Extremum Seeking for Parameter Identification, Implementation for Electron Beam Property Prediction

Alexander Scheinker and Spencer Gessner

Abstract—We report on an experiment performed at the Facility for Advanced Accelerator Experimental Tests (FACET), which is the first two kilometers of the Stanford Linear Accelerator Center (SLAC) linear accelerator, in which a new form of extremum seeking, one with known, bounded update rates, despite operating on an analytically unknown cost function, was utilized in order to provide a real time bunch length estimate of the electron beam. The approach was to simultaneously tune fourteen parameters, such as arbitrary klystron phase shifts and electron bunch energy, in order to match a simulated (LiTrack) bunch energy spread spectrum with the real time wiggler/scintillating YAG crystal signal. The simple adaptive scheme was digitally implemented using Matlab and the Experimental Physics and Industrial Control System (EPICS). The main result is the development of a non-intrusive, non-destructive real-time diagnostic scheme for prediction of bunch length, as well as other beam parameters, the precise control of which is very important for the plasma acceleration scheme being explored at FACET.

I. INTRODUCTION

A. Motivation

The Facility for Advanced Accelerator Experimental Tests (FACET) at the Stanford Linear Accelerator Center (SLAC) produces high energy electron beams for plasma wakefield acceleration [1]. For these experiments, precise control of the longitudinal beam profile is very important. A number of bunch length diagnostics are employed at FACET, including beam streaking with an x-band transverse deflecting cavity (TCAV), pyrometer measurement of optical transition radiation (OTR), produced by passing the relativistic electrons through regions of materials with varying dielectric constants, and bunch energy spread spectrum measurements with the real time wiggler/scintillating YAG crystal signal. Of the mentioned techniques, the TCAV measurement is most accurate, but is a time-consuming process, furthermore, both the TCAV and OTR approaches are destructive methods. The bunch energy spread measurement is the only real time approach which allows for an electron bunches’ length to be measured and for that same bunch to continue on to the plasma acceleration experiment, allowing for a study of the correlation between bunch properties and experimental results. To calculate the bunch length based on a bunch energy spectrum measurement, the detected spectrum is compared to a simulated spectrum created with the 2D longitudinal particle tracking code, LiTrack [2]. The short electron bunches desirable for plasma wakefield acceleration provide wide, non-gaussian spectra, which can uniquely be correlated to bunch length, if all of the various accelerator parameters which influence the bunch length and energy are accounted for accurately. Unfortunately, throughout the 2km facility, there exist systematic phase shifts of various high frequency devices, mis-calibrations, and time-varying uncertainties due to thermal drifts. Therefore, in order to effectively, accurately relate an energy spectrum to a unique bunch length, a very large parameter space must be searched and fit by LiTrack, which effectively limits and prevents the use of the energy spectrum measurement as a real time measurement of bunch profile.

At FACET, we coupled the above described technique with a new version of an extremum seeking (ES) algorithm, in order to provide a real-time estimate of the bunch profile [2], by adaptively identifying and tracking the many uncertain, time-varying parameters required by the LiTrack code. For the ES algorithm, the cost to be minimized was the $\chi^2$ residual between the measured (YAG) and simulated (LiTrack) spectra of the electron bunch. System parameters such as various arbitrary phase shifts and beam properties ($\beta$, dispersion) were the inputs to LiTrack. The adaptive scheme minimized the cost by varying an arbitrary number or parameters simultaneously. We simulate FACET with fourteen free parameters in code package called LiTrackES.

The form of ES described here is related to the scheme originally developed in 1922 [3], and first analytically studied in the 50s and 60s [4], [5], [6]. In particular, Meerkov’s 1967 analysis [7], utilizing quasi-steady states and asymptotic averaging [8] set the way for a study of the local and global stability of the scheme [9]. As an output optimizer for stable, controlled systems, ES has continued to develop throughout the years, examples include adaptive application for systems with parametric uncertainties [10], and time-varying controllers for discrete-time systems [11]. A broad overview of development is available in [12].

