Measurement of two-dimensional small angles based on interference fringes

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Abstract – A novel two-dimensional small angles measurement is introduced, which is based on the relation between the interference fringes shape (direction and width) and the angle contained by two lights of Michelson interferometer, and utilizes quadrant Si-photoelectric detector (QPD) as detection device. The status of interference fringes changes from static to dynamic, with modulating the wavelength of the laser-diode (LD) by sawtooth modulation. To ensure that while LD is shining, the change of the wavelength of the LD is linear, this paper adopted sawtooth modulation signal (500Hz) with a DC offset. Using High Speed A/D Convert Card and software to collect analogue signal, and data processing technique to got the swing angle. The test precision is 0.05°calculated by experiment data.

Keywords – interference fringes shape, laser-diode, two-dimensional small angle, saw-tooth modulation, quadrant Si-photoelectric detector

I. INTRODUCTION

The angle measurement has widely applied in the fields of alignment, assembly and precision control. It is conventionally performed by using either autocollimators or interferometers. Besides, the internal reflection method, the ring laser technique as well as the Moire technique are often used. In general, high accuracy and good repeatability for angular displacement measurement are required in many applications. Our research group has proposed a wavelength-modulation interferometer, whose structure is simple and compact. In the interferometer, with wavelength-modulating technique, the status of interference fringes changes from statically to dynamically, and the measurement of 2D small angles is converted to the measurement of direction and width of interference fringes, which can be detected by QPD.

The article is organized as follows, Section 2 describes the measurement principle of 2D small angles and the method how to receiving the static interference fringes by QPD. Some experiments are done in order to analyze the measurement precision in Section 3.

II. PRINCIPLE OF MEASUREMENT

A. Optical principle

Fig.1 shows the principle of measuring two-dimensional small angles, G is a spectroscope, M1, M2 are two reflected mirrors, and M1 was fixed in the pedestal of the instrument.

Fig.1. (a) The measurement principle of two-dimensional small angles, (b) Interference fringes
In Fig.1, the light originating from the LD passes through a spectroscope and is divided into two parts. The two interfering lights are produced by the reflection from the two mirrors M1, M2. When the measured mirror M2 is deviated, or rotated, the interference fringes on QPD will be changed. Therefore, the rotation angle of M2 is calculated by the direction and the width of the interference fringes.

In Fig.1, a fixed coordinate system OXYZ was set up for facilitate calculation, and $\alpha$, $\beta$, $\gamma$ are the angles that M was rotated around X-axis, Y-axis, and Z-axis. The relationship of the direction and width of the interference fringes and the angle of rotation is given as following ($\varphi$ is the rotated angle of the interference fringes).

$$
\begin{align*}
\tan \varphi &= \frac{\alpha}{\beta} \\
b &= \frac{\lambda}{2 \sqrt{\alpha^2 + \beta^2}}
\end{align*}
$$

where $\lambda$ is the wavelength of the LD.

Using Eq.(1), Fig.2 shows the simulations of interference fringes when the measured mirror M2 rotated X-axis and Y-axis with different angles.

Fig.2. The simulations of interference fringes.
(a) $\alpha=0$, $\beta=0$, (b) $\alpha=0.00005$, $\beta=0$, (c) $\alpha=0$, $\beta=0.00005$, (d) $\alpha=0.00005$, $\beta=0.00005$.

From Fig.2, it follows that the interference fringes was totally dark or bright, when M2 had not been rotated. As the mirror M2 is only rotated around X-axis, the direction of the interference fringes is vertical, and the width of the fringes is changed with the angle changing, as shown in Fig.2(b). The direction of the fringes is horizontal when M2 is only rotated around Y-axis. The angle of the interference fringes is 45° as $\alpha = \beta$, as shown in Fig.2(d).

