Finite-time Frequency Synchronization in Microgrids

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Abstract— This paper proposes a finite time frequency controller that synchronizes the microgrid frequency to the nominal frequency and shares the active power among distributed generators (DG) based on their active power ratings. The finite-time control accelerates the synchronization speed and provides the synchronization for microgrid frequency and DG active powers in a finite period of time. In addition, the proposed control is distributed; i.e., each DG only requires its own information and the information of its neighbors on the communication network graph. The efficacy of the proposed finite-time frequency control subsequent to islanding process and load changes is verified for an islanded microgrid test system.

I. INTRODUCTION

Microgrids are equipped with a control hierarchy to provide the frequency and voltage stability and restoration, and active and reactive power flow control. A microgrid can enter the islanded mode due to the pre-planned contingencies or unplanned disturbances. Primary control, as the lowest control hierarchy, maintains the frequency stability of the microgrid subsequent to the islanding process. However, the primary control alone may lead to slight deviation of frequency from its nominal value. To restore the microgrid frequency, the secondary control is commonly adopted [1]-[9].

The secondary control can be implemented either through a centralized or distributed structure. The centralized control structure requires a central controller with star communication network. The requirements for a central controller and star communication network reduce the system reliability and pose the control system to the single point of failure [10]-[13]. Alternatively, the distributed control framework has been exploited in recent works to improve the reliability of secondary controller and feature the peer-to-peer concept (Peer-to-peer concept means that the reliable operation of the microgrid is not affected by the malfunction of a single central or master controller.) [14]-[16]. Microgrid, as a collocation of several distributed generators (DG), is considered as a multi-agent system with each DG as an agent. The distributed control protocol on each DG would require only its own information and the information of its neighbors on the communication network [17]-[19].

Safety-critical loads in a microgrid require operation at the nominal frequency, e.g., 50 or 60 Hz. Therefore, it is of paramount value to accelerate the synchronization process and improve the convergence of the frequency control. The conventional distributed frequency control protocols (e.g., [17], [19]) lead to synchronization over an infinite time period with an exponential convergence rate. The convergence speed is dictated by the properties of the communication graph facilitating information exchange among agents.

For a general multi-agent system, corrective schemes have been proposed in the literature to speed up the convergence speed of the conventional consensus protocols. For example, [20] has improved the synchronization speed by adjusting the weights of the communication links among the agents. Most of the existing work modifies the communication network and increases the algebraic connectivity of the graph to increase the convergence speed [20]-[21]. Despite accelerating the synchronization process, such corrective schemes fail to synchronize the multi-agent system over a finite time horizon. This requirement has been achieved here through the finite-time control protocols [22]-[30].

The finite-time control protocols are inspired by the natural synchronization phenomena with only two states [25]. For example, in the synchronization of hundreds of fireflies, two states exist for each fly which corresponds to the on and off state of its photic organ [25]. Inspired by these phenomena, the finite-time control protocols have been recently presented in the literature that exploit binary signals based on the relative information measurements among the neighboring agents. These control protocols are robust against the perturbations and measurement errors, and are faster than the conventional distributed control protocols (e.g., [17]-[19]) [20]-[22].

In this paper, the convergence speed of the conventional distributed frequency control in [17] is improved by exploiting the finite-time control scheme. Finite-time
control is used to design fully distributed frequency control protocols for each DG to synchronize the DG frequencies to the nominal value and allocate the active power of DGs based on their active power ratings. The distributed finite-time control protocol on each DG uses binary signals based on the relative measured frequency and active power of each DG and its neighbors on the communication graph.

The rest of the paper is organized as follows: In Section II, the primary and secondary frequency control hierarchies of microgrids are presented. The preliminaries on graph theory are explained in Section III. In Section IV, the finite-time and distributed secondary frequency control framework is presented. The proposed controller is verified in section V on a microgrid test system. Section VI concludes the paper.

II. MICROGRID FREQUENCY CONTROL AND ACTIVE POWER SHARING

Figure 1 shows a microgrid containing inverter-based DG. Each inverter-based DG contains the primary DC power source, the inverter bridge, and the power, voltage, and current control loops. The control loops set and control the output voltage and frequency of the inverter-based DG. Inner voltage and current controller block diagrams are elaborated in [32]-[33] and omitted here for brevity. The power controller provides the voltage references $v_{od}$ and $v_{qd}$ for the voltage controller, and the operating frequency $\omega_i$ for inverter. Note that nonlinear dynamics of each DG in a microgrid are formulated on its own $d$-$q$ (direct-quadratic) reference frame. The reference frame of microgrid is considered as the common reference frame and the dynamics of other DGs are transformed to this common reference frame. The angular frequency of this common reference frame is denoted by $\omega_{com}$ ($\omega_{com}$ denotes the microgrid frequency).

