Quantitative Methods for Ranking Critical Events

Joseph DiVita and Robert L. Morris

Abstract—A technique using the Choquet integral will be investigated that will monitor the environment by sampling current utility values of criteria and aggregate this information to provide a means to determine the most suitable surface naval asset to be re-deployed in order to accomplish an unscheduled mission. This paper will present promising initial findings which supports the claim that the Choquet integral can be used to represent expert knowledge in the situation where the criteria are dependent. The results will be applied to the ranking of destroyers for re-deployment.

Index Terms—Choquet integral, Criteria, Fuzzy Measures, Prioritization, Ranking Alternatives

I. INTRODUCTION

The purpose of this research is to investigate a means to automate the ranking of alternative naval surface assets for re-deployment and tasking. In this scenario there are several naval assets, destroyers, which are currently performing missions. Each destroyer is scheduled to complete their current mission, or task, and then travel to a new location where they begin a new, scheduled task. The new, scheduled task is referred to as their 48 hour tasking, since it is the task they are scheduled to begin performing in 48 hours. An unscheduled task arises that must be completed as soon as possible, and the problem is which destroyer should be deployed to perform this new task? Thus a naval decision maker may be asked to rank the alternative destroyers from high to low where high in this context is the destroyer of first choice to perform the new mission and low is the destroyer of last resort to perform the new mission.

II. BACKGROUND

The Choquet integral is a fusion operator that allows one to aggregate interacting criteria to accomplish multi-criteria decision-making. Several researchers have demonstrated that the Choquet Integral may be used to prioritize alternatives [1] [2] [3] [4]. Each item that needs to be ranked, or prioritized, is referred to as an alternative. Associated with each alternative is a set of criteria. An alternative has a utility score on each criterion. The Choquet integral aggregates scores across all the criteria. It does this by taking into account the relative importance of each criterion and each possible subset of criteria. The importance of criteria is expressed in terms of a fuzzy measure.

Thus, let \( A \) be a set of alternatives, \( A = \{a, b, c, \cdots \} \), and let \( \mathcal{N} \) be a set of criteria, \( \mathcal{N} = \{c_1, \cdots, c_n\} \). Each alternative \( a \in A \), has a profile \( x^a = (x_1^a, \cdots, x_n^a) \in \mathbb{R}^n \), such that, \( x_i^a \) represents a score, or utility of \( a \) relative to the criterion \( c_i \). The scores for each criterion are real numbers and we assume that the scores for each criterion fall in the same range.

A discrete fuzzy measure, \( \mu \), on a set, \( \mathcal{N} \), is a set function whose domain is the power set of \( \mathcal{N} \), \( \mathcal{P}(\mathcal{N}) \), and whose range is the interval \([0,1] \):

\[ \mu: \mathcal{P}(\mathcal{N}) \rightarrow [0,1], \]

that satisfies the following conditions:

(i) \( \mu(\emptyset) = 0 \),

(ii) \( \forall S, T \subseteq \mathcal{N}, S \subseteq T \Rightarrow \mu(S) \leq \mu(T) \).

In our context, \( S \) and \( T \) are subsets of criteria. \( \mu(S) \) is the weight or importance of either a single criterion or a coalition of criteria. The relationship defined in (ii) is referred to as the monotonicity condition. It is defined to mean that the weight of the set of criteria, \( S \), can only increase if we add new criteria to the set \( S \). In this sense, a fuzzy measure is monotonic with respect to set inclusion. Fuzzy measures are generalizations of typical additive measures that are used in probability and measure (integration) theory.

The Choquet integral is defined as:

\[ C_\mu(x) = \sum_{i=1}^{n} [x_{\{i\}} - x_{\{i-1\}}] \mu(A_{\{i\}}), \]

where \( \{\cdot\} \) describes an ordering such that

\[ x_{\{0\}} = 0, \]
\[ x_{\{1\}} \leq x_{\{2\}}, x_{\{2\}} \leq x_{\{3\}}, \cdots, x_{\{n-1\}} \leq x_{\{n\}}, \]
\[ A_{\{i\}} = \{c_{\{1\}}, \cdots, c_{\{n\}}\}. \]

It is often convenient to express the Choquet Integral in terms of the Mobius representation of a fuzzy measure. A Mobius representation of a fuzzy measure \( \mu \), on a set \( \mathcal{N} \), is defined to be:

\[ \mu(T) = \sum_{S \subseteq T} a^\mu(S), \quad \forall T \subseteq \mathcal{N}, \]
where $T$ is a subset of $N$ and $a^\mu$ is a set function from the power set, $\mathcal{P}(N)$, to the real numbers, $\mathbb{R}$: $a^\mu: \mathcal{P}(N) \rightarrow \mathbb{R}$.