From Fig.2(c) and Eq.1, we know that the rotated angle $\beta$ is only relative to the width $b$ of the interference fringes. The relation between the width $b$ of the interference fringes and the rotated angle $\beta$ is showed in Fig.3.

Fig.3. (a) the relation between $b$ and $|\beta|$, (b) enlarged part of the relation between $b$ and $|\beta|$.

From Fig.3 (b), we can find when the width $b$ change from 1.5mm to 2.0mm, the rotated angle is linearly changing with $b$ increasing and the variation is approximate 15.18". Therefore, in the region $b \in [1.5, 2]$mm, receiving the interference fringes will be best. As the direction of the fringes keeps vertical and the width of the fringes changes, the width of the interference fringes is just depended on the rotated angle. Expanding Eq.(1), the expressions of the rotated angles $\alpha$, $\beta$, the direction $\varphi$ and the width $b$ of the interference fringes are written as follow.

$$
\begin{align*}
|\alpha| &= \frac{\tan \varphi \cdot \lambda}{\sqrt{2} \cdot b \cdot \sqrt{1 + \tan^2 \varphi}} \\
|\beta| &= \frac{\lambda}{\sqrt{2} \cdot b \cdot \sqrt{1 + \tan^2 \varphi}}
\end{align*}
$$

B. The methods of receiving and handling interference fringes

In traditional method, static interference fringes were received by CCD. In this measurement, we utilize a QPD as the detection device. The QPD, which is composed of four photodiode cells, can receive the moving interference fringes. Using sawtooth or triangle signals as the input electric current to modulate the LD, the wavelength $\lambda$ is
changed dynamically with the input current. As a result, the interference fringes are also changed dynamically, and the moving interference fringes can be received by the QPD.

Fig.5 shows the receiving of the interference fringes in QPD. The phase difference is related to the direction and width of the interference fringes, and it can be analyzed by the signals received with the QPD.

![Fig.5.Receiving the information of the interference fringe in QPD](image)

Assuming $S_1$, $S_2$, $S_3$, $S_4$ are respectively the electrical signal received by the first, second, third, and fourth quadrant, $S_{2-1}$ is $S_2$ minus $S_1$, and $S_{4-3}$ is $S_4$ minus $S_3$. Then the phase difference $\varphi_1$ between $S_{2-1}$ and $S_{4-3}$ is given as:

$$\varphi_1 = \frac{a \pi \cos \varphi}{b}$$

(3)

The phase difference $\varphi_2$ between $S_{2-3}$ and $S_{1-4}$ is given by:

$$\varphi_2 = \frac{a \pi \sin \varphi}{b}$$

(4)

where $a$ is the length of photodiodes cells side. From Eqs.(3) and (4), it follows that as the phase difference $\varphi_1$ and $\varphi_2$ have been known, then the direction $\varphi$ and the width $b$ of the interference fringes are easily obtained. And the rotated angle $\alpha$, $\beta$ can be calculated by using Eq.(2).

The static interference fringes will change to be dynamic with the technique of wavelength modulation. In this paper, the LD’s wavelength is modulated by sawtooth signal. The correlation coefficients of two signals received by two adjoining photodiode cells of the QPD can be got by Eq.(10). The phase difference can be calculated by Eq.(13) and the correlation coefficients.

The point on QPD is bright or dark determined by the ratio optical path difference $L$ dividing the wave length $\lambda$. From Fig.(1), we know that if the positions of all the optical elements are not changed, the optical path difference $L$ will be constant. In our designed measurement interferometer, the wavelength of laser is 650nm, and the distance of optical path difference exceeds 50mm. The wavelength of the LD is varying dynamically with the changed input current. As the voltage of LD increases 0.01v, LD’s wavelength will increases 1/100000.

If the voltage increased 0.01v, $L = 50\text{mm}$, $\lambda = 50\text{mm}$, we can obtain:

$$\frac{L}{\lambda} = 76922$$

The interference fringes have periodical change from above expression, and they move one cycle when the voltage of LD increases 0.01v.