The primary frequency control is locally implemented at each DG via the droop mechanism. Droop technique prescribes a desired relation between the frequency and the active power. The frequency and power droop
characteristics of the $i$-th DG are

$$\omega_i = \omega_{i_{ref}} - m_i P_i$$  \hspace{1cm} (1)$$

where $\omega_i$ is the angular frequency of the DG dictated by the primary control. $P_i$ is the measured active power at the DG's terminal. $m_i$ is the droop coefficients. $\omega_{i_{ref}}$ is the primary control references. The droop coefficients are selected based on the active and reactive power ratings of each DG [13].

The secondary frequency control chooses $\omega_{i_{ref}}$ such that the angular frequency of each DG synchronizes to its nominal value, i.e. $\omega_i \rightarrow \omega_{i_{ref}}$. Additionally, the secondary frequency control must share the active power among DGs based on their active power ratings, $P_{max}$, i.e., satisfying

$$\frac{P_1}{P_{max1}} = \cdots = \frac{P_i}{P_{maxi}} = \cdots = \frac{P_N}{P_{maxN}}.$$ \hspace{1cm} (2)

Since the active power droop coefficients $m_i$ are chosen based on the active power rating of DGs, $P_{maxi}$, (2) is equivalent to $m_i P_i = \cdots = m_P N P_N$. \hspace{1cm} (3)

III. PRELIMINARY OF GRAPH THEORY

The communication network of a multi-agent cooperative system can be modeled by a directed graph (digraph). A digraph is usually expressed as $Gr=(V_G, E_G, A_G)$ with a nonempty finite set of $N$ nodes $V_G = \{v_1, v_2, \ldots, v_N\}$, a set of edges or arcs $E_G \subseteq V_G \times V_G$, and the associated adjacency matrix $A_G =$ \{$a_{ij}\} \in R^{N \times N}$. A typical digraph is shown in Fig. 2. In a microgrid, DGs are considered as the nodes of the communication digraph. The edges of the corresponding digraph of the communication network denote the communication links. An edge from node $i$ to node $j$ is denoted by $(v_i, v_j)$, i.e., node $i$ receives the information from node $j$. $a_{ij}$ is the weight of edge $(v_i, v_j)$, and $a_{ij} > 0$ if $(v_i, v_j) \in E_G$, otherwise $a_{ij} = 0$. Node $i$ is a neighbor of node $j$ if $(v_i, v_j) \in E_G$. The set of neighbors of node $i$ is denoted as $N_i =$ \{$v_j | (v_i, v_j) \in E_G\}$. For a digraph, if node $i$ is a neighbor of node $j$, then node $j$ can get information from node $i$, but not necessarily versa.

The in-degree matrix is defined as $D_1 = diag \{d_1\} \in R^{N \times N}$ with $d_1 = \sum_{j \in N_i} a_{ij}$. The Laplacian matrix is defined as $L = D_1 - A_G$. A direct path from node $i$ to node $j$ is a sequence of edges, expressed as \{$(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_m}, v_{i_j})\}$. A digraph is said to have a spanning tree, if there is a root node with a direct path from that node to the every other node in the graph [34]. A digraph is detail-balanced if there exists positive column vector $p = [p_1, p_2, \ldots, p_N]^T$ such that $p_i a_{ij} = p_j a_{ji}$ $\forall i, j$.

IV. DISTRIBUTED FINITE-TIME CONTROL

The distributed finite-time frequency control is designed to synchronize the angular frequency of DGs, $\omega_i$ in (1), to the reference angular frequency, $\omega_{i_{ref}}$. Additionally, it should allocate the DG active powers based on their power ratings as stated in (3). It is assumed that DGs can communicate with each other through a prescribed communication digraph, $Gr$.

