Nonlinear Control Design Using Exact Linearization for Permanent Magnet Synchronous Generator

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Abstract—The paper presents the ways of modeling and analysis of exactly linearized permanent magnet synchronous generator (PMSG) coupled with boost chopper in wind generation systems. The primary focus of this paper is to use state of art approach for linearizing nonlinear model and to obtain control law. Exact feedback linearization via state feedback algorithm is the basic scheme that has been derived for the PMSG boost convertor nonlinear model.

Index Terms—permanent magnet synchronous generator (PMSG), Exact feedback linearization, wind system non linear model, Control Law

NOMENCLATURE

- $I_L$: Inductor Current.
- $R_s$: Resistance of the stator circuit.
- $L_s$: Stator inductance of stator.
- $p_s$: Number of pole pairs.
- $\Phi_{PM}$: Useful flux linkage.
- $\omega_e$: Electrical speed of the machine.
- $\omega_{ref}$: Reference electrical speed.
- $\theta_e$: Electrical Angle of rotor.
- $\Phi_w$: Integral curve for mapping.
- $W$: W space.
- $X$: X space.
- $Z$: Z space.

I. INTRODUCTION

The rapid development in Smart Grid and Renewable Energy has put power system planners in a competitive environment looking for the best possible solution for power system operation and control. Stabilized power system operation is the key feature for power supplying agencies and consumers satisfaction. Among all types of synchronous generators, permanent magnet synchronous generator (PMSG) has its own merits including small size, low moment of inertia, high reliability, no excitation requirement, which is a good choice for wind energy conversion system. PMSG coupled with wind turbine systems extracts maximum power from the wind by optimal adjustment of shaft speed. This design is used by small variable wind turbine for high controllability generation and stable delivering power.

PMSG with boost convertor model for wind energy system is studies in literature [1]-[4]. The generator section is coupled with rectifier circuits and DC-DC Chopper boost Convertor. Slow dynamic response is reported by authors in [2]-[7] using PI controller for PMSG. Optimal parameter adjustment for best performance in varying conditions is not easy to handle by PI controller and is obtained through hit and trial scheme. Nonlinear control schemes have worked well in the field of power electronics. Authors in [8] used the scheme of hysteresis current control which improved the dynamic performance of the overall system. Later on, sliding mode control topology [9] has been applied on different kinds of convertors which enhanced the characteristics features of the proposed model. Modern control strategies have changed the response of the system in terms of stability, reliability, optimal performance and improvement in dynamics. Complex mathematical computations, modeling, analysis and conceptual understanding are required for implementing nonlinear schemes to power convertors. Fuzzy logic scheme [10] is implemented for choppers successfully that changed the properties of the nonlinear system when convertors are interfaced with it. Model adaptive nonlinear control [11] and robust control [12] have been proven to have better performance for power convertors control in contrast to conventional schemes like proportion integral control (PID).

Exact linearization via state feedback transforms a nonlinear system into linear one with all disturbances decoupled from the output either in form of simple approach of linearization, zero dynamics design, disturbance decoupled method, or algorithms of exact linearization. Each scheme involves coordinate transformation from one space to other to completely convert the nonlinear system into Brunovsky normal form. In conventional schemes, the negligence of higher order terms sacrifices the optimal performance and accuracy and limits the
operation within a fixed boundary. A lot of literatures [13]-[18] are available for the implementation of this scheme to different types of energy systems, for example wind, solar, PMSG, doubly fed induction generator, etc. The idea of exact linearization was first presented by Brockett [19]. Necessary and sufficient conditions of multiple input cases for external global linearization were presented by Jakubczyk [20]. Simplified sufficient conditions for exact global linearization was done by Hunt [21].

The paper is divided into four sections. First section illustrates the case of single input single output feedback linearization conditions and concept of lie algebra involved. Second section provides whole mathematical modeling and analysis for PMSG coupled with boost convertor. The third section provides the optimal control law and last section includes the concluding remarks.

II. EXACT LINEARIZATION CONDITIONS FOR SISO SYSTEM

Consider a non-linear affine system as,

\[
\dot{X} = f(X) + g(X)u \\
y = h(X)
\]

(1)

Where \( X \) is the state vector such that \( X \in R^n \), \( u \) is the control variable while \( f, g \) are the \( n \) dimensional vector fields.