The function $a^\mu$ is referred to as the Mobius transform or Mobius representation of $\mu$ and is given by:

$$a^\mu(S) = \sum_{T \subseteq S} (-1)^{|S| - |T|} \mu(T), \quad \forall S \subseteq N,$$

where $|S|$ and $|T|$ are the cardinality of $S$ and $T$, respectively.

The Mobius representation captures the relative interaction among criteria. Consider a pair of criteria, $c_1, c_2$, then by definition $a^\mu(c_1, c_2) = \mu(c_1, c_2) - \mu(c_1) - \mu(c_2)$. Thus if $a^\mu(c_1, c_2) = 0$, there is no interaction between the criteria and they are independent. If $a^\mu(c_1, c_2) > 0$ then there is a positive interaction and the importance of the pair is superior to the importance of the sum of the criteria. If $a^\mu(c_1, c_2) < 0$ then there is a negative interaction, or redundancy between the criteria.

For a Mobius representation of a fuzzy measure, the following boundary and monotonicity conditions must be ensured:

$$a^\mu(\emptyset) = 0,$$

$$\sum_{T: i \in T} a^\mu(T) \geq 0, \quad \forall S \subseteq N, \forall i \in S,$$

$$\sum_{T \subseteq N} a^\mu(T) = 1.$$

In terms of the Mobius representation, the Choquet integral of an alternative associated with profile $x$, with respect to the measure $\mu$, is given by:

$$C_x^\mu = \sum_{T \subseteq N} a^\mu(T) \bigwedge_{i \in T} x_i,$$

where $\bigwedge$ denotes the minimum operator.

III. METHODOLOGY

A. Approach

In order to bound the re-deployment problem, several assumptions and constraints must be imposed on the scenario. First note the problem is both a prioritization problem and a scheduling problem. It is a scheduling problem since each destroyer already has a set of scheduled missions to perform with time constraints and those plans must be modified to meet a new set of time constraints.

We assume that the destroyers that are being considered for the new mission have slack time in their scheduling. Slack time [5] is illustrated in Fig. 1 for destroyer DDG1. If there is enough slack time, then a destroyer can finish their current task and travel to the new unscheduled task, finish the new task and still have time to travel to their 48 hour tasking.

It will be assumed that the new task must be completed. If the slack time is insufficient, or if there is no slack time at all, then one of the following must occur when a destroyer is assigned the new unscheduled tasking: 1) The destroyer must pre-empt their current task in order to make time for the unscheduled and 48 hour tasks; 2) The destroyer can complete their current task and the new unscheduled task but the deadline for the start time of the 48 hour task must slip; or 3) The destroyer pre-empts the current task and slips the 48 hour start time deadline.

The current scenario assumes that there is enough slack time so that each destroyer can complete its current task, travel to the newly scheduled task, complete that task, and then travel to the scheduled 48 hour task. However, all times, such as ‘the time on the current task’ or ‘time to travel to the new unscheduled task,’ are stochastic in nature. Thus the schedule reflects an estimate, a best guess, about when various tasks and travel times will be completed.

The decision maker must entertain the possibility that deadlines will be missed. Several factors may affect these time estimates and the actual outcome of the situation. For example, a task may be projected to take a long time, but the destroyer’s task readiness is better than baseline. This readiness may be a function of superior equipment or personal to perform the task. In this case, the decision maker expects the destroyer to perform the task quicker than the given estimated time. The decision maker may also expect that the variability around the estimated performance time will be smaller for the more capable destroyer. Likewise, a destroyer’s readiness may be subpar in which case the decision maker may expect a longer completion time and greater variability in performance times.

In this application of the Choquet Integral, an alternative is a destroyer. In order to apply the Choquet Integral, the set of criteria used to evaluate which destroyer should be redeployed must be specified. After interviewing experts, a set of fourteen criteria where identified.

