Air-to-Air Missile Vector Scoring

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Abstract—An air-to-air missile vector scoring system is proposed for test and evaluation applications. A linear six-state constant velocity (CV) dynamics model is used, consisting of missile position and velocity in a Cartesian coordinate system. Frequency modulated continuous wave (FMCW) radar sensors, carefully located to provide spherical coverage around the target, provide updates of missile kinematic information relative to a drone aircraft. Data from the radar sensors are linearized about a nominal measurement and fused with missile model predictions using an extended Kalman filter (EKF) algorithm. The performance of the system is evaluated through high-fidelity, six-degree of freedom (6DOF) simulations yielding sub-meter end-game accuracy in a variety of scenarios.

I. INTRODUCTION

The challenging task of estimating the navigation parameters of a missile has numerous applications. By accurately tracking an inbound missile, aircraft can dispense countermeasures at the critical moment and perform evasive maneuvers [1]. Additionally, a precise estimate of a missile’s flight path is vital for missile test and evaluation to ensure functionality and accuracy of weapons. In particular, the missile trajectory relative to the target aircraft is vital just prior to intercept.

This research proposes an end-game missile scoring system which relies on short range radio frequency (RF) sensors to update missile position and velocity just prior to intercept. Using this approach, multiple, active directional antennas are installed on a drone aircraft transmitting in a spherical pattern. During a live-fire test, the sensors’ measurements may be broadcast to the ground as an air-to-air missile approaches and passes the drone.

This paper utilizes an extended Kalman filter (EKF) to translate the kinematic measurements from the RF sensors into estimates of the missile’s position and velocity. Essentially, a target aircraft will be equipped with sensors which measure range and radial velocity of an incoming air-to-air missile. The target platform and sensor locations upon the target platform are assumed to be known. The air-to-air missile’s position is passed from an external tracking source with some uncertainty (discussed in Section IV).

The missile’s position is improved near the intercept point through an EKF based upon a constant velocity (CV) model of the missile dynamics and using relative range and radial velocity measurement updates.

In the remainder of the paper, Section II discusses the RF sensor type and location on the drone aircraft. Section III presents the missile dynamics model applied in the research along with the observation model for the sensor measurements. Section IV outlines the EKF algorithm. Section V discusses the approach applied to limit the impact of radar clutter. Section VI describes the truth model used to assess scoring performance. Section VII examines the performance for several target and missile geometries.

II. AIRCRAFT SENSOR CONFIGURATION

There are two categories of radar sensors capable of providing range and range-rate information: pulse-Doppler (PD) and Frequency Modulated Continuous Wave (FMCW) radar. PD radars are less suitable for short range applications due to a blind zone in close proximity to the sensors. Therefore, the results for this research are based on the performance of an existing automotive FMCW sensor with a range of 350 meters, a one-sigma range resolution of 0.01 × range and a range-rate resolution of 0.25 meters per second [2].

In order to overcome the short range of the radar sensors and high closure speed of the missile, each antenna transmits energy uniformly throughout its field of view rather than employing a sweeping pattern. The downside is measurements are limited to range and radial velocity, and no angular information is exploited.

The geometry of the sensor configuration is critical to the precision of state estimates. Missile position updates are based upon the principle of multilateration whereby range measurements from three or more sensors are used to calculate a 3-dimensional (3D) missile location [3]. Multilateration, which is also employed by Global Positioning System (GPS), is sensitive to changes in the geometry of the sensors relative to the target. For GPS the position dilution of precision (PDOP) quantifies the increase in position uncertainty caused by a suboptimal satellite configuration [4]. The lowest PDOP is achieved when the satellites are distributed uniformly about the target [4]. Unfortunately, the placement of radar sensors on the target aircraft is constrained by the dimensions of the aircraft. However, the general principle of maximizing the angular spacing of the sensors as viewed...
from the incoming missile still applies. The same concept is relevant to velocity calculation.

