Optimal Resonant Tank Design Considerations for Primary Track Compensation in Inductive Power Transfer Systems

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Abstract -- In this paper, design of a primary resonant tank for resonant converters compensation in an Inductive Power Transfer (IPT) system is analyzed and some criteria for selecting its reactive components are established. The optimization goal is to maximize the transferred power, while limiting the number of compensation components to no more than two reactive components. Additionally, VA limits of the inverter are taken into consideration. Current track requirements are relaxed and instead of constant track current, small track load dependency is allowed, which is acceptable in practice. The theoretical investigations and calculations, as well as simulations for two important practical cases considered are presented in the paper. It is shown that LC compensation structure allows maximum delivered power, but not for the case when the impedances of compensation elements are the same as it would be expected. Additionally, the impact of converter ratings and allowed load dependent track current variation on the optimal design are investigated through the set of numerical simulations.

Index Terms—Inductive power transfer, compensation tank design, resonant converter.

I. INTRODUCTION

Inductive power transfer (IPT) is a technique for transferring power with no physical contact between the source and the load [1, 2]. IPT concept is used in numerous industrial applications, where mechanical contact between the source and the load is not possible or should be avoided. Depending on the design, the system can be up to 90% efficient at distances of 100 mm [3]. Basic IPT system consists of three main components: a power supply (converter), a conductive track and a pick-up. The system typically operates at a high frequency (10-200 kHz range) to improve the system power transfer capabilities [4]. A multi pick-up system is shown in Fig. 1. Their specific parts will be analyzed and modeled in detail in the following sections.

Additional flexibility of an IPT system is its capability to deliver power to more than one pick-up. Multi-pickup IPT structures can be found in range from tens watts, for supplying low voltage sensors and transducers, to tens kilowatts for supplying factory conveyers and automated guided vehicles (AGV) [4]. Some important multi-pickup characteristics and requirements are summarized as follows:

The number of powered pick-ups on a track can vary during operation. It is especially true when an IPT system is used for supplying a moving platforms or an AGV, when the number of vehicles can vary from zero to some maximum number, denoted in the text below as $N_{\text{max}}$.

It is desirable that number of active pick-ups on a track does not have influence on primary track current $I_t$, which further means unchanged power delivery capabilities to each active pick-up. In some practical applications it is acceptable to have some small variation in the pick-up input power when the load (number of active pick-up) on the track changes. For example, it can be acceptable to have 5% variation in transferred power per pick-up when number of active pick-ups changes from 1 to $N_{\text{max}}$, especially when the pick-up has its own power conditioner which is almost always true for high power IPT systems.
This paper will consider IPT system supplied by a voltage fed full bridge single phase resonant converter. While the maximum voltage is determined by the voltage of DC link, the maximum current is typically limited by active switches current rating at specified switching frequency. Therefore, a practical design procedure must take into consideration current and voltage limitation of resonant converter, or in other word its VA power rating.

The number of elements in a practical compensation circuit varies from application to application. More elements allow the following additional criteria to be satisfied: constant track current, converter zero current switching (ZCS) operation, minimized VA rating of convertor, relaxed rating requirements for compensation elements etc. On the other hand, a resonant tank designed with minimum number of reactive components (one or two) has a lower cost, mass and volume, as well as higher reliability. Taking this into account, only one and two component resonant tank structures are analyzed in this paper. One of them must be a capacitor while the second one (if present) is an inductor.

The subject of this paper is to find the optimal one or two-element compensation circuit that maximizes the power delivery (specified by maximum rms of output voltage and current fundamental) to an arbitrary number of nominally loaded pick-ups. In other words, for an existing resonant converter (specified by maximum output voltage and current as well as constant switching frequency) and a priori specified track and pick-up, an optimal LC compensation structure will be designed, which will maximize the power delivered to pick-up and minimize the power variations when the number of active pick-up increases or decreases.

In the text below, four compensation structures are analyzed. Two similar analyses are made for the high power converter and long track (IPT systems for charging AGVs), as well as low power and short track (IPT system for supplying sensors). It will be shown that the LC compensation circuit offers the highest transferred power, keeping power per pick-up variation in an a-priori specified range. The results are given relative to the “matched LC” compensation structure where impedances of L and C compensation elements are the same and which is able to provide a load independent track current.

