Maximum Rates of Frequency Scanning and Mechanical or Electronic Stirring for Distortionless Signal Generation Inside Electromagnetic Reverberation Chambers

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Abstract—We present upper bounds for the rates of frequency scanning and mechanical or electronic stirring inside a reverberation chamber. The bounds are obtained by imposing quasi-stationarity of the field to hold, allowing avoidance of nonlinear distortion of the analog or digital excitation (test) signal injected into the chamber. The interior field is represented as a dynamic stochastic process causing random hybrid modulation of the excitation field.

Keywords—EMC measurements, reverberation chambers, quasi-stationarity, frequency scanning, mode stirring.

I. INTRODUCTION

Mode-tuned and mode-stirred electromagnetic reverberation chambers (MT/MSRCs) are finding increasingly widespread use in EMC for immunity, emissions, shielding, and absorption measurements and testing of electronic devices. The second edition of the generic international standard IEC 61000-4-21 on reverberation chambers for EMC measurements [1] is currently under development.

In applying MT/MSRCs to immunity testing, an issue of economic importance is the reduction of the duration of a test cycle. Major contributions to the cycle time are the time for one revolution (period) of the mode stirrer or mode tuner and the rate of frequency scanning at a single stir state. Here, we investigate the maximum stirring speed and maximum rate of scanning across a specified frequency interval such that nonlinear distortion of a narrowband excitation (test) signal used in immunity testing is prevented. This involves analogies, as well as important differences compared to the issue of dwell time of mechanical MT vs. MS, because reliable immunity testing requires a sufficiently slow sweeping.

Whilst the main application and focus in this paper is immunity testing, the results are also relevant to emissions testing, even though in this case the rate of any frequency variation is governed by the EUT. In addition, this issue is of considerably wider importance in connection with signal distortion/integrity and EM interference/robustness of modern ultra-wideband (UWB) wireless digital communication systems.

II. QUASI-STATIONARITY FOR FREQUENCY SCANNING

The condition that will be imposed here is for the field to be quasi-stationary at all times. To this end, the magnitude of the rate of change of the instantaneous frequency of a CW (time-harmonic) excitation signal must not exceed the squared halved minimum bandwidth of the overall measurement system, at this frequency [2], [3]:

$$\left| \frac{df(t)}{dt} \right| \ll \frac{\pi}{2} (\Delta f)^2.$$  (1)

For a monotonically increasing $f(t)$ and assuming that the Q-bandwidth of the chamber $\Delta f \equiv f/Q(f)$ is dominant (i.e., smaller than the susceptibility bandwidth of the EUT) at this frequency, (1) becomes

$$\frac{dt}{df} \gg \frac{2}{\pi} \left[ \frac{Q(f)}{f} \right]^2 = \frac{512\pi^3}{c^6} V^2 f^4 (CCF)^2,$$  (2)

in which CCF denotes the chamber calibration factor [1]. The overall time $T$ required to perform a complete frequency scan that maintains quasi-stationarity of the cavity field at all times is then

$$T \gg \int_{f_0}^{f_1} \left( \frac{dt}{df} \right) df.$$  (3)

If this limit is observed, then test results obtained for a continuously varying frequency sweep or using electronic frequency stirring are equivalent with those obtained with a discretely (stepwise) changing frequency.

For the designing or the chamber or the experiment, it is useful to have an a priori calculable theoretical estimate available that is based on the parameters of an idealized

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chamber and conditions of operation. Substituting (2) into (3) and making use of the theoretical HF asymptotic expression \( Q(f) = 3V\mu_0/[2S\mu_0\delta(f)] \) (cf., e.g., [4]) yields

\[
T > \frac{9V^2\mu_0\sigma^2}{2S^2\mu} \ln \left( \frac{f_1}{f_0} \right) \equiv T_{\text{min}}(f_0, f_1). \tag{4}
\]

The lower limit \( T_{\text{min}} \) holds under the condition of pure frequency modulation for the dynamic field. In general, however, frequency scanning in an overmoded cavity produces amplitude fluctuations as well, i.e., the overall field exhibits hybrid amplitude-plus-frequency modulation [3]. The resulting additional fluctuations demand an even slower sweeping to be observed in order for quasi-stationarity to hold.

