Abstract

Over the coming decades, aviation operations are predicted to rise steadily, increasing the burden on already congested and constrained airports and terminal areas. It has long been recognized that a major factor governing the safe minimum separation distance between aircraft is the hazard associated with the wake generated by the preceding aircraft.

A flight deck display that allows the pilot to "see" the neighboring aircraft, as well as its wake, may allow for a decrease in spacing and an increase in airport and airspace capacity, while maintaining safe separation distances from wake vortices. This research evaluates a model based on aircraft parameters that are readily available in flight to predict aircraft wake vortices in real time. Using this model, a number of graphical representations of wake symbology were created for use in a 3-D perspective view flight deck display. This research determines the feasibility of this method of wake vortex display and ascertains the most favorable symbology for use in flight deck operations.

Introduction

There currently is no means in place in the National Airspace System (NAS) that warns pilots of potential wake vortex encounters except the experience and foresight of air traffic controllers. The need for a warning system is especially critical during the approach and departure phases of flight when aircraft frequently follow in-trail of other aircraft. During visual meteorological conditions (VMC) pilots can see and avoid other aircraft and estimate the location of the wake, but the descent and dissipation rates of wakes vary from aircraft to aircraft and it is difficult to make an accurate estimation. The aviation community has thus far been fortunate that the recommended aircraft spacing guidelines, when strictly adhered to, have been successful in preventing wake encounters. Occasionally, the pilot is either unable to maintain proper spacing or unaware that the spacing is inadequate and tragedy strikes.

During instrument meteorological conditions (IMC) when pilots cannot see other aircraft, the recommended spacing, as issued by Air Traffic Control (ATC), is the only way to protect one aircraft from the wake of another. In IMC pilots cannot see one another and certainly cannot judge the location of the wake. They rely entirely on ATC to keep them clear of the wake of the preceding aircraft. Although these procedures insure safety, they have an adverse impact on capacity.

As a backup to the conventional ATC system and to aid with collision avoidance, commercial aircraft are equipped with airborne collision avoidance systems, such as the Traffic Alert and Collision Avoidance System (TCAS), that provide ground independent protection from midair collisions. If these collision avoidance systems could display even a prediction of wake vortices, along with the traffic, then pilots would be better informed of the potential danger and could make conscious decisions to avoid the wake.

Potential Benefits of a Wake Vortex Avoidance System

Capacity

Apart from the obvious benefit of increased safety during all phases of flight, a wake vortex avoidance system can impact the NAS in other advantageous ways. In 2000, ATC handled 46.1 million aircraft and that number was projected to jump to 85.1 million in 2005 [1].

During approaches under Visual Flight Rules (VFR), pilots are usually given the responsibility to maintain appropriate separation from other aircraft.

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and there are no minimum separation distances. In most cases, minimum separation is determined by the time it takes the preceding aircraft to land and clear the runway. Under IFR, ATC takes direct responsibility for aircraft separation and applies the recommended spacing based on aircraft weight categories. There is no fundamental reason to believe that aircraft wake vortices behave any differently under IFR than they do under VFR. If the pilot could reliably “see and avoid” the preceding aircraft and its wake in IMC, responsibility for separation under IFR could be returned to the pilot and airports could return to operations with near VMC capacity.

**ATC Display**

Since the information used to predict the wake of neighboring aircraft would be broadcast, anyone in the vicinity of the transmission can receive it, including ATC. This information could be used to drive a similar display, giving air traffic controllers the same visual information as the pilots and aid them in issuing avoidance advisories. ATC would then be able to see the wakes of the aircraft in their sector and issue wake vortex alerts as they do traffic alerts. This would give new meaning to the advisory “caution wake turbulence” as it would be based on visual confirmation that a wake is, in fact, in close proximity to another aircraft and eliminate the routine issuance of advisories to aircraft following larger aircraft.

**Wind Vector**

In order to have the best accuracy in the wind data, the wind vector should be a part of the data stream from the wake generating aircraft. In addition to providing ATC with the necessary wind data for the above wake display, the wind vector of all of the aircraft within a sector could be automatically collected by ATC and used as a better indication of current wind conditions within that sector, as well as, used to create a more accurate wind model for forecasting future winds.

**Enabling Technology**

**Global Positioning System (GPS)**

Existing surveillance technology is limited in both accuracy and coverage. Radar accuracy depends on an aircraft’s distance from the radar, whereas GPS is uniform worldwide [2]. Based on the accuracy of radar, current lateral radar separation standards are 3 nm in terminal airspace and 5 nm in en route airspace.