The text below is organized as follows: In Section II all possible compensation structures are discussed and only the structures that are acceptable for a voltage-fed resonant converter are selected. In section III a searching algorithm for a compensation tank design is developed and tested through two sets of simulation in Section IV applying the algorithm first on high power and than on low power IPT systems. Because the obtained results strongly favor LC compensation structure, further analysis is based on it. After that the influences of converter power rating and allowed load dependent track current variation on maximum power delivered per pick-up are investigated. Finally, some conclusions are summarized in Section VI.

II. COMPENSATION TANK SELECTION

Because of relatively high resonant circuit quality factor Q, approximate steady-state AC analysis approach for converter modeling is used. It allows neglecting the high voltage harmonics [4], [6] and modeling inverter output voltage as a sinusoidal signal at switching frequency f_s:

\[ V_{\text{inv}}(t) = V_{\text{inv,max}} \sin(2\pi f_s t). \]  

Primary load in an IPT application can be modeled with track inductance L_t, track resistance R_t, and the load transferred from the pick-up to the primary side R_L. The transferred impedance depends on the type of secondary circuit compensation, value of used pick-up elements, mutual inductance (related to track to pick-up misaligning) and it can have LR, CR or pure R nature. Transferred impedance depends on the frequency of the primary signal, and when this frequency varies, it can change not only the value but also the nature by becoming dominantly inductive or capacitive. This paper primarily deals with constant frequency. Furthermore, without losing generality in the analysis, purely resistive transferred impedance is considered. For example, tuned series compensated pickups represent this situation. Additionally, it is assumed that transferred resistivity has discrete nature: when the pick-up is on the track (active), it introduces R_{L,L} in the primary side, while the transferred value is zero when the pick-up is switched off or far from the track.

Considering one or two components in the resonant tank, there is a total of eight different types of resonant tank configurations [5] shown in Fig. 2. However, the specification of IPT load and type of converter eliminates some of the shown combinations. For example, any configuration that connects a capacitor directly to the output of the convertor requires current instead of voltage source converter. Therefore combinations B, F, H are omitted. The configuration G increases the reactive energy that circulates through switches, and therefore it is not considered. Furthermore, arrangement J has the inductor directly connected to the resonant converter, which is not useful for power transfer. This results in a clamping effect on the inductor voltage caused by the converter, which basically excludes the inductor from the resonant circuit by reducing scheme J to the tank A. For the tank K track inductivity can undertake the role of compensation inductor by reducing it to one-capacitor realization (A).

Finally, the number of acceptable combinations is reduced to 4 (A, C, D, E) and they are investigated in the paper. In order to allow easier remembering, the names of accepted configurations are related to their structures:

- tank A – C compensation circuit,
- tank C – LC compensation circuit,
- tank D – CL compensation circuit,
The analysis and design of resonant tank for IPT load is carried out for all four accepted configurations, but the procedure is explained in detail only for LC. For the other three tanks only the results and appropriate diagrams are given. A simplified model of LC compensated multi pick-up IPT system is given in Fig. 3. In the model, $L_x$ and $C_x$ represent inductive and capacitive elements of resonant tank, while $N R_{1,1}$ models the power transferred to pick-ups.

At first glance, matching two impedances at the switching frequency has real perspective to give excellent results. The expected advantages of that method are illustrated in the Fig. 4. According to the figure, Norton’s theorem application reveals that for the constant switching frequency the track current is constant and independent of the number of vehicles on the track (transfer load resistance). In that case, the track current has the value of:

$$I_1 = \frac{V_{in}}{\omega L_t}$$  \hspace{1cm} (2)

Furthermore, if the resonant tank reactances are matched with track reactance (Fig. 4), the converter output current will be in phase with the output voltage, which offers great perspective in achieving ZCS conditions [4]. Although matching $L_t$, $L_x$ and $C_x$ seems as the best solution, it often gives suboptimal solution for a practical case when maximum power delivered to track and pick-up is required. The reasons are listed below:

1. Track inductance $L_t$ is typically very big (for the first approximation its value in $\mu$H corresponds to the length of track in meters). Because of that, the case when $L_t$ and $L_x$ are equal will result in relatively low value of track current (2), far for the maximum value which can be obtained for some other impedance ratios. This problem can be solved by introducing the third compensation element: a capacitor in series with the track to “reduce” track inductivity and scale down the impedance seen by
The power delivered to each of the pick-up on track (power per pick-up) is:

\[ P_{\text{PTI}} = \frac{P_{\text{inv}}}{N} = \frac{R_{\text{inv}} V_{\text{inv}}^2}{(R + NR_{\text{inv}})^2 \left(1 - \alpha L C_s\right)^2 + \omega^2 \left(L_s + L - \alpha L C_s L + L_s\right)^2}. \]  

If we assume that \( N \) is an independent variable, \( P_{\text{PTI}} \) is a decreasing function. It means that the value of power delivered to each of active pick-ups decreases more and more when new pick-ups start to use track power. Our goal is to select \( L_s \) and \( C_s \) to elevate the described problem by keeping this deviation inside the specified range \( \Delta P_1 \).

\[ \left| \frac{dP_{\text{PTI}}}{dN} \right| = \frac{2 R_{\text{inv}}^2 V_{\text{inv}}^2 (R + NR_{\text{inv}}) \left(1 - \alpha L C_s\right)^2}{\left[(R + NR_{\text{inv}})^2 \left(1 - \alpha L C_s\right)^2 + \omega^2 \left(L_s + L - \alpha L C_s L + L_s\right)^2\right]^2}. \]  

It is worth to note here that for matched \( L_s \) and \( C_s \) impedances (condition: \( 1/\omega C_s = \alpha L_s \) is satisfied), the equations (5) – (7) are greatly simplified:

\[ I_{\text{unmch}} = \frac{V_{\text{inv}}}{\omega L_s}. \]

\[ P_{\text{PTI,unmch}} = \frac{P_{\text{inv}}}{N} = \frac{R_{\text{inv}} V_{\text{inv}}^2}{\omega^2 L_s^2}. \]

\[ \left| \frac{dP_{\text{PTI}}}{dN} \right| = 0 \]

Based on derived expressions and specifications, an algorithm is developed for \( L_s \) and \( C_s \) calculation. The algorithm has a general nature and can be used for any application with similarly defined conditions. The algorithm can be described as a series of steps:
1. Define the limits for capacitor and inductor and inside those limits form all possible pairs \((L_s, C_s)\) with predefined resolution \((\Delta L_s, \Delta C_s)\);
2. Sort over the pairs which give the slope defined by (7) inside the specified range \( \Delta P_1 \);
3. For any of the chosen pairs calculate value of converter output current for full range of load changes and disregard each pair which causes current higher than limit \( I_{\text{unmch}} \);
4. Among the remaining pairs choose that one which gives the highest value of power per pick-up for the case when the most likely number of pick-ups \((N = N_l)\) is active.

For other three accepted configurations the main formulas (for output power per vehicle and for the slope of that curve) are given in Table I.
Table I: Formulas for power and slope for LC, C, CL and C||L topologies

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Formula</th>
</tr>
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<tbody>
<tr>
<td>LC configuration:</td>
<td>[ P_{\text{IPT},1} = \frac{P_{\text{IPT}}}{N} = \frac{R_{L1} \left( \omega_s C_x \right)^2 V_{\text{inv}}^2}{\left( 1 - \omega^2 L C_x \right)^2 + \omega^2 C_x^2 \left( R_L + N R_{L1} \right)^2} ]</td>
</tr>
<tr>
<td>C configuration:</td>
<td>[ \frac{\text{d} P_{\text{IPT},1}}{\text{d} N} = \frac{2 R_{L1}^2 \left( \omega_s C_x \right)^2 V_{\text{inv}}^2 \left( R_L + N R_{L1} \right)}{\left( 1 - \omega^2 L C_x \right)^2 + \omega^2 C_x^2 \left( R_L + N R_{L1} \right)^2} ]</td>
</tr>
<tr>
<td>CL configuration:</td>
<td>[ P_{\text{IPT},1} = \frac{P_{\text{IPT}}}{N} = \frac{R_{L1} V_{\text{inv}}^2}{\left( 1 - \alpha L C_x \right)^2 + \alpha^2 \left( L + L - \alpha L C L \right)^2} ]</td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

IV. SIMULATION RESULTS

Described searching procedure is applied on two different IPT systems. The parameters of the systems are given in Table II. The first simulated system is a high power, long track (app. 108 m), low frequency (20 kHz) IPT system typically used for factory conveyers and AGV supplying. The other system is low power, short track (app. 16.2 m), high frequency (100 kHz) IPT system for sensor supplying. The maximum number of pick-ups in both case is the same \((N_{\text{max}}=25)\) as well as the resistance that models power transferred to pick-up \((R_{L1}=0.2 \, \Omega)\).