While ES saw many applications as an optimization scheme, for a-priori stable, controlled systems, it was recently modified, in order to apply it for the stabilization of uncertain, open-loop unstable systems [13]. The mechanism behind this new form of ES, that of introducing high frequency oscillations into a system’s dynamics, is closely related to the field of vibrational control, such as stabilizing...
the vertical equilibrium point of a pendulum by quickly oscillating its pivot point [14], vibrational control in general has been an active area of research [15], [16], [17].

The study of the dynamics of the new ES controllers, for stabilization, has benefited from results pertaining to highly oscillatory systems [18], [19], [20], [21], along with results relating the stability of highly oscillatory systems to their averages [22]. The new form of ES has been demonstrated as a tool for the stabilization of open-loop unstable, uncertain systems [23], including a pendulum’s vertical equilibrium point [24]. Also, a modified, non-differentiable form has been developed, in which has the benefit that the system’s control efforts settle to zero as equilibrium is approached [25].

B. Overview of Main Results

Recently, a new form of ES [26], [27], [28] has been developed, one with analytically guaranteed update rates and control efforts, due to unknown cost function entering the scheme’s dynamics as the argument of a sine or cosine functions. In this work, we apply this new form of ES to the standard parameter estimation problem [29], we consider a system of the form

\[
\begin{align*}
\dot{x} &= f(x, p, t) \\
\dot{p} &= \hat{f}(\hat{x}, \hat{p}, t)
\end{align*}
\]

(1)

where \(f(x, p, t)\) represents the actual, uncertain dynamics of a system being studied, with uncertain parameters \(p(t)\), and \(\hat{f}(\hat{x}, \hat{p}, t)\) represents a simulation of the system’s dynamics, where the actual system parameters, \(p(t)\), are approximated by virtual parameters \(\hat{p}(t)\), which are typically referred to as observers. Our goal is to identify the parameters \(p(t)\), by adaptively tuning the virtual parameters \(p(t)\), based on a cost function, \(C\), whose values depend on the comparison between an actual, uncertain, noisy measurement \(y = h(x, t)\) and a simulation of that same measurement \(\hat{y} = \hat{h}(\hat{x}, t)\). The adaptive law is then chosen as

\[
\dot{\hat{p}} = \sqrt{\alpha_i \omega_i} \cos (\omega_i t + k_i C(y(x, p), \hat{y}(x, \hat{p}))),
\]

(3)

which, according to the results of [26], [27], [28] has average dynamics

\[
\dot{\hat{p}} = -\frac{k_i \alpha_i}{2} \frac{\partial C}{\partial \hat{p}},
\]

(4)

performing a gradient descent to a locally minimizing value of \(C\), in which case the output of the simulation matches that of the real system. Furthermore, depending on the actual analytic forms of \(C\), \(h\), and \(f\), such a convergence may provide a prediction of actual parameter values \(p_t\), based on their virtual observers \(\hat{p}_t\).

Remark 1: The \(\cos(\cdot)\) terms in (3) may be replaced by \(\sin(\cdot)\) functions, or the two can be mixed together. The only requirement, for the convergence of the scheme, is that the perturbing functions are orthogonal in the frequency domain, such as \(\sin(\omega_i)\) and \(\sin(\omega_j)\), where \(\omega_i \neq \omega_j\), or \(\cos(\omega_i)\) and \(\sin(\omega_i)\).

II. BACKGROUND ON AVERAGING RESULTS AND SEMIGLOBAL PRACTICAL ULTIMATE BOUNDEDNESS

A. Averaging results

We start with the following averaging result for highly oscillatory periodic systems, as developed in various forms and generalizations in [18], [19], [20], [21].

\[\begin{align*}
\dot{x} &= f(x, t) + \sum_{i=1}^{n} g_i(x, t)(\omega_i t) \cos ((\omega_i t)^2 + t) \\
&- \sum_{i=1}^{n} h_i(x, t)(\omega_i t) \sin ((\omega_i t)^2 + t),
\end{align*}\]

