Abstract—This paper is concerned with the advanced control of a transfer system with a vibration mechanism, and gives special consideration to both track to a moving object and damping vibration. The system is constructed with two-degrees-of-freedom control in order to track to the moving object. Furthermore, in order to construct a transfer system for fast tracking and vibration damping, the controller is designed using a Hybrid Shape Approach that considers both time and frequency characteristics. The effectiveness of the proposed control system is shown through experiments in liquid container transfer.

I. INTRODUCTION

Automatic pouring systems have recently been applied to the pouring process in the casting industry. In these systems, the mold line is stopped at a fixed position, where the molten metal is poured from the ladle into the mold, as shown in Fig.1. After the pouring, the mold line is transferred again. The next mold is moved forward at the front position of the ladle and the mold line stops again. In this process, sand molds are sometimes destroyed by unsuitable acceleration of the line. Moreover, stopping the line reduces productivity. In order to solve these problems, a self-transfer-type automatic pouring system, such as that which synchronizes the movement of the ladle to the movement of the mold line (see Fig.1), is much needed. However, this system has the problem that sloshing (liquid vibration) of the molten metal in the ladle occurs when the ladle transfer accelerates. This residual vibration is generated by inappropriate transfer control. The sloshing causes the overflow of molten metal from the ladle and leads to contamination, which degrades the quality of the metal. Therefore, it is important to build an automatic pouring system that makes the transfer of a ladle track to the mold line with sloshing suppression.

Many studies have been published concerning the control of liquid container transfer with vibration damping [1]-[3]. For example, the $H_{\infty}$ control theory was applied to a liquid container transfer system in order to completely suppress sloshing and guarantee the system's robustness amid change in the static liquid level [1]. The control system designed using the Hybrid Shape Approach, on the other hand, has achieved slosh suppression in three-dimensional transfer motion [2]. Regarding the tracking control to the moving object, a lot of researches have been done up to the present. In [4], a tracking control system by an $H_{\infty}$ filter was applied to a visual tracking system using a camera and robot arm. Mao-Lin et al. studied the tracking control problem for a system with an uncertain dead-time [5]. This control system uses a two-degrees-of-freedom control system comprised of feedforward and feedback parts.

The above-mentioned studies have shown effective results in either tracking to a moving object or damping vibration. But few studies have satisfied both of these requirements. The development of a self-transfer-type automatic pouring control system that also suppresses sloshing would be particularly important. Yet, in the field of liquid container transfer, no studies have been reported on tracking control to a moving object.

Therefore, the purpose of this paper is to present a transfer control method that satisfies the above requirements. The present system makes the mother machine track into the moving object in sequence. Namely, as shown in Fig.1, the transfer of the ladle has to follow the transfer of the mold. The present problem is considered a kind of Master-Slave problem, which is a synchronous control problem.

The effectiveness of this control method is shown by applying it to a liquid container transfer system aimed at tracking to the moving object.

In this paper, a two-degrees-of-freedom control system is utilized in order to achieve tracking control to the moving object. The controller to dampen the sloshing is designed using the Hybrid Shape Approach proposed by the authors [2]. With respect to the two-degrees-of-freedom
control system, a dynamical model of a moving object is used as a reference model, and it is called the target system. The transfer model tracking to the moving object is used as the tracking model, and it is called the tracking system. Both the target system and the tracking system utilize feedback of position. The Hybrid Shape Approach is applied to these feedback systems. The controller designed through this approach allows the fastest possible transfer while meeting the given control specifications for the time and frequency characteristics. The Hybrid Shape Approach gives the proposed control system the ability to dampen vibration without the need for direct feedback of the vibration.

II. CONSTRUCTION AND STABILITY OF THE CONTROL SYSTEM

In this paper, the control problem is "to design a transfer system that tracks to the moving object as well as dampens vibration". To fulfill the requirement of tracking to the target system, i.e., the moving object or (i.e., the Master), a two-degrees-of-freedom control system is applied to the tracking system, i.e., the transfer system. A block diagram of the system is shown in Fig.2.