Differentiating the frequency-droop characteristic in (1) yields

$$\dot{\omega}_{ij} = \dot{\omega}_1 + m_i P_i = \dot{u}_i,$$ \hspace{1cm} (4)

where $u_i$ is an auxiliary control to be designed. Equation (4) is a dynamic system to compute the control input $\dot{\omega}_{ij}$ from $u_i$. It transforms the secondary frequency control of a microgrid, with $N$ DGs, to the synchronization problem for the following first-order, multi-agent system

$$\begin{align*}
\dot{\omega}_1 + m_1 P_1 = u_1 \\
\dot{\omega}_2 + m_2 P_2 = u_2 \\
\vdots \\
\dot{\omega}_N + m_N P_N = u_N.
\end{align*} \hspace{1cm} (5)$$

The control inputs $\dot{\omega}_{ij}$ solve the frequency synchronization problem asymptotically, if all DGs satisfy

$$\begin{align*}
\lim_{t \rightarrow \infty} [\omega_i - \omega_j] &= 0, \quad \forall i, j, \\
\lim_{t \rightarrow \infty} [m_i P_i - m_j P_j] &= 0, \quad \forall i, j.
\end{align*} \hspace{1cm} (6)$$

Alternatively, the control inputs $\dot{\omega}_{ij}$ solve the frequency synchronization problem over a finite time period, if there exists a settling time $T$ such that all DGs satisfy

$$\begin{align*}
\lim_{t \rightarrow T} [\omega_i - \omega_j] &= 0, \quad \forall i, j, \\
\dot{\omega}_i &= \omega_j, \quad \forall i, j, \quad \forall t \geq T, \\
\lim_{t \rightarrow T} [m_i P_i - m_j P_j] &= 0, \quad \forall i, j, \\
\dot{\omega}_i &= \omega_j, \quad \forall i, j, \quad \forall t \geq T.
\end{align*} \hspace{1cm} (7)$$

For the finite-time control design, the auxiliary controls $u_i$ are chosen based on the relative measured information of neighboring DGs on the communication graph $a$

$$u_i = -c_i (\sum_{j \in N_i} a_{ij} \text{sgn}(\omega_i - \omega_j) + g_i \text{sgn}(\omega_i - \omega_{i_{ref}}))$$

$$-c_i \sum_{j \in N_i} a_{ij} \text{sgn}(m_i P_i - m_j P_j),$$ \hspace{1cm} (8)
where $c_i$ is the control gain. It is assumed that the pinning gain $g_i \geq 0$ is nonzero for only one DG that has the reference frequency $\omega_{\text{ref}}$. $\text{sgn}(.)$ is the sign function defined by

$$\text{sgn}(r) = \begin{cases} 1 & r > 0 \\ 0 & r = 0 \\ -1 & r < 0 \end{cases}$$

The block diagram of the distributed finite-time secondary control is shown in Fig. 3.

To show the effectiveness of the control protocol in (8), the following relationship between the output active power of each DG and its angular frequency is used. The active power of each DG can be written as

$$P_i = \frac{V_{oi}}{X_{ci}} \left| V_{bi} \right| \sin(\delta_i) = h_i \sin(\delta_i),$$

where $\delta_i$ is the angle of the DG reference frame with respect to the common reference frame. $V_{oi}$, $V_{bi}$, and $X_{ci}$ are shown in Fig. 1. The term $h_i$ can be assumed to be constant since the amplitude of $V_{oi}$ and $V_{bi}$ change slightly around the nominal voltage. Since $X_{ci}$ is typically small, $\delta_i$ is also small and, hence, $\sin(\delta_i)$ is approximately equal to $\delta_i$.

Considering these assumptions and differentiating (10) yields

$$\dot{P}_i = h_i (\omega_i - \omega_{\text{com}}) + \sum_{j \in N_i} a_{ij} \text{sgn}(\omega_i - \omega_{j}),$$

Equation (11) provides a direct relationship between the differentiated output power and angular frequency of DGs with respect to the common angular frequency of microgrid.