Research conducted by Gazit [2] determined that the current minimum separation distance between aircraft can be safely reduced using GPS technology while keeping the same safety level that is provided by radar surveillance. Gazit concluded that GPS based aircraft tracking can yield a significant improvement in the quality and extent of ATC surveillance coverage.

GPS enables aircraft to broadcast horizontal position to approximately ±15m and when the Wide Area Augmentation System (WAAS) becomes operational to ±3m. A data link system already in limited use that is an excellent match with GPS technology is the Automatic Dependent Surveillance-Broadcast (ADS-B) system.

**Automatic Dependent Surveillance-Broadcast (ADS-B)**

Automatic Dependent Surveillance-Broadcast (ADS-B) is a means of providing the pilot with the necessary knowledge of airborne traffic without the need for interrogation. ADS-B is a passive system that periodically broadcasts its state vector and other information to provide a real-time operational data interface. Any user, either airborne or ground-based, within range of the broadcast (~150 nm), can receive and process the ADS-B surveillance information.

The ADS-B system provides a more accurate account of airborne traffic than conventional surveillance radar and can provide coverage where radar either cannot, such as low altitudes and in mountainous areas, or does not. The digital technology of ADS-B can be made small and light enough for use in GA aircraft, which would allow smaller aircraft to be equipped with flight deck displays similar to commercial carriers.

The Capstone project in SW Alaska, part of FAA’s Safe Flight 21 program, has outfitted up to 150 commercial aircraft operating in passenger, mail, and freight services with ADS-B equipment [3]. The new technology provides the ability to data link information such as weather, terrain, and traffic directly to the cockpit for the first time and

2.A.3-2
pilots' reviews of this new technology have been extremely positive.

Nominal Wake Model

In order to model the wake vortices on a flight deck display, the follower aircraft must know the characteristics of the generating aircraft and/or its wake. One way to model the wake is to have aircraft equipped with onboard sensors and actually measure the wake, using these measurements to display the wake. There are, however, no reliable, all weather, low cost operational wake vortex sensors [4]. Although pulsed lidar has been used for the direct detection of turbulence 15-30 seconds ahead of an aircraft, the state of the art of onboard instrumentation has not yet reached a level where sensing or measuring the wake in-flight is possible and the cost of outfitting aircraft with these sensors would be prohibitive. Another way to model the wake for a flight deck display is to predict the wake using mathematical models.

Mathematical modeling of wake vortices is extremely complex since it is dependent on the aerodynamic characteristics of the generating aircraft, atmospheric conditions, the proximity of the aircraft to the ground, and the time elapsed since generation. The model used in this research is based on theoretical equations simplified to use parameters readily available from current onboard instrumentation and based on the assumption that some form of aircraft-to-aircraft data link (i.e. ADS-B) will soon become a standard. The description of the model that follows is based on nominal dimensions. The model as depicted on the display will have dimensions based on a safety margin determined by the uncertainty analysis and will be larger.

Wake Terminology

A brief description of wake terminology at this point will lend to a better understanding of the material in this paper. While all aircraft generate a wake, the aircraft generating the particular wake in question is referred to as the generating aircraft and the other aircraft (whose intent is to avoid that particular wake) is referred to as the follower aircraft.

The parameters used in this paper are as follows (Figure 1): \( b_g \) is the wingspan of the generating aircraft; \( b' \) is the wake vortex span, the distance between the centers of the two counter-rotating vortex cores and is a function of the aircraft configuration, wing loading, and angle of attack; \( L \) is lift; \( W \) is weight; \( \gamma \) is the circulation of each vortex of a vortex pair, often referred to as the vortex strength; \( V \) is the freestream velocity or true airspeed; \( u \) is horizontal component of wind speed (but not crosswind component); and \( \rho \) is air density.

Figure 1. Coordinate System With Respect To The Wake Generating Aircraft And Wake Terminology

The Wake Vortex Hazard

Under most atmospheric conditions, the wake is quickly transported out of the approach corridor or decays at a relatively fast rate. Under conditions of negligible atmospheric turbulence (calm conditions), the wake may stall in the approach corridor. It was with these conservative meteorological conditions in mind that the current FAA wake vortex spacing and avoidance procedures were developed.

A wake vortex encounter could result in a mild disturbance or a catastrophic upset. The degree of upset depends mainly on the relative sizes of the generating and encountering aircraft and point of entry into the vortex field. The most hazardous motion due to an encounter is an overpowering rolling moment near the center of a vortex core. A Lear Jet-23 used in wake encounter research was observed to roll 360 degrees when it encountered the center of the wake of a C-5A. In other types of aircraft, roll excursions of 40-80 degrees were not uncommon. Even with aircraft of similar size, it is necessary for the pilot to apply control motions quickly to offset the wake-induced motions.


