodulator (which also includes the IFFT and FFT operations) introduce dependency on other symbols. These operations are also the reason why this modeling process is challenging for OFDM systems. In Eqn. (2), $g$ is the gain mismatch and $\theta$ is the phase mismatch.

$$C_n' = cos(\frac{\theta}{2}) (C_n + \frac{g}{2} C_{n-n}) + jsin(\frac{\theta}{2}) (C_{n-n} + \frac{g}{2} C_n)$$  \hspace{1cm} (2)

Where $C_n'$ is the predistorted QAM symbol on nth subcarrier and $C_n$ is its corresponding transmitted symbol. Also $C_{n-n}$ is the transmitted symbol on the symmetric subcarrier. The details of the derivation of this received symbol representation has been shown in [7]. We use this result to show the overall impact and how we can move forward with our combined model. IQ mismatch causes cross interference between I and Q arms. Also in multi-carrier systems, the information on each sub-carrier is corrupted by the symmetric sub-carrier side band. The symmetric sub-carrier can carry any of the $M$ possibilities of QAM symbols with M-QAM modulation.

C. IIP3, AM/AM and AM/PM effect

Designing a linear power amplifier with high efficiency is always a challenge for OFDM systems. High peak to average ratio in OFDM signals causes high sensitivity to power amplifier non-linearity. Non-linearity has two effects in these systems. First of all, gain compression occurs for all the sub-carriers. Secondly, in multi-carrier systems, sub-carriers lose their orthogonality because of non-linearity. Thus, the information on each sub-carrier is polluted by the uncorrelated information from other sub-carriers. The received QAM symbols will be a function of the gain compression effect on its own sub-carrier and the data on all other neighbour carriers. In the literature, some statistical models have been used to estimate the effect of non-linearity on EVM [10], [11]. However, here we analytically calculate each of the symbols.

The model in Fig. 1 with an ideal IQ modulator (no gain and phase mismatch) is used for the analytical derivation of the system. Note that the non-idealities in the IQ modulator change the locations of the symbols from their ideal positions. However, for the non-linearity modeling step, there is no assumption on the location of symbols. Thus, the two analysis can be combined seamlessly.

In the first step, the real and imaginary part of the QAM symbols ($C_n$) are separated as Eqn. (3) shows. This separation is done to make the derivations easier to understand.

$$C_{nr} = \text{Re} [C_n] \quad , \quad C_{ni} = \text{Im} [C_n]$$  \hspace{1cm} (3)
Eqn. (4) shows the OFDM signal in terms of the sent QAM symbols for one OFDM symbols.

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (C_{nr}(k) + jC_{ni}(k)) \exp \left[ j \frac{2\pi ft}{T} \right] \quad 0 < t < T \quad (4) \]

Where \( N \) is the number of sub-carriers (\( N=64 \) in all the presented simulations and hardware measurements) and \( T \) is OFDM symbol duration. A copy of the last part of the signal has to be added to the beginning of the OFDM symbol that is called cyclic prefix (CP). CP is added to the beginning of each OFDM symbol to guarantee the robustness of the OFDM transmission against the inter symbol interference (ISI). In addition, adding CP converts the linear convolution between the multipath channel and the transmitted OFDM block to a circular convolution. This portion of the signal is ignored in the receiver side for digital demodulation. Thus, for adding CP only the time limit changes in Eqn. (4) to \(-T_d < t < T\), where \( T_d \) is the cyclic prefix duration.

In order to derive the PA input signal, a simpler model is shown in Fig. 3. I arm carries the real part of the \( x(t) \) signal while Q arm carries the imaginary part. The output of the demodulator is shown in Eqn. (5). While \( \omega_c \) is the LO frequency of the transmitter. Note that \( I(t) \) and \( Q(t) \) here already include the effect of IQ mismatches.

\[ RF_p(t) = \Re \left[ x(t) \right] \cos(\omega_c t) - \Im \left[ x(t) \right] \sin(\omega_c t) \]

\[ = \Re \left[ x(t) \exp(j\omega_c t) \right] \quad (5) \]