The criteria fall into four categories of overall, current, new, and 48 hour criteria. See Table 1 for a complete list of

<table>
<thead>
<tr>
<th>Overall</th>
<th>Current</th>
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<th>48 Hour</th>
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<td>TOnCurr</td>
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<td>RdO</td>
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Table 1. Destroyer task criteria.
tasking” (ImpCurr), and “ship’s readiness to complete current tasking” (RdCurr). The new tasking category includes “time needed to travel to the new tasking” (TToNew), “time needed to complete the new task” (TOmNew), “importance of the new tasking” (ImpNew), and “readiness of the ship to perform the new tasking” (RdNew). Finally, the 48 hour category includes “time needed to travel to the scheduled 48 hour tasking” (TT048), “time needed to complete the 48 hour tasking” (TOm48), “importance of the 48 hour tasking” (Imp48), and “readiness of the ship to perform the 48 hour tasking” (Rd48). For each alternative destroyer, all of the criteria are rated with a score in the interval [0,1].

It is assumed that there are 6 tasks which the destroyers may be called upon to perform. The tasks, in order of importance, are as follows: 1. Ballistic Missile Defense (BMD); 2. Anti-Submarine Warfare (ASW); 3. Force Protection (FP); 4. Anti-Surface Warfare (ASuW); 5. In-Port (IP); 6. In Transit or Available (A).

A baseline score of 0.5 is assumed for criteria RdO, NetOps, C&P, RdCurr, RdNew, and Rd48. Here a baseline readiness score would lead one to expect the corresponding task to be performed close to the stated performance time. A readiness score greater than the baseline indicates that one may expect quicker performance for the corresponding task. A score below the baseline value means slower task performance. For example, if a destroyer’s current readiness, RdCurr, is 0.6, one would expect a decrease in the amount of time it would take the destroyer to complete the current task.

For simplicity the scores on criteria that entailed time estimates were reduced to just three states: short, medium, and long. For tasks, the specified short, medium, or long durations would be expected when the corresponding readiness contained the baseline value of 0.5.

It will be assumed that some subset of combinations using short, medium, and long durations will be used to generate our alternative scenarios. A sample alternative choice can be seen in Fig. 2 where DDG1 and DDG2 are currently performing a BMD task. The duration TToNew for DDG1 is long but the same duration for DDG2 is medium. From the new task to the 48 hour task, the duration for DDG1 is medium while duration TTo48 for DDG2 is long. Given the three durations and the two travel times of the scenario, there are 36 possible pairs of combinations (9 choose 2). In addition, there are 3 pairs where the durations are equal for both the first and second legs of the trip. Thus there are a total of 39 distance combinations.

Once the alternative scenarios are determined, expert ranking will be used to determine the relative importance (weight) of each criterion and the degree to which they interact. In previous research, DiVita and Morris [4] demonstrated that an approach proposed by Marichal and Roubens [3] could be implemented to successfully obtain the weights necessary to compute the Choquet integral. The key to the approach is that experts are asked to make pair-wise comparisons of the relative importance of alternatives and criteria along with a determination of the pair-wise criteria interaction. This data is used to generate a linear program whose solution is the set of weights for each criterion and subsets of criteria that are necessary to compute the Choquet integral.

B. Procedure

In order to solve the general problem with $n$ criteria, $2^n$ coefficients (Mobius weights) must be determined. For $n \geq 3$, this would require experts to make judgments concerning the interaction of sets of three or more criteria. Such judgments are difficult and often unnatural for the expert to make. Experts are comfortable making judgments about the interaction of pairs of criteria (see [3] for details). Therefore a 2-order model approximation of the Choquet integral

$$C_\mu(x) = \sum_{i \in N} a(i) x_i + \sum_{\{i,j\} \subseteq N} a(i,j) (x_i \wedge x_j), \quad (1)$$

is utilized where $x \in \mathbb{R}^n$ is an alternative profile. The number of unknown coefficients is reduced to $n + \binom{n}{2}$. For 14 criteria, only 105 coefficients are required compared to 16384 coefficients for the general problem.

For the 2-order model approximation, the Mobius boundary and monotonicity conditions become:

$$a(\emptyset) = 0,$$

$$\sum_{i \in N} a(i) + \sum_{\{i,j\} \subseteq N} a(i,j) = 1, \quad [1]$$

$$a(i) \geq 0, \forall i \in N, \quad [n]$$

$$a(i) + \sum_{j \in T} a(i,j) \geq 0, \forall i \in N, \forall T \subseteq N \setminus i. \quad [n(2^{n-1})-1]$$

The number of constraints each condition generates is listed beside the condition. Again, assuming 14 criteria, boundary and monotonicity conditions lead to 114,689 constraints.