In the steady state, the left side of (11) is equal to zero. Setting the left side of (11) equal to zero yields

$$\omega_i = \omega_{\text{com}} \forall i.$$  

Equation (12) shows that all DG frequencies synchronize to the microgrid frequency, $\omega_{\text{com}}$, in steady state. Without loss of generality, it is assumed that only $g_1$ is nonzero, i.e., DG 1 is the only DG that has the access to the reference frequency information, $\omega_{\text{ref}}$. In the steady state, the left side of (5) is equal to zero. Placing (8) into (5) and considering (12) yields

$$g_1 \text{sgn}(\omega_1 - \omega_{\text{ref}}) + \sum_{j \in N_1} a_{1j} \text{sgn}(m_{1j}P_1 - m_{1j}P_j) = 0, \text{ for } i = 1,$$

where $c_i$ is the control gain. It is assumed that the pinning gain $g_i \geq 0$ is nonzero for only one DG that has the reference frequency $\omega_{\text{ref}}$. $\text{sgn}(.)$ is the sign function defined by

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$$g_1 \text{sgn}(\omega_1 - \omega_{\text{ref}}) + \sum_{j \in N_1} a_{1j} \text{sgn}(m_{1j}P_1 - m_{1j}P_j) = 0, \text{ for } i = 1,$$
\[ a_i \operatorname{sgn}(m_{p_i} P_i - m_{p_i} P_i) + \sum_{j \in N_i} a_{ij} \operatorname{sgn}(m_{p_j} P_j - m_{p_j} P_j) = 0, \text{ for } i \neq 1, \]  
\[ \text{(14)} \]

Equation (14) shows that the set \( \{m_{p_1} P_1, m_{p_2} P_2, \ldots, m_{p_N} P_N\} \) can be considered on a communication digraph with \( m_{p_1} P_1 \) as the leader node. This subgraph is named as \( G' \). All nodes have access to the leader \( m_{p_1} P_1 \) through the matrix \( \bar{G} = \text{diag}\{a_{i1}\} \). Since the original digraph \( G \) has a spanning tree with \( m_{p_1} P_1 \) as its root node, at least one of the diagonal terms in \( \bar{G} \) is non-zero. Theorem 9 in [25] can be used to show that the control protocol in (8) synchronizes all \( m_{p_i} P_i \) to \( m_{p_1} P_1 \) which satisfies (2), or, equivalently, (3). This theorem assumes that \( G' \) contains a spanning tree and a set of detail-balanced subgraphs.

According to (13), having all \( m_{p_i} P_i \) synchronized to a common value shows that \( \omega_1 \) synchronizes to \( \omega_{ref} \) and then, according to (12), all DG angular frequencies synchronize to the reference frequency, \( \omega_{ref} \).

V. CASE STUDIES

An islanded microgrid is used to verify the effectiveness of the proposed frequency control framework. Figure 4 illustrates the single line diagram of the microgrid test system with four DGs. The nominal voltage and frequency of microgrid are 380 V and 50 Hz, respectively. The loads and lines between buses are modeled as series RL branches. The specifications of DGs, lines, and loads are summarized in Table 1. In this table, \( K_{PV}, K_{IV}, K_{PC}, \) and \( K_{PV} \) are the voltage and current controller parameters based on the detailed models discussed in [18]. In the following, the simulation results are presented for two different cases. The first case shows the effectiveness of the finite-time frequency control subsequent to islanding process while the second case verifies the performance of secondary control subsequent to load changes in an islanded microgrid.

A. Case 1

First, it is assumed that DGs communicate with each other through the communication digraph depicted in Fig. 5. The associated adjacency matrix of this digraph is

\[ A_G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 4 & 0 \end{bmatrix}, \]  
\[ \text{(15)} \]

DG 1 is the only DG connected to the leader node with the pinning gain of \( g_1 = 2 \). The communication digraph shown in Fig. 5 contains a spanning tree starting from DG 1 and ending to DG 4. Excluding DG 1 that has the information of the reference frequency, the remaining subgraph contains a spanning tree and a detail-balanced subgraph (the subgraph containing DG 3 and DG 4). As discussed in Section IV, these graph specifications ensure the synchronization of the microgrid frequency in a finite time. The control gain \( c_i \) in (8) is set to 8.