The inequality (4) shows that the minimum sweep period depends logarithmically on the ratio of \( f_1 \) to \( f_0 \). Hence, increasing \( f \) allows for progressively faster scanning. We shall refer to this optimum scanning strategy as adaptive scanning. Figure 1 shows plots of the lower limit in (4) as a function of \( f_1/f_0 \) for selected values of the characteristic size \( \ell = V/S \) of a nonmagnetic cubic chamber with interior volume \( V \), inner surface of area \( S \) and wall conductivity \( \sigma = 10^4 \text{S/m} \). Keeping the chamber size to a minimum has a beneficial effect on reducing the minimum required scan duration, apart from obvious benefits in minimizing power amplification requirements for attaining a minimum test level.

![Figure 1. Minimum scan period \( T_{\text{min}} \) (in units of s/dec) as a function of the ratio of upper and lower frequency limits of the scan band, for indicated selected values of the characteristic length (in units m) of a cubic cavity with wall conductivity \( \sigma = 10^4 \text{S/m} \) and permeability \( \mu = \mu_0 \).](image)

### III. QUASI-STATIONARITY FOR TESTING AT LISTED DISCRETE FREQUENCIES

For testing at a number of listed discrete frequency points, the theoretical maximum rate \( s_{\text{max}} \) (in units decade/s) obviously equals the reciprocal of the number of discrete frequencies \( n \) per frequency decade multiplied by the dwell time \( \delta t \) per frequency point:

\[
s_{\text{max}} = \frac{1}{n\delta t}. \tag{5}
\]

where it is assumed that the time needed to move between frequency points (frequency hopping) is ideally zero. In practice, \( n\delta t \) is to be piecewise augmented by the lower limit in (3) or (4) between each pair of listed test frequencies if quasi-stationarity of the field in between these frequencies must be maintained. In this case, (5) serves as a theoretical upper limit (maximum maximorum) that cannot be realistically attained. If the \( n \) frequencies per decade \( (n>1) \) have linearly equidistant spacings \( \delta f \), then the maximum scan rate, including specified dwelling at each test frequency, becomes

\[
s'_{\text{max}} = \frac{1}{n\delta t + (n-1)T_{\text{min}}'}. \tag{6}
\]

with

\[
T_{\text{min}}'(f_0, f_0 + \delta f) \approx \frac{81V^2\mu_0^2\sigma}{2(n-1)S^2\mu}. \tag{7}
\]

for \( n \gg 10 \). This strategy is suboptimal, because it does not take advantage of the fact that the maximum permissible scan rate increases (linearly) with increasing instantaneous frequency (cf. (1) with \( \Delta f \sim \sqrt{f(t)} \)), unlike in adaptive scanning for which the rate does increase linearly.

If quasi-stationarity of the field in between test frequencies is not required, then one must nevertheless still add a sufficiently long settling time at each test frequency – prior to applying the field and performing the test – in order for the field to reach steady-state conditions at \( f_1 = f_0 + \delta f \), following each (nonstationary) partial scan between \( f_0 \) and \( f_0 + \delta f \).

A set of example values of typical maximum sweep rates given by (6) is shown in Table I. For example, 100 discrete frequencies per decade with a dwell time of 10 ms per frequency requires a sweep rate slower than 0.1703 decade per second. All values were computed for a signal distortion level [3] specified as 100%. Depending on whether more or less signal distortion is permissible in a specific application, the listed scan rates may be increased or should be reduced accordingly.