**Hazardous Region**

This research defines an area that, if known, the pilot would avoid entirely. The horizontal and vertical cross-sectional limits (y,z plane) of this hazardous area can be defined as that region outside of which an encountering aircraft has the onboard control authority to overcome the wake-induced forces and have a reserve for necessary aircraft maneuvering and flight-path corrections.

Considerable research on wake vortices was conducted by Rossow [5,6,7,8] in order to understand the interaction between the wake and the encountering aircraft and to determine an acceptable level of wake intensity. Rossow made a comparison between the wake-induced rolling moment of the wake vortex and the aileron-induced rolling moment of the follower aircraft. Plotting lines of constant rolling moment coefficient of the wake, Rossow was able to determine those regions where the wake-induced rolling moment exceeds the maximum aileron-induced rolling moment of the follower aircraft. He determined that, for his models at least, the hazardous portion of the wake could be wholly contained within a box (y,z plane) the size of $2b_x \times 1b_z$ (Figure 2).

![Figure 2. Cross-Section Of Wake Vortices Defining The Dimensions Of A Wake Plane [10]](image)

**Vertical Motion of the Wake**

**Self-induced Descent Velocity ($w$)**

The circulation ($\Gamma$) of each individual vortex of a wake vortex pair is equal in magnitude and opposite in rotational direction. The center of each vortex is in the field of velocity induced by the other. Thus, in the absence of atmospheric effects, the vortices move downward by mutual induction with equal uniform velocity. This movement is called downwash and the velocity at which it moves is called the self-induced descent velocity ($w$) or sink rate.

The self-induced descent velocity ($w$), determines how fast the wake may descend behind the generating aircraft and out of the flight path of the follower aircraft. The time-averaged value for the vortex sink rate is derived from the vortex strength ($\Gamma$) and the vortex span ($b'$) (Equation 1) [9].

$$w = -\frac{\Gamma}{2\pi b'}$$  

where: $\Gamma = \frac{L}{\rho V_m b'}$

In level unaccelerated flight, weight (W) is typically substituted for lift (L). Combining the two previous equations and making the substitution for lift, the equation for sink rate becomes:

$$w = -\frac{W}{2\pi \rho V_m b'^2}$$  

(2)

**Descent Distance ($z_{w/g}$)**

Observations of actual wakes have shown that vortices often stop their descent at some level ($z_{\text{limit}}$) below the generating aircraft and then remain near that level until they dissipate (Figure 3). Until this point is reached, the vertical position of the wake ($z_{w/g}$) is determined by the elapsed time ($t$) since the wake plane was deposited (Equation 3).

$$z_{w/g} = wt$$  

(3)

![Figure 3. Theoretical Wake Descent Profile](image)
transport aircraft descend about 500-1000 ft, depending on aircraft type, before their downward motion slows or stops. In full-scale flight tests conducted by Condit and Tracy [10] for Boeing aircraft, it was shown that, for the aircraft studied, the vortices settled to approximately 900-1000 ft below the wake-generating aircraft. In research conducted by Crow and Bates [11], it was rare for the wake to descend more than 6 ft. For most commercial transports, 6 ft is between 500 and 1000 feet. Therefore, based on the literature, this research assumes that the wake levels off at:

$$z_{\text{limit}} = 6 \text{ ft}$$ (4)

Lateral Drift ($y_{\text{wg}}$)

In addition to the vertical motion of the wake due to mutual induction, the vortices are also transported horizontally by the wind and influenced by ground effect. At altitude, vortices move with the wind as a pair until they dissipate and although this movement is not restricted to the $y_g$ direction, $y_{\text{wg}}$ is used as the symbol for lateral drift (movement of the wake in the $x_g-y_g$ plane). Lateral displacement of the vortex with respect to the generating aircraft is represented by the wind speed ($u$) and elapsed time ($t$).

$$y_{\text{wg}} = ut$$ (5)

Lifespan

The atmospheric conditions that exist at the time and place a wake is deposited are the most significant contributors to the breakup and dissipation of the wake. A vortex deposited in an area where there is little or no atmospheric disturbance can last over an hour and hundreds of miles behind the generating aircraft. The issue is not how long the vortex can last, but how long it takes the wake to reach a non-hazardous state.