Lie bracket and lie derivative operation performed as:

\[
[\dot{f}, \dot{g}](X) = \nabla g \cdot \dot{f} - \nabla f \cdot \dot{g}
\]

(2)

Consider lie derivative of scalar function \( \lambda(X) \) along \( f(X) \) as,

\[
L_f \lambda(X) = \sum_{i=1}^{n} \frac{\partial \lambda(X)}{\partial x_i} f_i(X)
\]

(3)

Following are the conditions to be satisfied:

1. \((a)L_{I_x}L_{I_x}^T h(X) = 0, k < r - 1, \forall x \in \Omega \)
2. \((b)L_{I_x}L_{I_x}^{r-1} h(X) \neq 0 \)
3. \(D[e_1 e_2 ... e_n] \theta(X), r(D) = n \) (4)

Where \( r(D) \) is the rank of the matrix \( D \) and \( n \) is the degree of the state vector.

III. NONLINEAR MATHEMATICAL MODEL OF PMSG-BOOST CONVERTER

The state space model of PMSG-Boost Convertor [22] is given as:

\[
\dot{I}_e = k_1 I_L + k_2 \omega_e \sin(\theta_e - 60) + k_3 \\
\dot{\omega}_e = k_4 I_L \sin(\theta_e - 60) + k_5 \omega_e + k_6 \\
\dot{\theta}_e = \omega_e
\]

(5)

Where,

\[
k_1 = -\frac{2R}{2L_e + L}, k_2 = -\frac{\sqrt{3} \Phi_{ms}}{2L_e + L}, k_3 = -\frac{v_e}{2L_e + L} \\
k_4 = \frac{\sqrt{3} p^2 \Phi_{ms}}{J}, k_5 = -\frac{F}{J}, k_6 = \frac{p_m T_m}{J} \\
k_7 = \frac{v_e}{2L_e + L}
\]

Selecting state variables as,

\[
x_1 = I_e, x_2 = \omega_e, x_3 = (\theta_e - 60)
\]

Finally, the system equations become,

\[
\dot{x}_1 = k_1 x_1 + k_2 x_2 \sin x_1 + k_3 \\
\dot{x}_2 = k_4 x_1 \sin x_1 + k_5 x_2 + k_6 \\
\dot{x}_3 = x_2
\]

(6)

The vector fields as stated in Eq. (1) is as follows,

\[
f(X) = \begin{bmatrix} k_1 x_1 + k_2 x_2 \sin x_1 + k_3 \\ k_4 x_1 \sin x_1 + k_5 x_2 + k_6 \\ x_2 \end{bmatrix}
\]
and composite transformation space. Coordinate transformation \( T \) is defined as,

\[
g(X) = \begin{bmatrix} k_7 \\ 0 \\ 0 \end{bmatrix}, \quad y = h(X) = x_3 \quad \text{(7)}
\]

Now, applying the procedure for exact linearization algorithm step by step. Since the studied system is a third order affine nonlinear system, performing lie bracket operation as follows,

\[
ad_{g} \Rightarrow \begin{bmatrix} k_7 k_1 \\ k_4 k_1 \sin x_i \\ 0 \end{bmatrix}
\]

\[
ad^2_{g} \Rightarrow \begin{bmatrix} \frac{k_7 k_1}{k_4 k_1} k_1 + k_4 k_1 \sin^2 x_i \\ k_7 k_1 k_1 \cos x_i + k_4 k_1 \sin x_i + k_4 k_1 \sin x_i \\ k_7 k_1 \sin x_i \end{bmatrix}
\]

Formation of \( n \) linearly independent vector fields \( \overline{D}_1, \overline{D}_2, \overline{D}_3 \) and selecting necessary scalar functions such that each independent vector field is solved to get simplified model,

\[
\begin{bmatrix} \overline{D}_1 \\ \overline{D}_2 + k_1^{(1)}(X)[g(X)] \\ \overline{D}_3 + k_1^{(2)}(X)[g(X)] + k_1^{(3)}(X)[ad_{g}g] \\ k_1^{(3)}(X)[ad^2_{g}g] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}
\]