### IV. MEASURED RESULTS

The following measurements refer to the large reverberation chamber at NPL having dimensions \( 6.55 \times 5.85 \times 3.5 \text{m}^3 \). Fig. 2 shows the measured CCF for the range 100 MHz \( \leq f < 1500 \text{ MHz} \), obtained using a pair of nominally identical in-band log-periodic antennas averaged over five receiver locations. The asymptotic frequency dependence \( (\propto f^{-5/2}) \) is seen to be rapidly approached as frequency is increased. Fig. 3 shows \( T_{\text{min}} \) estimated from (2) and (3) based on the measured CCF between 800 MHz and 1500 MHz \( (f_1/f_0 = 1.875) \), showing fairly good agreement with the theoretical ideal characteristic.
Table I. Maximum overall scan rate $s'_{\text{max}}$ (in units dec/s) for specified dwell time $\delta t$, specified number of test frequency points $n$ per decade when maintaining a constant scan rate $1/T_{\text{min}}(f_{0}, f_{0}+\Delta f)$ throughout $[f_{0}, 4f_{0}]$.

<table>
<thead>
<tr>
<th>$s'_{\text{max}}$ (dec/s)</th>
<th>$\delta t=5\mu s$</th>
<th>$\delta t=10\mu s$</th>
<th>$\delta t=3\text{s}$</th>
<th>$\delta t=10\text{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=2$</td>
<td>0.7680</td>
<td>0.7564</td>
<td>0.1369</td>
<td>0.0469</td>
</tr>
<tr>
<td>$n=3$</td>
<td>0.5187</td>
<td>0.5107</td>
<td>0.0915</td>
<td>0.0313</td>
</tr>
<tr>
<td>$n=10$</td>
<td>0.2835</td>
<td>0.2757</td>
<td>0.02983</td>
<td>0.00966</td>
</tr>
<tr>
<td>$n=30$</td>
<td>0.2256</td>
<td>0.2113</td>
<td>0.01059</td>
<td>0.00329</td>
</tr>
<tr>
<td>$n=100$</td>
<td>0.2053</td>
<td>0.1704</td>
<td>0.00328</td>
<td>0.00099</td>
</tr>
<tr>
<td>$n=300$</td>
<td>0.1994</td>
<td>0.1248</td>
<td>0.00110</td>
<td>0.00033</td>
</tr>
<tr>
<td>$n=1000$</td>
<td>0.1972</td>
<td>0.0664</td>
<td>0.00033</td>
<td>0.00010</td>
</tr>
</tbody>
</table>

Figure 2. Measured chamber calibration factor CCF (in dB) as a function of frequency, with comparison to theoretical $f^{-5/2}$ law, for $[f_{0}, f_{1}]=[0.1\text{ GHz}, 1.5\text{ GHz}]$.

Figure 3. Experimentally estimated value and theoretical minimum value of scan period $T_{\text{min}}$ (in units s) for adaptive scanning across $[f_{0}, f]$ with $f_{0} \leq f \leq f_{1}$ as a function of $f/f_{0}$ for a cubic cavity with $V/S=0.82\text{ m}$, $\sigma_{\mu}/\rho = 4.7 \times 10^{7} \text{ S m}$, and $[f_{0}, f_{1}]=[0.8\text{ GHz}, 1.5\text{ GHz}]$.

V. Maximum Rate of Mechanical Stirring

If the stirrer rotates at a sufficiently slow speed and the chamber is reverberating sufficiently adequately (i.e., has a sufficiently large Q-value at $f$), then the following inequalities hold:

$$T_{s} \ll \tau_{\text{ch}} \ll \tau_{\rho} \quad \text{and} \quad T_{s} \ll \tau_{\text{Rx}} \ll \tau_{\rho},$$

where $T_{s} = 1/f$ is the period (in units s) of the CW excitation, typically of the order $\tau_{\text{ch}} = Q(f)/(2\pi f)$ is the effective (mode-averaged) decay time of the unstirred chamber, typically of the order of milliseconds (ms); $\tau_{\rho} = 2\pi/(N\Omega)$ is the time constant of the stirring process, which depends on the speed of revolution of the stirrer $\Omega$ (in units rad/s or rps) and on the maximum number of independent samples $N(f)$ generated in mode-tuned operation, with the same paddle wheel in the same chamber, $\tau_{\text{Rx}}$ is the time constant of the sensor (probe, antenna, receiver), typically of the order $\mu$s to ms.