The time after which the wake is no longer a hazard refers to the degree of upset or disturbance that might be experienced by an encountering aircraft; in other words, the time after which an aircraft can safely enter the wake of another without causing loss of control, aircraft damage, or discomfort to the occupants. Pilot simulations conducted under VFR and IFR conditions by several researchers were used to infer when non-hazardous conditions existed [5]. It was found that pilots considered roll angles of 7 degrees or less as non-hazardous at about 100 feet AGL under VFR conditions and at about 200 feet AGL under IFR conditions. The study does not address, however, where along the vortex wake this non-hazardous state may exist.

Since the purpose of this research is to give the pilot the means to avoid that region of airspace in which the hazardous portion of the wake is located, the issue for the display is to determine how long to keep the wake on the display. Translating the various spatial in-trail distances into the time domain, 5 nm translates to a 2.0-2.3 minute in-trail spacing and 3 nm is equivalent to a 1.2–1.4 minute in-trail spacing at approach speed.

Garodz, Lawrence, and Miller [12] conducted research on Boeing 747, 727, 707 and DC-9 aircraft and they concluded that the maximum vortex lifespan is 2.6 minutes above the earth’s boundary layer and 2 minutes within the earth’s boundary layer depending on aircraft type. The earth’s boundary layer is on the order of 3000 ft depending on atmospheric conditions and location on the earth. Crow and Bates [11] conducted flight tests with five different aircraft ranging in size from a Boeing 747 to a Cessna 170. The results indicate that the lifespan of the wake of a Boeing 747, the heaviest civilian transport with the largest vortex strength, is roughly 130 seconds (2.2 min) in negligible turbulence. Taking the longest lifespan as a worst case, the nominal wake will be at least 2 minutes.

Modeling the Wake on the Display

The wake is modeled on the display as cross-sectional slices or wake planes (Figure 4). The dimensions of these wake planes are based on the cross-section of the hazardous region as defined by Rossow [6.7.8], $2b_g \times 1b_g$, plus uncertainty.

Figure 4. Graphical Representation Of Wake Hazard Area.
An individual wake plane appears on the display and its movement is subject to its own sink rate and the wind vector as measured at that point. A wake plane is deposited at a specific time interval, and moves independently of all other wake planes. As the wake plane ages, it increases in size according to the uncertainty in the $y_g$ and $z_g$ direction.

**Uncertainty Analysis**

The quantities that affect the motion of wake vortices and, therefore determine the sources of the uncertainty in the prediction are the aircraft locations, size, location, and movement of the hazardous region, and time at which the vortex is no longer a hazard.

Compiling the available literature on wake vortex behavior, it is clear that there is a high degree of uncertainty, especially when the equations of motion are simplified. One of the goals of this research is to test the simplified equations. At this point, it is the philosophy of this research to err on the conservative side, which results in larger uncertainties.

**Location of Generating Aircraft**

Advances in technology have enabled individuals to determine their location and navigate with a precision far greater than was ever possible with radar. The location of the generating and follower aircraft is determined onboard each respective aircraft using GPS technology. ADS-B is then used by the generating aircraft to broadcast its position and velocity vector to the follower aircraft, which then compares this with its own location and velocity to determine their relationship to each other. The uncertainty in the location of the aircraft will depend on how well the actual positions are known and how well this information is communicated to other aircraft.

**GPS Position**

For this research, GPS accuracy is based on the use of the Wide Area Augmentation System (WAAS) and its associated research conducted at Stanford University. Considering a worst-case geometry acceptable for use, Table 1 illustrates representative one-sigma errors of GPS/WAAS position and velocity (north and east ground speed).

**Table 1. GPS/WAAS Position and Velocity Accuracies**

<table>
<thead>
<tr>
<th></th>
<th>Vertical Pos.</th>
<th>Horizontal Pos</th>
<th>Horizontal Vel.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.25 m</td>
<td>1.58 m</td>
<td>0.415 m/s</td>
</tr>
<tr>
<td></td>
<td>(7.38 ft)</td>
<td>(5.18 ft)</td>
<td>(0.81 kts)</td>
</tr>
</tbody>
</table>

**ADS-B Data Link**

The accuracy of position and velocity are also affected by the resolution of the data link message. The ADS-B UAT (Universal Access Transceiver) used in research in the WAAS Laboratory at Stanford University has the following resolution for position and velocity:

**Table 2. ADS-B Position and Velocity Resolutions**

<table>
<thead>
<tr>
<th></th>
<th>Vertical Pos.</th>
<th>Horizontal Pos</th>
<th>Horizontal Vel.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.62 m</td>
<td>2.5 m</td>
<td>0.26 m/s</td>
</tr>
<tr>
<td></td>
<td>(25 ft)</td>
<td>(8.2 ft)</td>
<td>(0.5 kt)</td>
</tr>
</tbody>
</table>

**Total Position Error**

Using the root square sum method, the total position error can be determined from the GPS accuracies and the ADS-B resolutions (Table 3).