Table II: Parameters of simulated IPT systems

<table>
<thead>
<tr>
<th>Inverter DC supply voltage, (V_{\text{DC}}) [V]</th>
<th>High power IPT system</th>
<th>Low power IPT system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter output voltage, rms, (V_{\text{inv}}) [V]</td>
<td>230/6</td>
<td>24</td>
</tr>
<tr>
<td>Max. inverter current, rms, (I_{\text{inv,max}}) [A]</td>
<td>250</td>
<td>4</td>
</tr>
<tr>
<td>Switching frequency, (f_s) [kHz]</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Track inductance, (L_t) [μH]</td>
<td>108</td>
<td>16.2</td>
</tr>
<tr>
<td>Track resistance, (R_t) [mΩ]</td>
<td>300</td>
<td>45</td>
</tr>
<tr>
<td>Maximum number of pick-ups, (N_{\text{max}})</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>The most likely num. of vehicle, (N)</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Reflected res. per pick-up at (f/f_s, R_{L1}) [Ω]</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Reflected react. per pick-up at (f/f_s, L_{x1}) [μH]</td>
<td>0.267</td>
<td></td>
</tr>
<tr>
<td>Max. slope of (P_{\text{IPT},1} = f(N)) curve, (\Delta P_{\text{IPT},1}) [W]</td>
<td>5</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table III: Optimal values of compensation elements.

<table>
<thead>
<tr>
<th>Compensation structure</th>
<th>High power IPT system</th>
<th>Low power IPT system</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{\text{xxx}}) [μF]</td>
<td>(L_{\text{xxx}}) [μH]</td>
<td>(C_{\text{xxx}}) [μF]</td>
</tr>
<tr>
<td>LC matched</td>
<td>1.5768</td>
<td>40.16</td>
</tr>
<tr>
<td>(C)</td>
<td>1.45</td>
<td>49.4</td>
</tr>
<tr>
<td>(C)</td>
<td>3.03</td>
<td>98.4</td>
</tr>
<tr>
<td>(C</td>
<td></td>
<td>L)</td>
</tr>
</tbody>
</table>

Carrying out the described procedure for the parameter given in the Table II, the optimal value for elements of analyzed resonant tank are calculated and summarized in the Table III. Using optimal values for \(L_t\) and \(C_x\), both IPT systems are simulated and the results are summarized in Fig. 6 to Fig. 11. Fig. 6 and Fig. 7 show power delivered per pick-up as a function of number of active pick-ups on the track. The results are normalized with the maximum power per pick-up when LC compensation parameters are matched. Normalization values do not depend on track load and have the constant values of \(P_{\text{IPT,1,match}} = 2020.4\,\text{W}\) for high power system and \(P_{\text{IPT,1,match}} = 2.63\,\text{W}\) for low power system.

Fig. 6: High power IPT system - normalized power per pick-up vs number of active pick-ups.

Fig. 7: Low power IPT system - normalized power per pick-up vs number of active pick-ups.
In order to offer a better understanding of the obtained results, some additional diagrams and results are given in Fig. 8 - 11. Fig. 8 and Fig. 9 show *rms* of inverter current fundamental, while Fig. 10 and Fig. 11 show the inverter currents at the moment of switching. In all four cases number of pick-ups on the track is an independent variable. These diagrams are important to explain the impact of designed compensation circuits on converter losses, because the *rms* of inverter current determines conduction losses, while the same current in the moment of switching primarily dictates converter switching losses.

Finally, the maximum reactive powers of all compensation elements for different compensation circuits are summarized in Table IV.