![Fig. 2. Two-degrees-of-freedom control system](image)

In this system, the transfer function $P_M(s)$ represents a motor model from control input $u_m$, i.e., the motor voltage, to output $y_m$, i.e., the position of the target plant. Servo control of the target plant can be accomplished by the feedback control of the position. Meanwhile, the transfer function $P_T(s)$ represents the motor model of the tracking plant, where $u_T$ is a control input of motor voltage and $y_T$ is output, which is the position of a tracking system. The reference trajectory $r_m$ is given to the target system as the command input. Then, the position output $y_m$ of the target plant is applied to the tracking system as the command input. In addition, the position $y_m$ is applied to the inverse model $P_T^{-1}(s)$ of the tracking plant, and then the feedforward control input $u_{FF}$ is obtained for the tracking system. The condition of using the inverse model $P_T^{-1}(s)$ is limited, where the order of the denominator in the target closed-loop system is equal or higher than the order of the denominator the tracking system $P_T(s)$. This condition is available for reason, which in transfer systems, $P_T(s)$ and $P_M(s)$ are transfer functions of motor. So, transfer function of motor from the voltage to the position is generally first order system with an integrator. Therefore, $P_T(s)$ and $P_M(s)$ is the same order system, and the number of order of the target closed-loop system included $P_M(s)$ is equal to $P_T(s)$ or higher order than $P_T(s)$. The reference trajectory $r_T$ is given to the tracking system when the system shifts at a different target point of the moving object in sequence, while $r_T = 0$ when the tracking system follows the movement of a target point of the moving object. $K_M(s)$ and $K_T(s)$ denote, respectively, the feedback controller of the target system and the tracking system. Hence, the tracking system constitutes a two-degrees-of-freedom control system comprised of feedback control $u_{FB}$ and feedforward control $u_{FF}$. Now, the feedback controllers $K_M(s)$ and $K_T(s)$ are each designed using the Hybrid Shape Approach in order to suppress vibration. Section 4 describes the design of this control system through the Hybrid Shape Approach.

One of the purposes of this system is that the position $y_T$ of the tracking system follows to the position $y_m$ of the target system. By using two-degree-of-freedom control system with feedforward term having inverse model $P_T^{-1}(s)$ to tracking plant, transfer function from $y_m$ to $y_T$ is one. Moreover, modeling error to feedforward term is compensated by feedback term. Therefore, $y_T$ of the tracking system can track to $y_m$ of the target system.

![Fig. 3. Two-degrees-of-freedom control system by rewriting Fig.2](image)

The control system described above can be transformed into the block diagram shown in Fig.3. In Fig.3, $G_M(s)$ is a closed-loop transfer function from the reference trajectory $r_m$ to the position $y_m$ of the target plant in the target system, described by

$$G_M(s) = \frac{P_M(s)K_M(s)}{1 + P_M(s)K_M(s)}$$

As for the stability of the overall system, comprised of the target and tracking systems, the following two conditions must be satisfied:

1. The feedforward terms $G_M(s)$ and $G_M(s)P_T^{-1}(s)$ are stable.
   If each of $G_M(s)$ and $P_T^{-1}(s)$ is stable, $G_M(s)P_T^{-1}(s)$ becomes stable. Therefore, if each of $G_M(s)$ and $P_T^{-1}(s)$ are stable, the feedforward terms become stable. Assuming that $P_T^{-1}(s)$ is stable. This is because, in transfer systems, most of the tracking plant $P_T(s)$ is a transfer function of the motor. Therefore, there is generally no pole on the inverse model of $P_T(s)$. Consequently, since the closed-loop transfer function $G_M(s)$ in the target system can be stabilized, the stability of feedback terms is guaranteed.

2. The closed-loop system constructed by $P_T(s)$ and $K_T(s)$ is internally stable.
   If $P_T(s)$ and $K_T(s)$ are both stable and the closed-loop system constructed by $P_T(s)$ and $K_T(s)$ is stable, the closed-loop system constructed by $P_T(s)$ and $K_T(s)$ becomes the internally stable.
For the conditions given above, the stability of the present two-degrees-of-freedom control system is guaranteed, if the stability of the tracking system and that of the target system are both guaranteed. Therefore, it is possible to independently design the tracking system and the target system.

III. EXPERIMENTAL APPARATUS OF LIQUID CONTAINER TRANSFER SYSTEM

A schematic diagram of an experimental apparatus of a liquid container transfer system is shown in Fig. 4. The radius of the three-dimensional cylindrical container $R$ is 0.12[m] and its height is 0.3[m]. The object liquid in the present experiments is water, with a static liquid level $h_s$ of 0.16[m]. This paper uses straight paths in the liquid container transfer following the movements of the object.