**Table 3. Total Position Error**

<table>
<thead>
<tr>
<th></th>
<th>Vertical Pos.</th>
<th>Horizontal Pos</th>
<th>Horizontal Vel.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.9 m</td>
<td>2.9 m</td>
<td>0.490 m/s</td>
</tr>
<tr>
<td></td>
<td>(26.1 ft)</td>
<td>(9.7 ft)</td>
<td>(0.95 kts)</td>
</tr>
</tbody>
</table>

**Location and Movement of the Hazardous Region**

**Self-Induced Descent Velocity**

As can be seen from Equation 2, the self-induced descent velocity ($w$) or sink rate of the vortex is dependent on the weight ($W$) and airspeed ($V_a$) of the generating aircraft, the spanwise distance between the vortex centers ($b'$), and the air density ($\rho$). Of these, weight is the overwhelming factor in the uncertainty of $w$.

The weight of an aircraft changes throughout its flight regime from its maximum weight at takeoff to its minimum weight at landing. In order to accommodate the uncertainty due to the aircraft's weight, which is not included in the ADS-B message, $w$ for the approach corridor is calculated using a nominal weight based on the mean between the maximum landing weight (MLW) and the
operational empty weight (OEW). The spread between the mean and these values is the error bound. When calculating the nominal weight for departure and en-route phases of flight, gross take-off weight (GTW) and OEW are used.

In association with weight, load factor is another major contributor to the uncertainty of \( w \). It was assumed that weight could be substituted for lift in Equation 2 based on level unaccelerated flight (or a load factor of 1 g). This can be assumed about 95% of the time or better. During corrections for minor changes from trimmed flight, the maximum variation in load factor is about 10% or between 0.9 and 1.1 g. During a missed approach procedure, the maximum acceleration is about 1.2 gs. During turning flight, the maximum bank angle is usually limited to 30 degrees or less and generally no more than the number of degrees of heading change. In a level turn 30 degrees translates to about 1.15 gs and in a climbing turn to about 1.2 gs. It is estimated that a deviation between 0.9 and 1.1 g will fall within a 2-3 sigma distribution[13].

Since \( b' \) is a function of the aircraft configuration and angle of attack and the configuration of the aircraft changes many times during flight, \( b' \) cannot be assumed to be constant. For commercial aircraft in all configurations \( b' \) is not greater than 0.8 \( b \) and not less than 0.75 \( b \). The nominal value for \( b' \) is taken as 0.7753 (half way between these two points) with an error bound of 0.025\( b \).

Density (\( \rho \)) is determined from the GPS altitude (\( h \)) using typical equations for determining standard air density and the uncertainty is therefore a function of the vertical GPS position error and temperature.

The velocity used to calculate \( w \) is the freestream velocity or true airspeed. True airspeed is calculated from the GPS ground speed as broadcast by ADS-B and wind speed (\( u \)), which can be measured by the onboard air data computer to a resolution of \( \pm 1 \) kt. As a result, the uncertainty in true airspeed is a function of the GPS horizontal velocity error and the air data computer resolution. Table 4 is a summary of the factors contributing to the uncertainty in \( w \).

### Table 4. Factors Contributing to Uncertainty in Self-Induced Descent Velocity (\( w \))

<table>
<thead>
<tr>
<th></th>
<th>( W )</th>
<th>( n )</th>
<th>( b' )</th>
<th>( u )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/c specific</td>
<td>( \pm 0.1 ) g</td>
<td>( \pm 0.025b )</td>
<td>( \pm 1 ) kt</td>
<td>( \approx -0.001\rho )</td>
<td></td>
</tr>
</tbody>
</table>

### Propagation of Uncertainty

The uncertainty in the vertical location of the wake was derived from a method of linear error propagation based on a first order Taylor series as described in Mehn[14,15] and Coleman and Steele[16]. The uncertainty in each variable was propagated through Equation 3 and is on the order of 20-40% of \( z_{\text{w_k}} \) depending on the type of aircraft and whether the nominal weight is based on MLW or GTW. Table 5 illustrates the relative contribution of each variable used to derive \( z_{\text{w_k}} \) for the Boeing 747 at 5000 ft AGL using MLW and approach airspeed. Weight and load factor become more equal in their contribution if the weight is known to a resolution of 25,000 lbs (see An Argument for Additions to ADS-B below).