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TABLE I

SPECIFICATIONS OF THE MICROGRID TEST SYSTEM

<table>
<thead>
<tr>
<th>DGs</th>
<th>1 and 2 (9 kVA rating)</th>
<th>3 and 4 (7 kVA rating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_p )</td>
<td>( 14.1 \times 10^3 )</td>
<td>( 18.75 \times 10^3 )</td>
</tr>
<tr>
<td>( n_Q )</td>
<td>( 1.3 \times 10^{-3} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>( R_c )</td>
<td>( 0.03 , \Omega )</td>
<td>( 0.03 , \Omega )</td>
</tr>
<tr>
<td>( L_c )</td>
<td>( 0.35 , mH )</td>
<td>( 0.35 , mH )</td>
</tr>
<tr>
<td>( R_f )</td>
<td>( 0.1 , \Omega )</td>
<td>( 0.1 , \Omega )</td>
</tr>
<tr>
<td>( L_f )</td>
<td>( 1.35 , mH )</td>
<td>( 1.35 , mH )</td>
</tr>
<tr>
<td>( C_f )</td>
<td>( 50 , \mu F )</td>
<td>( 50 , \mu F )</td>
</tr>
<tr>
<td>( K_{PV} )</td>
<td>420</td>
<td>390</td>
</tr>
<tr>
<td>( K_{IV} )</td>
<td>15</td>
<td>10.5</td>
</tr>
<tr>
<td>( K_{PC} )</td>
<td>20000</td>
<td>16000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lines</th>
<th>Line 1</th>
<th>Line 2</th>
<th>Line 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{l1} )</td>
<td>0.23 , \Omega )</td>
<td>( 0.35 , \Omega )</td>
<td>( 0.23 , \Omega )</td>
</tr>
<tr>
<td>( L_{l1} )</td>
<td>318 , \mu H )</td>
<td>1847 , \mu H</td>
<td>318 , \mu H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loads</th>
<th>Load 1</th>
<th>Load 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{L2} ) (per phase)</td>
<td>3 , \Omega</td>
<td>( R_{L2} ) (per phase)</td>
</tr>
<tr>
<td>( L_{L2} ) (per phase)</td>
<td>6.4 , mH</td>
<td>( L_{L2} ) (per phase)</td>
</tr>
</tbody>
</table>
It is assumed that the microgrid is connected to the main grid at the point of common coupling (PCC), shown in Fig. 4. Microgrid is islanded form the main grid at \( t = 0 \), and the secondary frequency control is applied at \( t = 0.6 \) s. In practical applications, the frequency control should be applied immediately after the disturbance occurrence. Here, it is delayed by 0.6 s to highlight the effectiveness of the proposed controller. Figure 6 shows frequencies and output power ratios of DGs, \( m_P P_i \), before and after applying the finite-time frequency control. Once the microgrid is islanded, its frequency drops. The primary control keeps the frequency in a stable range. However, the secondary control is required to restore the microgrid frequency to 50 Hz. As seen in Fig. 6(a), the frequency synchronization is provided in less than 0.3 s. Additionally, Fig. 6(b) shows that the frequency control shares the active power among DGs based on their active power ratings in less than 0.7 s.

To compare with the proposed finite-time control, the results from the existing distributed frequency controller in [17] are shown in Fig. 7. Comparing the results in Figs. 6 and 7, one can observe that the finite-time controller provides a faster synchronization for microgrid frequency and active power of DGs.

B. Case 2

It is assumed that the microgrid is islanded from the main grid at \( t = 0 \), and the frequency control is applied at \( t = 0.6 \) s. At \( t = 1.5 \) s, the Load 1 resistance changes from 3 \( \Omega \) to 1 \( \Omega \), and Load 2 resistance change from 2 \( \Omega \) to 1 \( \Omega \), respectively. It is assumed that DGs communicate with each other through the communication digraph depicted in Fig. 5. DG 1 is the only DG connected to the leader node with the pinning gain of \( g_1 = 2 \).

Figure 8 shows frequencies and output power ratios of DGs, \( m_P P_i \), before and after applying the finite-time frequency control. The primary control keeps the frequency in a stable range subsequent to islanding process. At \( t = 0.6 \) s, the frequency control is applied to restore the microgrid frequency to 50 Hz. At \( t = 1.5 \) s, the load change is applied which result in in deviations in frequency and active power ratio of DGs. As seen in Fig. 8, the finite-time frequency control effectively controls the frequency and active power of DGs, and its performance does not deteriorate subsequent to sudden load changes.

VI. CONCLUSION

A finite-time and distributed control is proposed to control the frequency of microgrids. The proposed controller synchronizes the microgrid frequency to the nominal frequency and shares the active power among distributed generators (DG) based on their ratings, in a finite time. The proposed control is distributed and implemented by a sparse communication network, where each DG only requires its own information and the information of its neighbors on the communication network graph. The finite-time control improves the synchronization speed of the
conventional distributed frequency control.

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