This research assumes an F-16 aircraft is the platform for the vector scoring system. This aircraft has an approximate length and wingspan of 16 meters and 10 meters, respectively. To reduce errors in missile navigation states, seven antennas are located on the aircraft as follows: one directional antenna on the top and bottom of the nose section, one directional antenna on the top and bottom of each wingtip and an omnidirectional antenna on the aircraft tail. Using this configuration, missile trajectories that approach in-plane with a wings level aircraft will create problems for scoring, but any trajectories from above or below will have excellent sensor visibility.

III. System Model

A Kalman Filter is a commonly used recursive, data processing algorithm which provides statistically optimal estimates of the states of a stochastic system. Implementing a KF requires the development of a system dynamics model and observation model. The dynamics model is designed to capture the typical behavior of the system in order to predict the changes in states of interest between measurement updates. The observation model provides the mathematical relationship between measurements and system states required to improve missile estimates with sensor data. Utilizing these models the KF updates the state estimates by optimally weighting the dynamics and observation models according to their uncertainties.

A. Dynamics Model

This research applies a basic CV linear model to predict missile motion between measurement updates. Using a Cartesian coordinate frame, the CV model incorporates six navigation states to characterize the missile position and velocity yielding the state vector

\[
x(t) = [x \ y \ z \ vx \ vy \ vz]^T \tag{1}
\]

The general continuous time linear form of the CV dynamics model is [5]

\[
\dot{x}(t) = Fx(t) + Gw(t) \tag{2}
\]

where

\[
F = \begin{bmatrix}
0_{3x3} & I_{3x3} \\
0_{3x3} & 0_{3x3}
\end{bmatrix} \tag{3}
\]

\[
G = \begin{bmatrix}
0_{3x3} \\
I_{3x3}
\end{bmatrix} \tag{4}
\]

and the strength of the noise vector \(w(t)\) is defined by

\[
E[w(t)w(t + \tau)] = Q = \begin{bmatrix}
q & 0 & 0 \\
0 & q & 0 \\
0 & 0 & q
\end{bmatrix} \tag{5}
\]

\[1\] Continuous acceleration and 3D coordinated turn models were also evaluated, but did not demonstrate superior performance and required more states leading to greater computational burden.

The variable \(q\) is adjusted during filter tuning to improve performance.

In an inertial reference frame, the CV model assumes a constant velocity along each axis while acceleration along each axis is modeled by an independent, zero-mean, Gaussian, white noise [5]. For this research the propagation time steps are only 10 msec, and the missile is tracked for a short duration of less than one second. Therefore, a flat Earth is assumed and missile propagation is performed in a local-level navigation frame with an origin on the surface of the Earth.

B. Observation Model

Range and range-rate measurements are a nonlinear function of system states. Therefore, this research utilizes a nonlinear measurement model with additive, independent, zero-mean, Gaussian, white noise in the general form [6]

\[
z_k = h[x_k] + v_k \tag{6}
\]

where \(z\) is the measurement vector, \(h[\cdot]\) is a nonlinear operator, \(v\) is the noise vector, and the subscript \(k\) indicates the time index of the measurement.

The range measurement from the \(i\)th sensor is related to the system states according to

\[
ri = \sqrt{(x - xi)^2 + (y - yi)^2 + (z - zi)^2} \tag{7}
\]

The sensor coordinates, \([xi yi zi]\), are easily defined in the aircraft body frame, but must be converted into the same reference frame as the missile state vector. In this research, a local level Earth-fixed navigation frame is used exclusively.

In addition, radar radial velocity measurements from the \(i\)th sensor are defined by

\[
v_i = \frac{(vx - xi)(y - yi) + (vy - yi)(z - zi) + (vz - zi)(x - xi)}{\sqrt{(x - xi)^2 + (y - yi)^2 + (z - zi)^2}} \tag{8}
\]

IV. Extended Kalman Filter

A conventional KF is inadequate to deal with the non-linearities present in the system observation model. An EKF deals with this issue by linearizing about a nominal measurement. The linearization is accomplished by using a first-order Taylor series approximation.