In order for the cavity field to remain in the same steady-state as during mode-tuned operation (ignoring stepping transitions), an upper limit for the permissible stirring rate is given by the order-of-magnitude relation

$$\Omega(f) \ll \frac{\pi c^3}{4\sqrt{1-\frac{\pi f^2 Q(f)V}{\sqrt{N(f)}}}}.$$  

For example, with $V=100\text{ m}^3$, $f=1\text{ GHz}$, $Q(f)=10^4$ and $N(f)=1000$, we require $\Omega(f) \ll 0.72\text{ rps}$.

Like for mode-tuned testing, mode-stirred immunity testing requires the chamber to meet the field uniformity requirement. In addition, the stirring process must be sufficiently slow to enable the equipment under test (EUT) to respond adequately. The same considerations for the response time of the receiver or sensor apply to the response of a band-pass-type single-resonance EUT, in which case the condition becomes

$$\Omega(f) \ll \frac{c^3}{8\sqrt{1-\frac{\pi f^3 \tau_{\text{EUT}}(f)V}{\sqrt{N(f)}}}}.$$  

where $\tau_{\text{EUT}}(f)$ is the characteristic time constant of the EUT at $f$. For example, for a chamber with $V=100\text{ m}^3$, $f=1\text{ GHz}$, $N(f)=1000$, and $\tau_{\text{EUT}}(f)=1\mu$s, we require $\Omega(f) \ll 1.15\text{ rps}$. Regarding $\tau_{\text{EUT}}$, if the bandwidth $\Delta f_{\text{EUT}}$ is larger than the chamber Q-bandwidth $\Delta f$ at $f$, then (9) applies. However, in EMC the frequency response and time constant of an EUT are usually unknown, or there may be several such constants with different orders of magnitude. The EUT bandwidth may be measured or estimated by other means, e.g., by determining its $10\% - 90\%$ rise time $T_{r}$ for a step or pulse excitation, from which $\Delta f_{\text{EUT}} = 0.38/T_{r}$ and $\tau_{\text{EUT}} = Q_{\text{EUT}}/\omega_{\text{EUT}} = 1/(2\pi \Delta f_{\text{EUT}})$, for a resonant-type EUT.

If the test involves a modulated excitation signal, then the necessary condition to be fulfilled is that the period of the signal does not exceed the correlation time, leading to

$$\Omega < \frac{B}{2\pi N},$$

where the upper limit defined by (9) has to be scaled down if the permissible level of distortion has to be lowered [3].
where $B$ is the width (in units Hz) of the excitation band $[f - B/2, f + B/2]$ or $\Delta f$, whichever is the smaller.

VI. Averaging Effects

Typically, the EUT performs a weighted averaging of the mode-stirred cavity field, which reduces the magnitude and rate of perceived field fluctuations and the maximum test level compared to mode-tuned operation, under steady-state conditions. This increases the correlation length and decreases the number of independent samples $N$ as perceived by the EUT. It may then be necessary to reduce the stirring speed to make such averaging effects negligibly small in order to maintain the specified maximum test level. On the other hand, averaging intervals that are exceedingly long relative to the rate of fluctuation of the mode stirred field may be used to reduce field fluctuations, to the extent that immunity testing is performed with reference to the mode-stirred average value instead of the mode-tuned maximum value. Testing with respect to the average value has the disadvantage of producing a considerably lower test level than the maximum level (therefore requiring additional power amplification), but has the advantage of having a significantly lower uncertainty, even for large values of $N$ for the tuner sweep.

VII. Conclusions

In this paper, we addressed the maximum rates of frequency scanning and mechanical stirring, by requiring the field to remain quasi-stationary at all times in order to establish equivalence with test results obtained for mode-tuned and discrete-frequency operation. The time constant of the EUT is an important quantity when deciding on the maximum rate. It may therefore be useful to measure or estimate its (frequency-dependent) value by other means, possibly at lower field strength levels, before subjecting the EUT to an EMC test in a MSRC or continuous frequency sweeping in a MTRC.

REFERENCES