### Table 5. Relative Contribution of Error Sources to \( w \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Bias (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>( W )</td>
<td>70.00</td>
</tr>
<tr>
<td>Load Factor</td>
<td>( n )</td>
<td>21.64</td>
</tr>
<tr>
<td>Vortex Span</td>
<td>( b' )</td>
<td>8.13</td>
</tr>
<tr>
<td>Time</td>
<td>( t )</td>
<td>0.07</td>
</tr>
<tr>
<td>Wind Direction</td>
<td>( WD )</td>
<td>0.07</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>( u )</td>
<td>0.06</td>
</tr>
<tr>
<td>North Velocity</td>
<td>( NV )</td>
<td>0.02</td>
</tr>
<tr>
<td>East Velocity</td>
<td>( EV )</td>
<td>0.01</td>
</tr>
<tr>
<td>Altitude</td>
<td>( h )</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Lateral Drift

Any uncertainty in the wind measurement will generate uncertainty in the lateral size and movement of the wake plane. It is therefore important that the wind vector be measured accurately in order to keep the size of the wake plane as small as possible. The air data computer used in this research measures the wind to a 1 kt resolution in speed and a \( \pm 5^\circ \) resolution in direction.

The wind vector is not currently part of the ADS-B message, consequently, the wind vector as measured by the follower aircraft is used to calculate the drift of the wake due to the wind. The
difference in the winds between the generating aircraft and the follower aircraft introduces an additional uncertainty to the size and motion of the wake plane. A study conducted by Lincoln Laboratory at MIT suggests that the rms vector error of the wind speeds between two aircraft is \(2 - 3 \text{ m/s (4-6 kts)}\) when horizontal separation is \(3 - 8 \text{ km (1.5-4 nm)}\) [17]. Table 6 summarizes the factors contributing to the uncertainty in lateral drift.

The position of each wake plane, throughout its lifespan, is determined by the lateral \((u)\) and vertical \((w)\) velocities and the time \((t)\) elapsed since placement. There is an uncertainty in time due to the latency in the ADS-B message, which propagates through Equations 3 & 5. This latency is estimated to be 1.0183 seconds.

### Table 6. Factors Contributing to Uncertainty in Lateral Drift

<table>
<thead>
<tr>
<th>Instrumentation Error</th>
<th>Wind Speed RMS Vector Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 m/s (±1 m/s)</td>
<td>±2-3 m/s (±4-6 kts)</td>
</tr>
</tbody>
</table>

The uncertainties in lateral drift are propagated through Equation 5 in the same manner as the uncertainty in vertical location. Table 7 summarizes the relative contribution of each of the variables used to derive \(y_{WLR}\). The uncertainty in lateral drift is on the order of 1-20% of \(y_{WLR}\) depending on the wind speed and direction.

### Table 7. Relative Contribution of Error Sources to Lateral Drift (Instrumentation Error Only)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Bias (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Direction</td>
<td>WD</td>
<td>64.51</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>WS</td>
<td>35.12</td>
</tr>
<tr>
<td>Time</td>
<td>t</td>
<td>0.25</td>
</tr>
<tr>
<td>East Velocity</td>
<td>EV</td>
<td>0.09</td>
</tr>
<tr>
<td>North Velocity</td>
<td>NV</td>
<td>0.02</td>
</tr>
</tbody>
</table>

### Size of Hazardous Region

The uncertainty in the wind vector and the sink rate become visible on the flight deck display as an uncertainty in the size of the hazard area. The uncertainty in the wind causes an increase in the horizontal dimensions over time and the uncertainty in the sink rate causes an increase in the vertical dimensions over time.

This can be illustrated through an example using the Boeing 747. Starting with the uncertainty in weight, the Boeing 747-400P has a MLW of 574,000 lbs and an OEW of 399,000 lbs. The nominal weight, then, for this exercise is 486,500 lbs, with an error bound of ±87,500 lbs. The wingspan is 211.42 ft, so the nominal vortex span \((b')\) is 163.85 ft, with an error bound of ±5.26 ft. The heading is 030 degrees, approach speed is 146 kts, altitude is 5000 ft MSL, and the wind is from 070 degrees at 15 kts.

### Vertical Dimensions for Boeing 747 Wake

Using the above conditions and propagating the uncertainty through Equation 2, the sink rate for the Boeing 747-400P is \(-5.72 \text{ ft/s (±1.23 ft/s)}\) (or 21.5% uncertainty). After 60 seconds a wake plane will have descended 343 ft ±74 ft. This uncertainty will be visible on the display by increasing the vertical dimensions of the hazard area (wake plane) by 148 ft. The vertical dimensions of the nominal wake plane for a Boeing 747-400P is 211 ft (one wingspan). After 60 seconds, the vertical dimension of the wake plane will be 359 ft, in order to accommodate the uncertainty.