![Fig. 4. Schematic diagram of experimental apparatus of a liquid container transfer system](image)

In Fig. 4, the tracking system is a part of the liquid container transfer system, while the target system is a part of the moving object. The purpose of the tracking system (the container) is to shift the liquid container to the specified target point (1st, 2nd or 3rd target) of the moving object, and then track it to the movements of its target points. The present work will contribute as basic research toward the establishment of an automatic pouring system in a continuously moving mold line, although the control of the tilting motion of a ladle will also be necessary to accomplish such a system. In the future, the moving object in Fig. 4 will correspond to the mold line in Fig. 1, and each target in Fig. 4 will correspond to one of the molds in Fig. 1. Both the container and the moving object are moved independently using AC servo-motors with ball screws, and the velocity and position of the container and the moving object are controlled by adjusting the voltage applied to each motor. The maximum velocity and maximum acceleration of the motor used in the liquid container transfer system are 0.8[m/s] and 2.0[m/s$^2$], respectively, and those of the motor used in the moving object are 0.5[m/s] and 1.0[m/s$^2$]. The positions of the container and the moving object are detected by encoders fitted to each motor. In order to evaluate only the experimental results, level sensors were installed in this apparatus. Displacement of the liquid level is detected through changes in resistance between two stainless electrodes.

For the AC servo-motors on the container and the moving object, the transfer function $P_T(s)$ from the input voltages $e(t)$ to the positions $y(t)$ of the container, and the transfer function $P_M(s)$ from $e(t)$ to $y(t)$ of the moving object, are respectively given in the form of the following first-order lag model with an integrator.

$$P(s) = \frac{Y(s)}{E(s)} = \frac{K_m}{s(T_m s + 1)}$$

where $T_m$ is the time constant and $K_m$ is the gain. These parameters were identified by adding a step-wise input to the apparatus. As a result, the parameters on the tracking plant (the container transfer system) $P_T(s)$ were obtained as $K_{mT} = 0.1063[\text{m/sV}], T_{mT} = 0.012[s]$, and the parameters on the target plant (the moving object) $P_M(s)$ were obtained as $K_{mm} = 0.1652[\text{m/sV}], T_{mm} = 0.0040[s]$.

In general, the natural frequency $f_n$ for a (1,1)-mode sloshing of perfect fluid in a cylindrical container is represented by the following equation:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{R}} \frac{\cosh^{-1}\left(\frac{h_s}{R}\right)}{h_s}$$

where $h_s$ is the static liquid level, $R$ is the radius of the cylindrical container, $g$ is the gravitational acceleration, and $\cosh^{-1}$ is the least positive root for a first-order derivative of the first kind Bessel function ($\epsilon_1 = 1.841$). In this study, the target liquid level $h_s$ is 0.16[m] and the radius of the cylindrical container is $R = 0.12[m]$. Hence, the natural frequency $f_n$ is calculated as 1.9379[Hz] (12.17[rad/s]).

IV. CONTROL SYSTEM DESIGN BY HYBRID SHAPE APPROACH ON LIQUID CONTAINER TRANSFER

In order to suppress sloshing, this system is designed using the Hybrid Shape Approach. This approach (which is described in detail in [2]) is a design method for starting control and vibration damping, and satisfies hybrid specifications in both the frequency domain (gain margin, phase margin, vibration characteristics, and so on) and time domain (transient response, settling time, overshoot, restriction of control input, and so on) by using real-time feedback of only the container's position data.

In fact, in a liquid container transfer system, sloshing is caused by self-transfer movement under circumstances in which there is no disturbance. To suppress sloshing, it is important to shape the frequency response of the controller to be notch-typed at the resonance frequency. Therefore, it is theoretically possible to suppress vibration by controlling acceleration, and it is not necessary to directly feedback liquid vibration. Therefore, the dynamical model of sloshing is not necessarily required in the control design.

In this approach, sloshing can be damped by furnishing the controller with a notch filter at the resonance frequency of sloshing. The method to design this system using the Hybrid Shape Approach is described as follows.

A. Control system

Since it has been described in the previous section that the tracking system and the target system can be designed independently, each control system is described by block diagrams in Fig. 5.
B. Selection of controller elements

In order to satisfy the desired specifications, the controller is formulated as follows.

\[ K(s) = \prod_{i=1}^{n} K_i(s) \]  

Eq.(2) is a servo system with an integrator. Thus, according to the Internal Model principle, a proportional control \((P\) control) system is sufficient to avoid the offset. Therefore, the proportional gain \((K_P)\) is selected as the first element of the controller \(K(s)\) given by Eq.(4) in the tracking system as well as in the target system.

\[ K_1(s) = K_P \]  

In order to reduce the influences of higher-mode sloshing and noise, a low-pass filter \((K_2)\), which makes the controller low gain at a high frequency domain, is selected as the second element of the controller in each system.

\[ K_2 = \frac{1}{T_l s + 1} \]  

A notch filter \((K_3)\) is selected as the final element of the controller in order to suppress sloshing.

\[ K_3 = \frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{s^2 + \omega_n s + \omega_n^2} \]  