To ensure that while the LD is shining, the wavelength of LD was also be altered, and the current signal (converted into voltage signal) of QPD also is linear to the LD’s static and dynamic characteristics, the sawtooth modulation signal is.

$$I(t) = 2.302 + 0.06(t - 0.002k)$$

$$t \in [0.002k, 0.002(k + 1)]$$

(5)

The method can make the interference fringes moving, and also inhibit the interference from background light. The change of the wavelength of LD is linear by sawtooth modulation. And the frequency of the sawtooth-wave signal is 500 Hz, and the sawtooth modulation voltage is showed in Fig.6.

![Fig.6. Sawtooth modulation signal](image)

With sawtooth modulation, signals received by QPD can be expressed as:

$$S_1(t) = S(1 + \beta_1 t)(S_0 + S_1 \cos (t + \alpha_1))$$

$$S_2(t) = S(1 + \beta_2 t)(S_0 + S_1 \cos (t + \alpha_2))$$

$$S_3(t) = S(1 + \beta_3 t)(S_0 + S_1 \cos (t + \alpha_3))$$

$$S_4(t) = S(1 + \beta_4 t)(S_0 + S_1 \cos (t + \alpha_4))$$

(6)

where $S$ is the DC component of the output intensity, $\beta_2$ is the modulation factor of the laser wavelength, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ are respectively the initial phase in QPD, $S_0$ and $S_1$ are respectively the coefficients of the DC component and amplitude of the DC component for the interference signal, with ignoring the intensity modulation. The relations between the phase difference $\varphi_1$, $\varphi_2$ and $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ are given by:

$$\varphi_1 = \frac{\pi + (\alpha_1 + \alpha_2) - (\alpha_3 + \alpha_4)}{2}$$

$$\varphi_2 = \frac{\pi + (\alpha_1 + \alpha_2) - (\alpha_3 + \alpha_4)}{2}$$

(7)
The experiments using QPD receiving signals has been done, and the result is showed in Fig. 7:

![Fig.7. Signals received by the QPD](image)

In Fig. 7, the signal of the light intensity is superposed by sawtooth signal and AC signal. The sawtooth signal is resulted from the light intensity of LD, and the AC signal from modulating LD’s wavelength by sawtooth modulation of LD. The cycles of AC signal in sawtooth signal are the cycles of periodical change of interference fringes. As the distance of optical path difference is longer, the period is larger. Sawtooth signal is the information of the light intensity, is inutility to solving the question of the phrase difference. We can eliminate them by subtraction circuit. \( X_1, X_2 \) and \( Y_1, Y_2 \) are the results of subtraction of two signals in vertical or horizontal line of the photodiode cells, and they given as:

\[
\begin{align*}
X_1 &= S_1(t) - S_2(t) \\
&= S(1 + \beta_1 t)S_1 \{ \cos(t + \alpha_1) - \cos(t + \alpha_2) \} \\
X_2 &= S_1(t) - S_2(t) \\
&= S(1 + \beta_2 t)S_1 \{ \cos(t + \alpha_1) - \cos(t + \alpha_2) \}
\end{align*}
\]

\[
\begin{align*}
Y_1 &= S_1(t) - S_2(t) \\
&= S(1 + \beta_1 t)S_1 \{ \cos(t + \alpha_1) - \cos(t + \alpha_2) \} \\
Y_2 &= S_2(t) - S_1(t) \\
&= S(1 + \beta_2 t)S_1 \{ \cos(t + \alpha_1) - \cos(t + \alpha_2) \}
\end{align*}
\]

With correlation analysis of \( X_1, X_2 \) and \( Y_1, Y_2 \), the correlation coefficients of \( X_1 \) and \( X_2 \), or \( Y_1 \) and \( Y_2 \) will be known. The phase difference can be calculated by the correlation coefficients. The correlation coefficient \( \rho \) is express

\[
\rho = \frac{\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)}{\sqrt{\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)^2 \cdot \sum_{i=1}^{n} (X_{2i} - \bar{X}_2)^2}}
\]