Figure 5 illustrates how the vertical dimension of the wake plane increases with time due to uncertainty. The wake planes descend at a constant rate, \(w\), until they reach a distance of \(6b'\) below their point of origin at which time they level off continuing to increase in size \((z_{lim})\) is about 1000 ft for the Boeing 747.

### Figure 5. Theoretical Descent Profile

2.A.3-8
Lateral Dimensions for Boeing 747 Wake

Considering only uncertainty due to instrumentation error and propagating this uncertainty through Equation 5, the lateral drift of the wake with respect to the generating aircraft is $-10.4 \text{kts} \pm 1.17 \text{kts}$ (or 11.2% uncertainty). After 60 seconds, a wake plane will have drifted 1061 ft to the left of the flight path $\pm 119$ ft. This means that the lateral dimension of the wake plane depicted on the display will have increased by 238 ft to accommodate the uncertainty in the drift.

If the uncertainty of the wind between aircraft locations is included ($\pm 4$ kts), the uncertainty in the lateral drift will be $\pm 3.01$ kts (or 28.9% uncertainty). The drift uncertainty will have increased to $\pm 307$ ft after 60 seconds and the lateral dimension of the depicted wake will have increased to 614 ft.

Figure 6 is a plot of the lateral uncertainty, which illustrates how the lateral dimension of the wake plane will increase with time due to instrumentation error alone, instrumentation error plus the 4 kt wind error, and instrumentation error plus the 6 kt wind error. As can be seen in the plot, it is critical to know the wind speed to an accuracy of better than $\pm 1$ kt in order to keep the lateral dimension of the depicted wake plane under about 1000 ft after 3 minutes.

Figure 6. Uncertainty in Lateral Drift

Wake Vortex Lifespan

Since there is inadequate research in this area on which to form a model or prediction of the lifespan of the wake, there is little useful information on which to base an uncertainty analysis. All estimates in the literature seem to point to approximately a 2-minute lifespan for the hazardous portion of the longest wakes. Because the uncertainty in the wake lifespan is so questionable, it seems reasonable to defer to a standard safety margin of 50% and paint the wake vortex on the display for 3 minutes.

An Argument for Additions to ADS-B

The accuracy of weight, the major contributing factor to the uncertainty in sink rate (Table 5), could be improved by including weight in the ADS-B message. Table 8 illustrates the improvement in uncertainty for the sink rate of the Boeing 747-400P based on a nominal weight for the approach corridor. The published MLW and OEW for the Boeing 747-400P and an error bound of 87,500 lbs yields an uncertainty of about 21% of $w$. If the weight is known to an accuracy of 25,000 lbs, the uncertainty could be improved to about 13%. After 3 minutes that would be a savings of about 200 ft in the depth of the depicted hazard area. Figure 7 illustrates this graphically. An accuracy of less than 25,000 lbs does not significantly improve the uncertainty of $w$ (Table 8). It should be noted that these values vary with altitude.

<table>
<thead>
<tr>
<th>Weight</th>
<th>$w$</th>
<th>Uncertainty</th>
<th>%$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87,500 lbs</td>
<td>$-5.72$ ft/s</td>
<td>$\pm 1.2298$</td>
<td>21.5</td>
</tr>
<tr>
<td>25,000 lbs</td>
<td>$-5.72$ ft/s</td>
<td>$\pm 0.7345$</td>
<td>12.8</td>
</tr>
<tr>
<td>15,000 lbs</td>
<td>$-5.72$ ft/s</td>
<td>$\pm 0.6958$</td>
<td>12.1</td>
</tr>
<tr>
<td>10,000 lbs</td>
<td>$-5.72$ ft/s</td>
<td>$\pm 0.6832$</td>
<td>11.9</td>
</tr>
<tr>
<td>500 lbs</td>
<td>$-5.72$ ft/s</td>
<td>$\pm 0.6731$</td>
<td>11.7</td>
</tr>
</tbody>
</table>

If the uncertainty were based on a departure or en route scenario, then the nominal weight would be 599,500 lbs calculated from GTW and OEW. The error bound in this situation would be 190,475 lbs with an uncertainty of 38.0% of $w$. The improvement using 25,000 lb increments is more dramatic (21.3%) with a savings after 3 minutes of about 400 ft (Figure 8).
Figure 7. Uncertainty in Sink Rate based on Accuracy of Weight MLW

Figure 8. Uncertainty in Sink Rate based on Accuracy of Weight at GTW

A major factor contributing to the uncertainty in lateral drift is the wind speed. By adding the wind vector to the ADS-B message, the uncertainty in the wind speed between aircraft locations can be significantly reduced. The savings in the lateral dimension of the depicted wake plane would be about 1000 ft after 3 minutes (Figure 6). Adding the wind vector to the ADS-B message will also significantly reduce lateral drift error for ATC, which has no source of wind to drive a wake display otherwise. The error without knowledge of the wind vector will be, of course, the wind itself.