(5)

where \(r_i \neq r_j, \forall i \neq j\), and the functions

\[
\begin{align*}
f(x, t) &\in \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n \\
g_i(x, t) &\in \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n \\
h_i(x, t) &\in \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n
\end{align*}
\]

are continuous and Lipschitz, and \(D f_i, D^2 f_i, \partial g_i / \partial x, \partial h_i / \partial x, \sum_i \partial h_i / \partial x \in \mathbb{R}^{n \times n}, \sum_i \partial h_i / \partial x \in \mathbb{R}^{n \times n} \) are continuous and bounded, where \(D = \frac{\sqrt{\nu}}{\pi} \). For \(T \in [0, \infty)\), and any compact set \(K \subset \mathbb{R}^n\) for any \(\nu, \delta > 0\), there exists \(\omega^*\) such that for all \(\omega > \omega^*\), the distance between the trajectory \(\bar{x}(t)\) of the system

\[
\dot{x} = f(\bar{x}, t) - \frac{1}{2} \sum_{i \neq j} \left( \frac{\partial h_i}{\partial x_i} g_i - \frac{\partial g_i}{\partial x_i} h_i \right), \quad \bar{x}(0) = x(0),
\]

(9)

and \(x(t)\) of system (5) has bound

\[
\max_{t \in [0, T]} |x(t) - \bar{x}(t)| < \delta.
\]

B. Stability Results of Moreau and Aeyels

We recall the following definitions as in Moreau and Aeyels [22]. In what follows, given a system

\[
\dot{x} = f(t, x),
\]

(11)

\(\psi(t, t_0, x_0)\) denotes the solution of (11) which passes through the point \(x_0\) at time \(t_0\). In conjunction with (11), we consider systems of the form

\[
\dot{x} = f(t, x),
\]

(12)

whose trajectories are denoted as \(\psi^\epsilon(t, t_0, x_0)\).

Definition 1: Converging Trajectories Property: The systems (11) and (12) are said to satisfy the converging trajectories property if for every \(T \in (0, \infty)\) and compact set \(K \subset \mathbb{R}^n\) satisfying \(\{(t_0, t_0, x_0) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n : t \in [t_0, t_0 + T], x_0 \in K \} \subset \text{Dom} \psi\), for every \(\nu, \delta > 0\) there exists \(\epsilon^*\) such that for all \(t_0 \in \mathbb{R}\), for all \(x_0 \in K\) and for all \(\epsilon \in (0, \epsilon^*)\),

\[
|\psi^\epsilon(t_0, t_0, x_0) - \psi(t_0, t_0, x_0)| < \epsilon, \quad \forall t \in [t_0, t_0 + T].
\]

(13)

Definition 2: \(\epsilon\)-Semiglobal Practical Uniform Asymptotic Stability (\(\epsilon\)-SPUAS): The origin of (12) is said to be \(\epsilon\)-SPUAS if it satisfies the following three conditions:

- Uniform Stability: For every \(c_2 \in (0, \infty)\) there exists \(c_1 \in (0, \infty)\) and \(\epsilon \in (0, \infty)\) such that for all \(t_0 \in \mathbb{R}\) and
for all $x_0 \in \mathbb{R}^n$ with $\|x_0\| < c_1$ and for all $\epsilon \in (0, \bar{\epsilon})$, 
\[ \|\psi^\epsilon(t, t_0, x_0)\| < c_2 \quad \forall t \in [t_0, \infty). \] (14)

- **Uniform Boundedness:** For every $c_1 \in (0, \infty)$ there exists $\epsilon_1 \in (0, \infty)$ and $\bar{\epsilon} \in (0, \infty)$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $\|x_0\| < c_1$ and for all $\epsilon \in (0, \bar{\epsilon})$, 
\[ \|\psi^\epsilon(t, t_0, x_0)\| < c_2 \quad \forall t \in [t_0, \infty). \] (15)

- **Global Uniform Attractivity:** For all $c_1, c_2 \in (0, \infty)$ there exists $\bar{T} \in (0, \infty)$ and $\bar{\epsilon} \in (0, \infty)$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $\|x_0\| < c_1$ and for all $\epsilon \in (0, \bar{\epsilon})$, 
\[ \|\psi^\epsilon(t, t_0, x_0)\| < c_2 \quad \forall t \in [t_0 + \bar{T}, \infty). \] (16)

With these definitions the following result of Moreau and Aeyels [22] is used in the analysis that follows.