Defining the mapping and solving set of partial differential equations,

\[
F(w_1, w_2, w_3) = \Phi_w^{\overline{D}_1} \circ \Phi_w^{\overline{D}_2} \circ \Phi_w^{\overline{D}_3}, X_0 = [x_{o_1} \ x_{o_2} \ x_{o_3}]^T
\]

\[
F_1(w_1, w_2, w_3) = w_1 + x_{o_1} = x_1 \rightarrow X = F(W)
\]

\[
F_2(w_1, w_2, w_3) = w_2 + x_{o_2} = x_2 \rightarrow X = F(W)
\]

\[
F_3(w_1, w_2, w_3) = w_3 + x_{o_3} = x_3 \rightarrow X = F(W)
\]

Inverse Mapping \( F^{-1} \) becomes,

\[
F^{-1}(f) = J_{\overline{D}_1}, f(X) \rightarrow X = F(W)
\]

\[
f^{(0)}(W) = \begin{bmatrix} f_1^{(0)}(W) \\ f_2^{(0)}(W) \\ f_3^{(0)}(W) \end{bmatrix}
\]

\[
F^{-1}(f) = \begin{bmatrix} k_1 x_1 + k_2 x_2 \sin x_3 + k_1 \\ k_2 x_1 \sin x_3 + k_2 x_2 + k_3 \\ k_3 \end{bmatrix}
\]

Now we derive relations for the coordinate transformation \( R_{w-1} \).

We will perform the coordinate transformation from \( W \) space to \( Z^{n-1} \) space. Required mapping will be calculated from \( X \) space to \( Z^{n-1} \) space. Coordinate transformation \( T \) is defined as,

\[
T = R_{w-1} F^{-1}. \quad \text{The relations among the coordinate transformation between } W, X, Z \text{ is explained in the Fig.1.}
\]

Defining the transformation \( R_i \) and composite transformation \( T \) as,

\[
R_i: z_i^{(1)} = f_i^{(0)} = k_1 x_1 \sin x_3 + k_2 x_2 + k_3
\]

\[
z_i^{(2)} = f_i^{(0)} = x_2
\]

\[
z_i^{(3)} = w_{3} = x_3 - x_{o_3}
\]
Fig. 1. Coordinate transformations between W, X, Z spaces

Final coordinate transformation $Z = F(X)$ for the conversion of nonlinear system into exactly linear system is,

$$z_1 = w_3 = x_3 - x_{30}$$
$$z_2 = f_3^1 = x_2$$
$$z_3 = f_2^1 = x_2 = k_4 x_1 \sin x_3 + k_2 x_2 + k_6$$  \(15\)

Exactly linearized system $X = F^{-1}(Z)$ is:

$$\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= v
\end{align*}$$

The control law is defined as,

$$v = h(X) = x_3$$
$$u = x_3 = -\frac{f_3^1(X) + v^*}{g_1^1(X)}$$  \(16\)

$$v^* = -1.75\omega_n (k_4 x_1 \sin x_3 + k_2 x_2 + k_6) + 3.25\omega_n^2 \Delta \omega + \omega_n \int \Delta \omega dt$$

$\Delta \omega = \omega_{ref} - \omega$; $\omega_n$ is the system natural frequency.

This is a complex equation for the control law involving many terms such as system parameters, generator parameters and certain constants. Each parameter depends upon the type of system used and generator controlled setup. The control law $u$ will make the speed constant of PMSG setup.

Simulation results of the derived nonlinear controller will be presented in future papers.

IV. CONCLUSION

This paper presented the state of art control law for the PMSG-Boost convertor for the applications of wind energy systems. Complex nonlinear model is transformed into a linear system thus providing a control law for most controlled wind mechanism. Proposed mechanism is the extension of work presented by A. Isidori in 1985 for the global linearization of nonlinear systems through state feedback. Steps were simple to transform but involves lengthy mathematical stuff and complex understanding.

$$u = \frac{-(k_4 \sin x_3 (k_4 x_1 + k_2 x_2 \sin x_3 + k_5) + k_3 x_2 + k_4 x_1 x_2 \cos x_3) - 1.75\omega_n (k_4 x_1 \sin x_3 + k_2 x_2 + k_6) + 3.25\omega_n^2 \Delta \omega + \omega_n \int \Delta \omega dt}{k_4 k_5 \sin x_3}$$  \(17\)
V. REFERENCES