It is known that the dominant mode of the sloshing of liquid in a container is a first-mode frequency [1]. In the tracking system, when the static liquid level \(h_s\) is 0.16[m], the first-mode natural frequency is \(f_n = 12.17[\text{rad/s}] (1.9379[\text{Hz}])\), as described in Section 3. Therefore, the parameters \(\omega_n\) and \(\zeta\) in Eq.(7) are given as \(\omega_n = 12.17\) and \(\zeta = 0.0001\). By introducing the notch filter into the controller \(K(s)\), the controller gain can be shaped such that 20[dB] is relatively reduced, at least at the natural frequency of the sloshing. The notch filter also makes it possible to suppress residual vibration without directly measuring the liquid vibration. On the other hand, in the controller \(K_M(s)\) of the target system, the same type of notch filter as that used in the tracking system is introduced. The reason is that the sloshing in the container is influenced by the feedforward input directly applied to the tracking plant. The feedforward input to the tracking system is given from the target system via the inverse model of the tracking plant, as shown in Fig.2. Finally, the transfer functions of the controllers on the tracking and target systems are given as Eq.(8).

\[ K_P(s) = \prod_{i=1}^{3} K_i = \frac{K_{PT}(s^2 + 2\zeta \omega_n s + \omega_n^2)}{(T_{IT} s + 1)(s^2 + \omega_n s + \omega_n^2)} \]

\[ K_M(s) = \prod_{i=1}^{3} K_i = \frac{K_{PM}(s^2 + 2\zeta \omega_n s + \omega_n^2)}{(T_{IM} s + 1)(s^2 + \omega_n s + \omega_n^2)} \]  

In Eq.(8), \(K_{PT}, T_{IT}\) and \(K_{PM}, T_{IM}\) are unknown parameters. These parameters are reasonably determined by solving an optimization problem.

C. Formulation of design specifications

In the Hybrid Shape Approach, various control specifications in the time and frequency domains can be given. The specifications of the controllers in both domains are formulated by making use of penalty functions, and then the controllers \(K_P(s)\) and \(K_M(s)\) are respectively calculated to satisfy those specifications. In the control design, Spec.(I) ~ Spec.(V) were given for each controller.

- Spec.(I) : The controller and closed-loop system are stable. Penalties are given in order to compensate for the stabilities of the controller and closed-loop system, if the following constraints do not hold:
  \[ \text{Re}[r_K] < 0, \text{Re}[r_{cl}] < 0 \]
  \[ K_P > 0, T_l > 0 \]  

- Spec.(II) : The controller gain should be less than 0[dB] at \(\omega_k = 314[\text{rad/s}] (50[\text{Hz}])\) in order to decrease the influences of the higher-order sloshing mode and noise. A penalty is given if the following constraint does not hold:
  \[ |K(\omega_k)| < 0[\text{dB}] \]  

- Spec.(III) : Due to the restriction of input magnitudes, the input voltage \(u\) should not exceed a magnitude of \(\pm10[\text{V}]\). A penalty is given if the following constraint does not hold:
  \[ \max|u| < 10[\text{V}] \]  

- Spec.(IV) : Maximum overshoot should not exceed a magnitude of \(10^{-3}[\text{m}]\). A penalty is given if the following constraint does not hold:
  \[ \max(O_s) < 10^{-3}[\text{m}] \]  

- Spec.(V) : The controller gain should be less than 0[dB] at the first-mode natural frequency \(f_n = 12.17[\text{rad/s}]\). A penalty is given if the following constraint does not hold:
  \[ |K(f_n)| < 0[\text{dB}] \]  

When it is necessary to design the frequency shaping more finely, more restrictions should be provided. In vibration control, residual vibration can be suppressed by means of the restriction of Eq.(13).
D. Formulation of an optimization problem

The following optimization problem using penalty terms is formulated with Eq.(9) ~ Eq.(13).

\[
\min_{K(s)} J = T_s + J_P
\]

where,

\[
J_P = \sum w_i \cdot \sum \frac{n}{m} \cdot \left(\frac{y_i - y_i(t)}{y_i^*}\right)^2
\]

\[
T_s = \min \{t \mid |y_f - y(t)| < y_e \}
\]

and \(t\) is the time, \(y(t)\) is the position of the container or the mold line at time \(t\), \(y_f\) is the target position, \(T_s\) is the settling time to the reference trajectory \(r_m\) in the target system and to the reference trajectory \(r_t\) in the tracking system and \(y_e\) is the admissible error set to 10^{-3}[m]. Minimizing \(T_s\) means that the trajectory of the moving object quickly reaches the reference trajectory \(r_m\), and the trajectory \(y_f\) of the liquid container fast-tracks to the trajectory \(y_m\) of the moving object. If none of the above constraint conditions are held, the penalty function \(w_i\) in Eq.(14) is added as \(w_i = 10^8\) to the cost function \(J\). The index \(i\) of \(w_i\) corresponds to the above specification number.