For the simulations performed in Section VII, the EKF is initialized using missile position and velocity information from an external source. Most air-to-air missile test and evaluation ranges include a system which monitors missile position for safety. For example, missile testing performed at Tyndall AFB utilizes the Gulf range drone control system (GRDCS) [7]. This system provides position accuracy of approximately 15 meters in the \(x\) and \(y\) axis and 45 meters in the \(z\)-axis. The data update rate is 20 Hz and velocity is determined by calculating the change in position over one time step. Based on the GRDCS system the missile state vector is set equal to truth with a random position and velocity error added to each axis from a zero-mean normal
distribution with the following standard deviations: $\sigma_x = 15$, $\sigma_y = 15$, $\sigma_z = 45$, $\sigma_{vx} = 10$, $\sigma_{vy} = 10$ and $\sigma_{vz} = 10$. Furthermore, the initial covariance matrix is set according to the GRDCS uncertainty as

$$P_0 = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{vx}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{vy}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{vz}^2 \end{bmatrix}$$ (9)

An alternate approach is to initialize the target purely based on sensor measurements through multilateration. A downside to this approach is it requires three measurements. If target detection probability is low, the multilateration procedure will not have sufficient information to initialize the target state vector [3]. Alternatively, if the sensors see large numbers of ghost targets, the multilateration process requires the tracking of multiple target hypotheses. Track scoring is one possible technique for making decisions on which track is the true target. However, in a multiple target and high clutter environment this approach becomes computationally burdensome. Fortunately, in target scoring applications with short range sensors, a low clutter environment is probable.

### A. Propagation

Since the dynamics model is linear, conventional KF algorithms are applied to propagate the state estimate. Using the Van Loan [8] method, (2) is translated into a the equivalent discrete time difference equation

$$x_{k+1} = \phi x_k + G w_k$$ (10)

where the state transition matrix, $\phi$, is

$$\phi = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ (11)

and the variable $T$ represents the propagation time step of 10 msec which is determined by the sensor measurement rate. Moreover, the discrete noise strength matrix is

$$Q_d = \begin{bmatrix} \frac{T^3}{3} & 0 & 0 & \frac{T^2}{2} & 0 & 0 \\ 0 & \frac{T^3}{3} & 0 & 0 & \frac{T^2}{2} & 0 \\ 0 & 0 & \frac{T^3}{3} & 0 & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & 0 & T & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 & 0 & T & 0 \\ 0 & 0 & \frac{T^2}{2} & 0 & 0 & T \end{bmatrix} q$$ (12)

Using these matrices, the *apriori* state estimate, $\hat{x}_{k+1}^-$, and uncertainty, $P_{k+1}^-$, are calculated according to [6]

$$\hat{x}_{k+1}^- = \phi \hat{x}_k$$ (13)
\[
\hat{x}_{k+1}^+ = \hat{x}_{k+1}^- + K_{k+1} \delta z 
\]

(28)

\[
P_{k+1}^+ = P_{k+1}^- - K_{k+1} H_{k+1} P_{k+1}^-
\]

(29)

where \( K_{k+1} \) is the Kalman gain defined by \[6\]

\[
K_{k+1} = P_{k+1}^- H_{k+1}^T [H_{k+1} P_{k+1}^- H_{k+1}^T + R]^{-1}
\]

(30)

This algorithm is repeated recursively to update the missile navigation states.

V. GATING AND DATA ASSOCIATION

The radar sensors for the missile scoring system are unlikely to see a large number of false returns in their operating environment for several reasons. First of all, the FMCW sensors are short range so the volume of airspace they observe is extremely limited. Secondly, a drone aircraft equipped with a missile scoring systems for missile test and evaluation is generally operated in visual meteorological conditions for safety. However, the algorithm used in this research does include two features to limit the impact of false radar returns: gating and data association.