Substitution Eq. (7) into Eq. (8). \( X_1 \) and \( X_2 \) are finally given as:

\[
\begin{align*}
X_1 &= 2S(1 + \beta_1 t)S_1 \sin\left(\frac{\alpha_1 - \alpha_2}{2}\right) \cos\left(\alpha_1 + \phi_1 + \frac{\alpha_1 + \alpha_2}{2}\right) \\
X_2 &= 2S(1 + \beta_2 t)S_1 \sin\left(\frac{\alpha_1 - \alpha_2}{2}\right) \cos\left(\alpha_1 + \phi_1 + \frac{\alpha_1 + \alpha_2}{2}\right)
\end{align*}
\]

When \( n \) was infinite, \( \bar{X}_1 = 0 \) and \( \bar{X}_2 = 0 \) from Eq. (8). With Substituting \( \bar{X}_1 = 0 \) and \( \bar{X}_2 = 0 \) into Eq.(10), the correlation coefficient \( \rho \) can be written as:

\[
\rho = \cos \phi_1
\]

The relation of the coefficient of correlation and the rotated angle \( \beta \) can be obtained. Using Eqs. (13), (4) and (2) and assuming \( \alpha = 0 \), Fig. 8 shows the simulation results.

![Fig.8. (a): Relation of \( \rho \) and \( |\beta| \) (0< b<2mm) (b): Relation of \( \rho \) and \( |\beta| \) (2mm< b)](image)

III. EXPERIMENTS

The experimental setup shown in Fig. 1 was constructed. The light source was an LD with a central wavelength of 650nm and an output power of 2.5mW. The frequency of sawtooth current modulation is 500 Hz, and the distance of optical path difference is 65mm.

In the experiment, using High Speed A/D Convert Card (250K), we acquired the signal of \( X_1, X_2 \) and \( Y_1, Y_2 \) at the same time with \( \alpha = 0 \). Each signal was composed of 60000 data sequences. We divided these data sequences into six parts for research, then each part has 10000 sets of data. According to Eqs. (13) and (12), we
have calculated the relations correlation coefficients $\rho$ and the phase difference $\varphi$ as shown in Table 1.

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<thead>
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<tr>
<td>1</td>
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<tr>
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<td>5</td>
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<tr>
<td>6</td>
<td>-0.74833</td>
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</table>

Table 1: Relations between $\rho$ and $\varphi$

The average and standard deviation of the phase different are: $\overline{\varphi} = 2.415$ rad, $\sigma = 0.000776$ rad. Substituting $\overline{\varphi}$ and $\overline{\varphi} + 3\sigma$ into Eq. (8), we got the width of the interference fringes $b = 2.415047$ mm and the incremental width of the interference fringes $\Delta b = 0.002328$ mm. Using similar methods, the incremental angle $\Delta \beta$ can be calculated to be 0.05". $\Delta \beta$ is the precision of this probe by $3\sigma$ Standards; The test scope respectively are -50"~50" calculated by experiment data.

IV. CONCLUSION

A new measurement for two-dimensional small angles is introduced, which detects 2D small angles through the direction and width of interference fringes shape, utilizes quadrant Si-photoelectric detector (QPD) as detection device. This probe may be implemented and the anticipated accuracy of it may be great by theoretical analysis. The status of interference fringes changes from static to dynamic, with modulating LD’s wavelength by sawtooth modulation. Using High Speed A/D Convert Card (250K) and Software to collect analogue signal, and correlation analysis technique to get the angle of rotation. The measurement is characterized by strong anti-interference capability, simple construction, do nothing with the intensity of light and the noise outside. The test precision and scope respectively are 0.05" and -50"~50" caculated by experiment data.

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