Now \( x(t) \) can be substituted in Eqn. (5) from Eqn. (4). Also we need to take the gain of the IQ modulator \( G_{i} \) into account. The result is shown in the following equation:

\[ RF_p(t)=G_i \Re \left[ \sum_{k=0}^{N-1} (C_{nr}(k) + jC_{ni}(k)) \exp \left[ j \frac{2\pi ft}{T} + \omega_c t \right] \right] \]

\[ = G_i \sum_{k=0}^{N-1} C_{nr}(k) \cos \left( \frac{2\pi ft}{T} + \omega_c t \right) - j C_{ni}(k) \sin \left( \frac{2\pi ft}{T} + \omega_c t \right) \]

\[ - T_d < t < T \quad (6) \]

The IIP3, AM/AM and AM/PM is modeled in the PA using polynomial model with complex coefficients. The \( RF_p(t) \) has to be substituted in this model.

\[ Y(t)=a_1 x(t) + a_3 x^3(t) \quad (7) \]

Phase distortion effect is introduced as the phase of the first and third order coefficients which translate to the delays in the RF signal as it is shown in Eqn. (8). \( \tau_1 \) is the delay due to the first order term while \( \tau_3 \) is due to the third order term.

\[ a_1 = |a_1| e^{j\phi_1} \]

\[ \phi_1 = \tau_1 \omega_c \quad (8) \]

In the next step, an ideal demodulation is performed on the PA output signal. After the down conversion, low pass filtering is done to remove high frequency components of the signal. Four functions are defined in Eqn. (9) to help the demonstration of the output signals. Here \( C_i(x) \) corresponds to the imaginary part of the QAM symbol on \( xth \) subcarrier, while \( C_r(x) \) is its real part.

\[ f_1(x,y,z) = C_i(x) C_y(x) C_z(x) \]

\[ f_2(x,y,z) = C_r(x) C_y(x) C_z(x) \]

\[ f_3(x,y,z) = C_i(x) C_y(x) C_z(x) \]

\[ f_4(x,y,z) = C_r(x) C_y(x) C_z(x) \quad (9) \]

Eqn. (10) shows the \( I'' + jQ'' \) signal at the input of the FFT block. The \( X_1 \) term includes the linear gain of the amplifier and the rest of the terms are the result of non-linearity and other modeled distortion effects. In addition, the gain of the IQ demodulator on the receiver side \( (G_r) \) is taken into account.

In order to calculate the QAM symbols, an FFT has to be performed on \( X_1[n] \) signal. The final results for the received QAM symbols are not shown because of the space limit. However, basically they have a similar format to the \( X_1 \) signal. In order to calculate any of the QAM symbols in the frame, the sequence of the symbols in the frame has to be multiplied and added together. The sequence will be different for each sub-carrier depending on their location.

Table I includes simulation results for some cases of different IIP3, and distortions cases while Fig. 4 demonstrate the maximum presented AM/AM and AM/PM versus the maximum power in each OFDM frame for the first case in Table I. The maximum distortion of the frames are averaged out over all 50 simulated frames and it is shown in Table I. Using the calculated QAM symbols, the EVM is calculated and compared with the simulated one which is measured with conventional scheme. EVM results are accurate with less than 0.05% error. This error is caused by accuracy of sampling in MATLAB.

**D. EVM computation process**

With the analytical model, the EVM can be calculated using the sent QAM symbols and the system imperfections of the device under test (DUT):
\[ X_r(n)=G_0 \left[ \sum_{k=0}^{N-1} X_1(n) + \frac{a_3G_i}{8(\sqrt{N})^3} \sum_{p=0}^{N-1} \sum_{z=0}^{N-1} X_2(n) + X_3(n) + X_4(n) + X_5(n) + X_6(n) + X_7(n) \right] \] 

\[ X_1(n)=\left( C_i(k) + jC_i(k) \right)e^{j(\frac{2\pi(k+p)z}{N} - \omega_c)}e^{-j(\frac{2\pi(k+p)z}{N} + \omega_c)} e^{j(\frac{2\pi(k+p)z}{N} - \omega_c)}e^{-j(\frac{2\pi(k+p)z}{N} + \omega_c)} \]

Fig. 5. Combined non-linearity and IQ imbalance effect (a) Received QAM symbols (b) Real part of calculated vs. received QAM symbols

- System parameters (IQ imbalance, IIP3, AM/AM, AM/PM and path gain) which are already measured.
- Generate enough number of random input symbols based on the standard.
- Calculate the received symbol using the proposed derivation.
- determine the Standard based parameters in Eqn. (1) like N and \( P_{in} \).
- Calculate EVM according to Eqn. (1).