Taking into account these results, the following conclusion can be drawn:

1. LC compensation circuit results in maximum power delivered to pick-ups in both high and low power cases, at the same time keeping the power to load sensitivity in the specified range. Other compensation structures (C, CL and C||L) give much less power, mainly because their inability to reject influence in load changes to track current and keep delivered power relatively constant.

2. Fig. 6 and Fig. 7 show that when some load dependence in delivered the power per pick-up is allowed (constant track current requirement is relaxed) detuned LC compensation structure (structure with different L and C impedances) is able to deliver at least 26% power more for high power system and 23% power more in...
case of a low power IPT system comparing to the matched LC structure. In other words, matched LC compensation circuit gives suboptimal solution when maximum transferred power is designer’s goal and absolutely constant track current is not required.

3. Additionally, Fig. 8 and Fig. 9 reveal that for optimal LC unmatched case increase in active power delivered by the converter it is achieved at the expense of reactive power, keeping the inverter current almost the same. It guarantees unchanged conduction losses in the converter. Furthermore, lower phase angle causes lower switching current especially at high load (Fig. 10 and Fig. 11). As a conclusion we can say that optimal compensation structure offers also higher overall IPT system efficiency comparing to the LC matched compensation.

4. Disadvantages of designed compensation (again comparing to the matched counterpart) are slightly higher VA rating of compensation inductor and capacitor and small but acceptable variations in delivered power. For full changes in track load this variations per pick-up are less than 63 W for high power system and 0.34 W for low power system, which represent 2.39 % and 9.54 % of their maximum power, respectively. Additionally, a disadvantage is leading nature of inverter current which typically causes higher hard switching losses than lagging counterpart, especially when MOSFET switches are used.

Obeying previously given explanations, the following analysis will be primarily based on LC compensation structure. In order to highlight the influence of maximum converter current $I_{\text{inv,max}}$ (converter VA rating) and allowed slope $\Delta P_1$ on average delivered power per pick-up, a set of additional simulation was done and the results are presented in Fig. 12 - 15. Fig. 12 and Fig. 13 show average delivered power as the function of maximum inverter current. The slope of $P_{\text{IPT,1}} = f(N)$ curves is kept constant at the values given it Table II. At the same time, the Fig. 14 and Fig. 15 represent the results when the specification of maximum inverter current is kept unchanged but slope $\Delta P_1$ varies. In all four figures, the numbers, written on the graph beside the markers, represent the relative improvement in power of optimal unmatched LC compensation tank comparing to the optimal matched one.

As it was expected, delivered power is almost linearly related to converter VA rating (maximum inverter current). Also, it increases with the increase in allowed power variation $\Delta P_1$. However, it is interesting to note that power improvement of unmatched LC compensation structure over the optimal matched one reaches some kind of saturation when either $I_{\text{inv,max}}$ or $\Delta P_1$ increases.

Finally, a Simulink model of whole high power IPT system is made and dynamic behavior of optimal matched and unmatched LC compensation structure is tested. The results of simulation are given in Fig. 16 – Fig. 18. In the model, 4 IGBTs are arranged in an H-bridge structure to form a resonant converter. The gate signals are generated in order to achieve the maximum output voltage (duty ratio equal 0.5). Track load is then changed after each 50 ms, in the way shown in Figure 16, and power per vehicle is measured and plotted.

As it was expected, in the steady state design with matched impedances it perfectly rejects any load influence on track current and delivered power per pick-up, but offers suboptimal design in terms of delivered power per pick-up.
V. CONCLUSION

In this article, four simple resonant tank compensation structures are analyzed for application in an unregulated IPT multi-pickup system. Main optimization goal was maximum power transferred to the load. Also some practical consideration (track current variation and limited power rating of resonant converter) are taken into account. The optimization procedure, based on a searching algorithm, was developed and tested through two sets of simulations on two different IPT systems. Additionally, the aspects of voltage and current ratings of chosen components were analyzed. The results show that optimal compensation structure is the LC compensation but with unmatched impedances of $L$ and $C$. In comparison to the optimal matched LC structure unmatched one offers 20% more delivered power. The results are simulated in order to prove steady state optimal nature as well as stable operation during transients. The explained compensation procedure is suitable especially for low power, low cost IPT applications where simple structure of compensation circuit and maximized transferred power are more beneficial than penalties paid for increased converter hard switching losses.

REFERENCES