Gating approaches the task of distinguishing true target observations from clutter by evaluating the distance between the expected target location and each measurement. Two types of gating, square and ellipsoidal, are applied sequentially in our missile scoring algorithm. Square gating is applied as a computationally cheap method to quickly eliminate observations far from the expected track location. To perform square gating the maximum eigenvalue of the residual covariance scaled by the selected gate size, \( \gamma \), is evaluated according to \[5\]

\[
e_{max} = \sqrt{\text{max}(\text{eig}(\gamma S))}
\]

(31)

The gate size is selected to achieve a desired probability that a true observation will fall within the selected gate. For example, choosing \( \gamma = 9.2 \) provides a 99 percent chance that the true observation is within the gate \[5\]. The residual covariance is calculated from the \textit{apriori} state covariance, \( P_k^- \), and the measurement model parameters using the formula \[5\]

\[
S_k = H_k P_k^- H_k^T + R
\]

(32)

After computing \( e_{max} \), each measurement, \( z_{ij} \), is compared to the expected target measurement, \( \hat{z}_i \), using the formula \[5\]

\[
\hat{z}_i - e_{max} \leq z_{ij} \leq \hat{z}_i + e_{max}
\]

(33)

The subscript \( ij \) refers to the \( j \text{th} \) measurement from the \( i \text{th} \) sensor. The variable \( z_{ij} \) is a measurement vector consisting of \( r_i \) and \( v_i \). The expected target measurement vector, \( \hat{z}_i \), is calculated by evaluating (7) and (8) at the \textit{apriori} state estimate. Every measurement outside this region is eliminated as a possible candidate for updating the target. If multitarget tracking is employed, these unused measurements are evaluated for updating alternate targets or initiating new targets.

Square gating only provides a coarse evaluation of potential observations because it’s based on the residual covariance in the worst case direction of the measurement space. Ellipsoidal gating is used to further reduce observations relevant to the target by comparing the residual norm of each measurement to the gate size. The residual norm, \( d_{ij}^2 \), is calculated by \[10\]

\[
d_{ij}^2 = \delta z_j^T S_i^{-1} \delta z_j
\]

(34)

If \( d_{ij}^2 > \gamma \), the observation is eliminated as an option for target update \[10\].

If multiple observations lie within the target’s ellipsoidal gate, data association is applied to select the most likely candidate for target update. Although numerous different options are available for data association, this research applies the simplest technique, the global nearest neighbor (GNN). This approach works well for our application involving a single-target in a low clutter environment. Based on GNN, the closest observation within each track’s ellipsoidal gate is selected to perform the update. The closest observation is assessed by comparing the residual norm, \( d_{ij}^2 \), for each observation surviving gating. \[10\]

During algorithm testing, radar clutter is simulated for individual sensors. During each measurement update, individual sensors return observations based upon the true missile position as well as clutter, i.e., false observations. The true observations are generated from truth data by adding random noise from a zero-mean, Gaussian distribution with variance defined by the sensor performance as described in Section II. For clutter, the number of false observations are chosen from random, uniformly distributed integers over the interval \([0,3]\). Each clutter measurement takes on a value chosen from a random, uniform distribution over the sensor’s minimum detection range through its maximum detection range.

VI. TRUTH MODEL

In order to provide realistic missile truth data in dynamic scenarios two separate simulation tools are employed: \textit{Profgen} and \textit{Argos 3.0}. \textit{Profgen} takes a vehicle through a series of maneuvers to produce kinematic outputs. These kinematic outputs, which track the motion of an aircraft in six degrees of freedom (6DOF), provide target information for use in \textit{Argos}. \textit{Argos} is a 6DOF missile simulation tool developed through collaboration between the National Air and Space Intelligence Center (NASIC) and the Air Force Research Laboratories (AFRL).

In \textit{Profgen} \[11\], the user specifies the dynamic capabilities of the vehicle and directs maneuvers by stringing together basic profile elements such as horizontal and vertical turns, jinks, accelerations and rolls. Since only the kinematic solution is of interest, \textit{Profgen} models the vehicle as a point-mass
body and considerations such as lift, drag, weight and thrust are substantially reduced. Profgen outputs are specified in one of two coordinate frames, WGS-84 or Standard Navigation Unit, with five associated reference frames: inertial, Earth-centered Earth-fixed (ECEF), geodetic, wander-level and body.

In this research, aircraft data is output in a WGS-84 ECEF reference frame and then externally converted into an Earth-centered North-East-Down (NED) local-level navigation frame for import into Argos and our EKF algorithm. Argos only utilizes target aircraft position and velocity information to compute missile trajectory. In contrast, the EKF requires truth data on aircraft position, velocity and attitude.