Beyond these Mobius constraints, 3 additional sets of constraints encapsulate expert knowledge of the decision making process. These sets include expert alternative ranking, expert criteria ranking, and expert opinion on criteria interaction. In each case a partial ranking may suffice to establish a linear program which embodies the alternative decision-making process. Expert knowledge is collected by requesting a subject matter expert to rank pair-wise comparisons of alternatives and criteria. The expert is also asked to provide criteria interaction data.

To generate a constraint from alternative ranking data, assume that alternative $a_1$ is preferred over $a_2$, then

$$C_\mu(a_1) - C_\mu(a_2) \geq \delta + \varepsilon$$
is appended to the set of alternative constraints. Here \( C_\mu(a_1) \) and \( C_\mu(a_2) \) represent the 2-order model approximation (1) of alternatives, \( \varepsilon > 0 \) is the optimization variable, and \( \delta \) is a positive tuning threshold.

For criteria constraints, if criterion \( c_j \) is preferred to criterion \( c_k \) then \( a(j) - a(k) \geq \varepsilon \) is added to the set of criteria constraints. The notation \( a(j) \) indicates the Mobius representation value of criterion \( c_j \).

Finally, a criteria interaction constraint is produced when two criteria are correlated. Hence the constraint \( a(i,j) \leq \varepsilon \) is included in the interaction set of constraints. Here \( a(i,j) \) is the Mobius representation value of the \((i,j)\) pair of criteria. Note that each constraint has one or more unknown Mobius terms.

The unknown weights or coefficients are determined by solving the linear program which maximizes the objective function \((z = \varepsilon)\) constrained by the 4 sets described above.

C. Experiment

In the re-deployment context an example of a pair-wise comparison is illustrated in Fig. 3. A decision maker is asked to choose which of the two destroyers, DDG1 or DDG2 should be assigned the new unscheduled task given the current set of conditions. These conditions are detailed in Table 2. For example, DDG1 is currently performing a high value task of BMD. DDG1 meets the requirement of baseline readiness and preparedness to perform this task and DDG1 is scheduled to complete the current tasking in a relatively short amount of time. In contrast, DDG2 is also performing a BMD task and it too meets baseline readiness requirements to perform the task; however, it is scheduled to complete the current task in a relatively longer period of time. Thus DDG1 will be able to reach the new unscheduled task sooner than DDG2. The problem however is that DDG1 doesn’t meet baseline requirements to perform the new unscheduled ASW task (see Table 2). Thus it should take DDG1 longer than expected to complete the new unscheduled task. Still we expect DDG1 to be able to complete the task and to get to the scheduled 48 hour with some slack time. That is, without missing the start deadline for the 48 hour task.

The problem is that there is more variability, when compared to baseline, associated with our estimate of when DDG1 will complete the new tasking. So the estimated completion times may be wrong. In contrast, DDG2 will finish its current job later and get to the new task later, but once it gets there it should complete the task in less than average completion time since its readiness to perform the ASW task is above the baseline (0.6). We expect DDG2 to get to its 48 hour task in time. There is less slack time when compared to DDG1, but there is less variability around our estimate as to when DDG2 will finish the new ASW task as opposed to our DDG1 estimate. Note that both the 48 hour tasks for DDG1 and DDG2 are ASuW, so the importance of the scheduled 48 hour task for each destroyer is being held constant. Given the above scenario, one can easily see the numerous permutations. For example, suppose DDG1’s 48 hour task was FP which is more important than DDG2’s ASuW, would that change the destroyer the subject matter expert would assign the new tasking to? In this manner a set of alternatives are generated for the expert to rank.

D. Conclusions

We have detailed a process that will determine whether the Choquet Integral can be used to predict expert ranking of alternative naval assets for re-deployment. The current problem uses fourteen criteria. This requires solving a very large linear program on the order of \(10^5\) constraints x \(10^2\) unknowns. Based on previous results [4] we expect the Choquet Integral to achieve high accuracy when compared directly to actual expert rankings of naval assets.

REFERENCES