Graphical Display of Wake Vortices

Since 1995, the Wide Area Augmentation System (WAAS) Laboratory at Stanford University has been developing and flight testing a 3-D Perspective View Tunnel-in-the-Sky Display. Research has shown that the equipment necessary to display the wake can be manufactured inexpensively making it affordable not only to commercial carriers, but to general aviation (GA), as well [18].

Early in the development of this display, the standard navigation instruments were integrated into the display to reduce the scan and increase the situational awareness of the pilot [19]. This early display had a blue sky and brown ground similar to standard attitude indicators. A later version incorporated terrain and terrain alerting symbology in order to make the display more representative of an “out-the-window” view and to address the hazard of controlled flight into terrain (CFIT) [20]. The latest version to complete the flight test phase addressed the issue of traffic awareness and implemented a Runway Incursion Monitor [21]. It uses ADS-B technology to determine the position of neighboring aircraft and shows them on the display. The most recent contribution to this effort is the graphical display of the wake vortices of airborne traffic.

Graphical Presentation

One of the most creative parts of this research was developing various ways to represent the wake planes on the display while keeping in mind the purpose, to improve the situational awareness of the pilot with respect to the wake of neighboring aircraft. The various representations evaluated in piloted simulations were developed balancing issues such as complexity, ability to see other traffic and terrain through the wake, and display clutter. Six different wake representations were chosen for the piloted simulations (Figure 9).

In deciding the color and shape of the wake plane, it was necessary that it be easy to distinguish as a hazard symbol. Therefore, red was an obvious choice for color. It was thought that the pilot might find it useful to know the relative age of a wake plane when encountering the wake. The variable color scheme was developed to give the pilot a
sense of whether he was near the beginning or end of the wake.

Because the nominal wake was defined as a rectangular shape ($2b_w \times 1 b_w$), an obvious choice for the shape of the wake plane was the rectangle. In addition, the rectangle is an easy geometry to dimension and modify for uncertainty. The oval shape was chosen because the rounded edges make it more distinct from the tunnel symbology.

![Figure 9. Wake Plane Representations](image)

While the open rectangle and open oval shapes allow for the best view through the wake, it is thought that they may not be as easy to see on the display and may be confused as a tunnel to fly through instead of an area to avoid. For this reason, the transparent fill was added to the rectangle and oval shapes for the evaluation.

A one second spacing between the wake planes makes them distinctly separate objects. It was thought that by connecting the wake planes together, the wake planes might appear more as a three-dimensional wake hazard area and discourage unwitting pilots from trying to fly between the wake planes.

**Flight Deck Display**

The following figures are graphics of the flight deck display captured during simulated flights.

![Figure 10. Display Symbology](image)

Figure 10 explains the display symbology. The tunnel guides the pilot along the flight path and is stationary in space. The nominal path indicator guides the pilot through the tunnel by means of the predictor symbol and moves with the predictor symbol. The predictor symbol predicts the position of the ownship 3 seconds forward in time.

![Figure 11 Wake Of Aircraft Departing The Same Runway Ahead Of Ownship, Drifting With Wind.](image)

Figure 11 shows the ownship departing the runway approximately one minute after another aircraft. While the other aircraft is no longer in view on the display, the wake of the other aircraft can be seen to the left of the predictor symbol having drifted with the wind (from 070 at 5 kts). The wake plane representation used in this display is a filled oval.
level of certainty that the hazard is fully contained within the displayed area.

By adding weight and wind vector to the ADS-B message, improvements can be made in the uncertainty of the prediction and the depiction of the wake on the flight deck display. Depending on altitude and airspeed the savings from these improvements are on the order of 1000 ft in width and 100-500 ft in depth after 3 minutes.

The final goal is to demonstrate the potential for decreased aircraft spacing and increased capacity. A flight deck display that allows the pilot to “see” the neighboring aircraft, as well as its wake, may allow for a decrease in spacing and an increase in airport and airspace capacity, while maintaining safe separation distances from wake vortices and give new meaning to the term “see and avoid.” During the flight tests, the ability of the pilot to reduce spacing between aircraft and maintain a safe distance from the wake will be demonstrated.

References


[17] Cole, R.E., 2000, based on conversations with Dr. Rod Cole, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, MS


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