Argos [12] provides numerous options for missile models and coordinate frames for performing simulations. The results in Section VII are obtained using an unclassified short-range missile and assuming a flat-Earth environment. The simulation is based in Matlab simulink and the resulting missile kinematic data is output in a NED local-level navigation frame. The true missile position is not known to the filter, but it’s used to generate noise corrupted sensor measurements and compute filter error.

VII. RESULTS AND ANALYSIS

The proposed missile scoring solution is tested in three simulated scenarios to establish performance. Scenario 1 is a non-maneuvering target attacked vertically from below. Scenario 2 involves a tail-aspect attack against an aware adversary performing an aggressive 9G descending break-turn into the shooter. Scenario 3 is also a tail-aspect attack, but the target performs a 7G vertical maneuver. The filter tuning is performed during scenario 2, since it represents the most dynamic scenario based on the degree of target maneuvers. EKF tuning resulted in dynamic noise of $q = 10,000$ and an observation noise matrix of

$$
R = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}
$$

These tuning parameters are fixed over all scenarios.

A. Scenario 1: Non-maneuvering Target

The results from scenario 1 are recorded in Figure 1. Figure 1(a) depicts the flight profile for the scenario. The target aircraft is wings-level, flying northbound at an altitude of 5000 meters and does not maneuver for the duration of the missile intercept. The shooter starts from a position one mile in front of the target on a southbound heading, at an altitude of 500 meters and a 70 degree pitch-up attitude. The true aircraft and missile trajectory are depicted for the entire simulation of approximately eight seconds. The missile estimate from the proposed scoring system is only depicted after the missile reaches the sensor range limit of 350 meters.

The root-sum-squared (RSS) position error for a single sample run is shown in Figure 1(b). The target is randomly initialized for the sample run in accordance with typical GRDCS range accuracies as discussed in Section IV. For the particular run shown, the position error is initially about 37 meters and the estimate improves to an end-game error on the order of centimeters. The continuous improvement in the position accuracy as the missile closes on the target is a result of better sensor geometry leading to improved observability of missile states. The final simulation time corresponds to missile impact with the target.

In addition, a 100 run Monte Carlo analysis is performed to evaluate the errors in the estimates of each of the six missile navigation states. Figures 1(c) and 1(d) record the results and the statistics are summarized in Table I. Despite large initialization errors in position, the ensemble mean error for all position states is less than a centimeter at impact and error standard deviation is minimal. The largest velocity state error appears in the y-axis where the ensemble mean is $-0.881 \text{ m/s}$ and error standard deviation is 13.327 m/s. Figure 1(d) reveals that the z-axis velocity estimates are significantly better than the x or y-axis. This is a result of reduced observability in the x and y-axis due to the geometry of the intercept. Since the missile is approaching the drone aircraft vertically along the z-axis of the NED frame, the missile’s velocity component in the x and y-axis direction is almost perpendicular to the aircraft sensors’ line-of-sight (LOS). Therefore, radial velocity measurements from the sensors do not provide precise measurements of the $v_x$ and $v_y$ states.

<table>
<thead>
<tr>
<th>Missile State</th>
<th>Mean Error</th>
<th>Error Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0.007 m</td>
<td>0.129 m</td>
</tr>
<tr>
<td>$y$</td>
<td>0.009 m</td>
<td>0.214 m</td>
</tr>
<tr>
<td>$z$</td>
<td>0.000 m</td>
<td>0.052 m</td>
</tr>
<tr>
<td>$v_x$</td>
<td>-0.455 m/s</td>
<td>5.781 m/s</td>
</tr>
<tr>
<td>$v_y$</td>
<td>-0.881 m/s</td>
<td>13.327 m/s</td>
</tr>
<tr>
<td>$v_z$</td>
<td>0.400 m/s</td>
<td>1.040 m/s</td>
</tr>
</tbody>
</table>

B. Scenario 2: Target Break-Turn

Figure 2 shows the results from scenario 2 where the drone aircraft makes a descending right hand break-turn as the shooter fires from a trail position. Figure 2(a) illustrates the the aircraft and missile 3D trajectory. The drone aircraft begins the simulation in a wings-level attitude, heading north, at an altitude of 5000 meters. After the simulation starts, the target rolls to approximately 120 degrees of bank and initiates a 9G, right-hand, descending turn. Missile impact occurs as the target is passing through an easterly heading at an altitude of about 4300 meters. The shooter fires at the outset of the scenario from a three mile trail position, heading north, at an altitude of 5000 meters.