There is no need for any EVM test set-up and capturing high number of symbols to measure the EVM, as the captured symbols can be directly calculated. Having these accurate relations between the system imperfections and the EVM, an accurate pass and fail limit can be obtained for any system characterization based on the EVM limitation.

E. Verification of the EVM calculation using simulations

In this section, we present simulation results to illustrate the accuracy of the derived mathematical relation. Fig. 5(a) shows the received symbols (dots) in presence of IQ mismatch (g = 20\%, \( \theta = 5^\circ \)) and non-linearity (IIP3 = 9dBm, PA input power=8.4dBm) in the system. The average of the maximum AM/AM distortion over 100 frames is 2.37dB while AM/PM is in a 17.5\% range.

Fig. 5(b) shows one example of the calculation accuracy under the combined effect of IQ mismatch and non-linearity. The calculated symbols follow the received ones with high accuracy.

Table II includes more cases of different IIP3, distortions and IQ mismatch values. EVM results are accurate with less than 0.4\% error. The error in symbols is quite small and it is caused by accuracy of up sampling and down sampling in MATLAB as it is explained before.

IV. HARDWARE MEASUREMENT RESULTS

The accuracy of the method and the derivation is also confirmed on a hardware platform.

Fig. 6 shows the measurement set-up. Vector signal generator (R&SSMBA1100) is used as the transmitter IQ modulator. IQ mismatch can be injected in the system, using this equipment. A commercial off-the-shelf amplifier (mini-Circuits zx60-2522M-S+) is used as the power amplifier in the transmitter path. The amplifier specifications are measured using traditional techniques. The gain of the PA is 21dBi and the IIP3 is 6.2dBm. A vector signal analyser (RkeSFSVR) is used as the golden receiver. The measurements has been done under presence of each imperfection separately and both at the same time on 2.4GHz frequency range.

Table III shows the result for only having IQ mismatch in the system (there is no amplifier in the path). Measured EVM values follow the calculated one with less than 0.5\% error.
Table IV shows the result for only non-linearity in the path. Thus, the IQ modulator is ideal. As we use the same amplifier for all the cases, the signal power at the input of the PA is changed by using different gain for the IQ modulator. Thus, we can model different levels of non-linearity in the system. Average AM/AM distortion is different for each case due to the change of signal power at the input of power amplifier. Also in this case, measured EVM follows the calculated one with less than 0.5% error. This is an acceptable error for EVM and it is caused by noise in the system and any unmodeled effects in the path.

Table V shows the EVM results for IQ mismatch and non-linearity in the system. In this step, the gain of the VSG is changed for each IQ mismatch setup to show how accurate the calculated EVM result follows the measured EVM result in all levels of non-linearity and any IQ mismatch values. Measured EVM follows the calculated one with less than 1% error. This is an acceptable error for EVM measurements.

V. CONCLUSIONS

An accurate analytical EVM calculation method has been presented in presence of combined effect of IQ mismatch and non-linearity. The received symbols are calculated directly using the information on system imperfections and the gain of the system from the sent QAM symbols. Thus, the EVM is calculated without actually making a single EVM measurement. In addition having accurate relation between EVM and system parameters enable us to define accurate pass/fail boundaries for the system imperfections according to EVM requirement of the system. The accuracy of the proposed technique is shown with simulations and hardware measurements.

ACKNOWLEDGEMENTS

This project is supported by Semiconductor Research Corporation with Task Number 1836.084. The authors would like to thank Rhode and Schwartz for donating the time on the measurement equipment.

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