**Theorem 2 ([22]):** If systems (12) and (11) satisfy the converging trajectories property and if the origin is a GUAS equilibrium point of (11), then the origin of (12) is $\epsilon$-SPUAS.

**Corollary 1:** If the origin of system (9) is GUAS, then the origin of system (5) is $\frac{1}{\omega}$-SPUAS.

**Proof:** By Theorem 1 the solutions of (5) and (9) satisfy the converging trajectories property for any $T \in [0, \infty)$. Since the origin of (9) is GUAS, by Theorem 2, the origin of (5) is $\frac{1}{\omega}$-SPUAS.

### III. Adaptive Tuning Method

Figures 1, 2, and 3 show the overall setup of the tuning procedure. A simulation of the accelerator, LiTrack, is run in parallel to the machines operation. The simulation was initialized with the best guess and any available measurements of machine settings (even measured values have arbitrary phase shift errors). The electron beam in the actual machine was accelerated and then passed through a series of deflecting magnets, as shown in Figure 2, which created x-rays, whose frequency distribution can be correlated to the electron bunch density. This non-destructive measurement is available at all times, and used as the input to the ES scheme, which is then matched by adaptively tuning machine parameters in the simulation.
Each parameter setting had its own influence on electron bunch dynamics, which in turn influenced the separation, amount of charge in, emittance, length, etc, of the leading and trailing electron bunches. Bunch properties correlated to an energy spectrum which was detected in the transverse deflecting cavity.

The parameters which were simultaneously tuned, in order to match the simulation’s output to that of the machine readings were:

- $p_1$: Electron beam bunch length
- $p_2$: Initial energy spread
- $p_3$: Number of particles
- $p_4$: Initial Gaussian asymmetry
- $p_5$: Amplitude of RF compressor
- $p_6$: RF compressor phase
- $p_7$: RTL compression
- $p_8$: RTL second order compression
- $p_9$: Klystron numbers 2-10 phase
- $p_{10}$: Klystron numbers 11-20 phase
- $p_{11}$: Ramp phase

$\hat{\psi}(f, p(n))$ was the actual, time-varying (due to phase drift, thermal cycling...) detected spectrum, and $\psi(f, p(n))$ was the LiTrack, simulated spectrum, which depends on parameter settings $p(n)$.

The problem was then to locate the minimum of the function $C(p, t) : \mathbb{R}^n \times \mathbb{R}^+ \rightarrow \mathbb{R}$, for $p = (p_1, \ldots, p_n) \in \mathbb{R}^n$, which we can measure the value of, but whose analytic form is unknown. The hope was that, by finding simulation machine settings which resulted in matched spectrums, we would also match other properties of the real and simulated beams, such as bunch length, something we could not simply do by setting the simulation parameters to the exact machine settings, due to unknowns, such as time-varying, arbitrary phase shifts.

The first step of the adaptive scheme was to choose physically realizable constraints for all parameters:

$$p_{\text{max}} = (p_{1,\text{max}}, \ldots, p_{m,\text{max}}),$$

$$p_{\text{min}} = (p_{1,\text{min}}, \ldots, p_{m,\text{min}}).$$

Implementing initial parameter settings $p(1)$, which are chosen based on the physics model and experience, allowed us to calculate $C(p(1))$. The iterative update scheme was...
then:
\[ p_i(n+1) = p_i(n) + \Delta \sqrt{\alpha} \cos (\omega_i n \Delta + kC(p(n))) , \] (18)
which is based on the finite difference approximation of the derivative:
\[ \frac{p_i(t + \Delta) - p_i(t)}{\Delta} \approx \frac{\partial p_i}{\partial t} = \sqrt{\alpha} \cos (\omega_i t + kC(p(t), t)) , \] (19)
which we may expand as
\[ \cos (\omega_i t + kC) = \cos (\omega_i t) \cos (kC) - \sin (\omega_i t) \sin (kC) \] (20)
and rewrite the \( p_i \) (\( 1 \leq i \leq n \)) dynamics as
\[ \dot{p}_i = \sqrt{\alpha} \cos (\omega_i t) \cos (kC) - \sqrt{\alpha} \sin (\omega_i t) \sin (kC) , \] (21)
which, according to Theorem 1, resulted in an average parameter and cost relationship of the form:
\[ \dot{p}_i = -\frac{k \alpha}{2} \frac{\partial C (\vec{p}, t)}{\partial \vec{p}_i} (\cos^2 (kC (\vec{p}, t)) + \sin^2 (kC (\vec{p}, t))) \]
\[ = -\frac{k \alpha}{2} \frac{\partial C (\vec{p}, t)}{\partial \vec{p}_i} , \] (22)
and therefore
\[ \dot{\vec{p}} = -\frac{k \alpha}{2} \nabla C , \] (23)
which is a gradient descent towards the minimum of \( C \).

The constraints were simply implemented by checking the updated parameters at each step and confining them to their

\[ p_{i,n} = \frac{2(p_i(n) - C_{p,i})}{D_{p,i}} , \] (24)

Fig. 5. The scaled parameters, confined to \( \pm 1 \) are kept within their respective bounds as they are perturbed and adapted in order to minimize the cost.

Fig. 6. The LiTrack prediction can be seen roughly following the trajectory of the destructive, lengthy TCA V measurement over the course of several hours.
where $C_{p,i} = \frac{p_{i,\text{max}} + p_{i,\text{min}}}{2}$ and $D_{p,i} = p_{i,\text{max}} - p_{i,\text{min}}$, bounding each parameter within $[-1, 1]$. We then performed the RR-update on the scaled parameters

$$p_{s,i}(n+1) = p_{s,i}(n) + \Delta \sqrt{T_{\text{RR}}} \cos(\omega_i n \Delta + k_i C(p(n))) ,$$

while forcing the scaled parameters to satisfy the constraints $-1$ and $1$. Finally, we transformed back into unscaled parameter values in order to calculate the cost for the next iteration:

$$p_i(n+1) = \frac{p_{s,i}(n+1) D_{p,i}}{2} + C_{p,i} .$$

Typical behavior of the scaled parameters and cost, during simulation, are shown in Figure 5.

In our implementation of the scheme, the entire data-gathering, simulation, parameter update loop had a period of approximately 0.6 seconds. As the ES scheme tuned parameters, constantly attempting to match the simulations output spectrum to that detected by the TCAV, a real time bunch-length prediction was provided.

IV. RESULTS

The algorithm and LiTrack were initialized with parameters that are assumed to be close to the real machine parameters. In general, the initial guess does not provide an accurate estimate of the measured energy spectrum, but the adaptive scheme evolves the simulation parameters to find the match. On average, it took about 500 iterations of the algorithm (roughly 5 minutes) to find a match. Once a match was found, the system was stabilized and remained locked to the measured energy spectrum. LiTrackES was found to be in agreement with the TCAV and pyrometer. The obvious advantage of LiTrackES over the other methods was that LiTrackES provided real time bunch profile. The pyrometer is sensitive to peak current rather than bunch profile and the TCAV cannot be used every shot. The experimental setup is shown in Figure 1. The results of the run are shown in Figure 6.

V. CONCLUSIONS AND FUTURE WORK

ES, together with LiTrack, as demonstrated, is now available full time in the control room of FACET, providing a real time estimate of the electron bunch length. We plan on expanding this procedure to describing other machine and beam characteristics and hope to one day use it as an actual feedback to the machine settings in order to tune electron beam properties.

REFERENCES