As recorded in Figure 2(b), RSS position error rapidly improves from about 28 meters to a few centimeters at impact for the single run shown. The results from a 100-run Monte Carlo analysis in Figures 1(c) and 1(d) illustrate errors in the navigation states. The ensemble statistics from this analysis are summarized in Table II.
The filter performance in estimating position states is similar to scenario 1. The velocity states exhibit larger error standard deviations in the y and z-axis. Once again, this is the result of observability issues. The missile is approaching the target along the x-axis of the NED frame as the missile estimate is performed. As a result, the aircraft sensors do not provide precise measurements of missile velocity along the y and z-axis since these velocity components are nearly perpendicular to the sensors’ LOS.

C. Scenario 3: Target Vertical Maneuver

The results from the last scenario are recorded in Figure 3. As shown in Figure 3(a), the target starts the simulation wings-level, northbound, at an altitude of 5000 meters. The shooter starts out two miles in trail with attitude and altitude identical to the target. When the simulation begins, the drone aircraft performs a 7G vertical pull-up while the shooter immediately fires.

In Figure 3(b) the missile position is randomly initialized based on typical GRDCS range accuracies to approximately 38 meters, but still corrects to a few centimeters during the 0.39 second missile estimate for this sample run. Figures 3(c) and 3(d) show the results from the Monte Carlo analysis. The worst velocity observability occurs in the y and z-axis since the missile approaches along the x-axis of the NED frame. The results from the Monte Carlo analysis are summarized in Table II.
This research proposed an extended Kalman filter algorithm to provide air-to-air missile scoring. Missile dynamics were modeled by assuming a CV missile and measurements of range and radial velocity are provided by sensors on the target drone. The drone set-up consisted of seven FMCW sensors located to insure spherical coverage and minimize error in missile flight path reconstruction. Basic gating and data association was implemented in the algorithm to deal with possible radar clutter.

Simulation results demonstrated centimeter level precision in position state estimates for three different target profiles: non-maneuvering, high-G break-turn and high-G vertical maneuver. The results are based on the performance of commercially available automotive sensors not optimized for this application. Higher performance sensors with a faster update rate and greater range resolution offer a further improvement in performance. Future research in this area involves comparing the performance of alternate nonlinear filters such as the unscented Kalman filter and particle filter. Additionally, since test and evaluation applications do not

in Table III.

<table>
<thead>
<tr>
<th>Missile State</th>
<th>Mean Error</th>
<th>Error Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>0.006 m</td>
<td>0.022 m</td>
</tr>
<tr>
<td>(y)</td>
<td>0.005 m</td>
<td>0.201 m</td>
</tr>
<tr>
<td>(z)</td>
<td>-0.015 m</td>
<td>0.245 m</td>
</tr>
<tr>
<td>(v_x)</td>
<td>-0.120 m/s</td>
<td>1.284 m/s</td>
</tr>
<tr>
<td>(v_y)</td>
<td>-0.364 m/s</td>
<td>18.527 m/s</td>
</tr>
<tr>
<td>(v_z)</td>
<td>0.763 m/s</td>
<td>19.897 m/s</td>
</tr>
</tbody>
</table>

In all scenarios, the end-game mean position error is on the order of centimeters with a worst-case standard deviation of 0.245 m. Furthermore, the range resolution of the FMCW sensors is one of the primary limitations on position accuracy. Since range accuracy is advertised as 2.5 percent of range, errors of 8.75 meters are expected at the scoring system’s maximum range.

VIII. CONCLUSION
require real-time scoring, backwards smoothing could be employed to offer additional performance improvement.

**References**