E. Computation of a controller

Feedback controller \(K(s)\) in Eq.(8) is derived to minimize the cost function given in Eq.(14). The simplex method [6] is adopted as the optimization method, where the reflection coefficient is \(\alpha = 1.0\), the expansion coefficient is \(\beta = 0.5\) and the contraction coefficient is \(\gamma = 2.0\). The solution obtained by the simplex method is not always the global optimal one. Hence, the initial simplex values and weights of the cost function are modified to obtain the desired characteristics of the controller. The initial simplex values were assigned to be \(K_{PT} = (40, 70, 1)\) and \(T_{PT} = (0.01, 0.005, 0.1)\) for the tracking system, and \(K_{PM} = (44, 25, 1)\) and \(T_{PM} = (0.01, 0.005, 0.1)\) for the target system. The reference trajectories \(r_t\) and \(r_m\) in Fig.5 were determined separately by integrating the maximum velocity and maximum acceleration of the present experimental apparatus on each system. In the design, the starting point was set to be \((x_T, x_M) = (0, 0)[m]\), and the endpoint was set to be \((x_T, x_M) = (1.0, 0.8)[m]\), because of the limitations of the apparatus. As a result, around 20 iterations were required for convergence, and it took about 20 seconds to compute an optimization problem using a personal computer (Pentium III 700MHz CPU). The characteristic roots of the closed-loop controller became stable, because all real parts of the roots were negative. Table I shows the results of the computation.

|TABLE I| VALUES OF COST FUNCTIONS AND PARAMETERS OF THE OBTAINED CONTROLLER ON EACH SYSTEM |
|---|---|---|---|
| | \(J = T_s\) | \(K_p\) | \(T_l\) |
|Tracking system | 2.26 | 15.70 | 0.050 |
|Target system | 2.30 | 16.57 | 0.053 |

The frequency response of the controller is shown in Fig.6. From this figure, it can be seen that the gain decreases at the resonance frequency (12.17[rad/s]) of the sloshing, and the specification is satisfied because it is less than 0[dB]. Also, it can be confirmed that it is less than 0[dB] at \(\omega_l = 314[\text{rad/s}]\) (50[Hz]). Since the gains are monotonously decreasing in the high-frequency domain, it is also clear that these controllers can decrease the influences of high-mode sloshing and noise. Furthermore, the high-speed transfer of the container is achieved because the controller is high-gain in the low-frequency domain.

From the simulation results of the tracking control by the two-degrees-of-freedom control system using the obtained controllers, it was confirmed that the container quickly tracked to the mold line, and the restriction of the magnitude of control input was satisfied. The results cannot be shown here due to the limitations of this paper. But the effectiveness of this system can be evaluated by the experimental results shown in the following section, because the experimental results are almost the same as the simulation.

![Bode diagram of the proposed controllers](image)

Fig. 6. Bode diagram of the proposed controllers

V. EXPERIMENTAL RESULTS

Fig.7 shows the experimental results of the two-degrees-of-freedom control system achieved by applying the proportional control (P control) without a notch filter. These results are shown for the sake of comparison with the results obtained using the Hybrid Shape Approach. The proportional gains of the tracking system and of the target system were respectively set to be \(K_P = 35.4\) and 37.8, to reach the same settling time as reached by the Hybrid Shape Approach. Fig.7 shows the positions and control inputs of the tracking system and the target system. The position of the tracking system is that of the container, and the position of the target system is that of the moving object, as shown in Fig.4. The initial tracking error is 0.27[m], where the container stays at the origin of the \(x_T\)-axis and the moving object stays at the position 0.27[m] of the \(x_M\)-axis. The first target position of reference trajectory \(r_m\) is given as 0.67[m], and then the reference is shifted such that \(r_m = 0.57, 0.47, 0.37\) and 0.27[m]. The sloshing detected by the level sensor drawn in Fig.4 is shown in Fig.7. As seen from Fig.7, the tracking control by P control is achieved, but the container causes a high degree of sloshing.

Fig.8, on the other hand, shows the experimental results of the two-degrees-of-freedom control system using the Hybrid Shape Approach. The figure clearly shows that the
tracking system tracks to the target system. The tracking error after the time of 1.5[s] is under 0.001[m]. Therefore, the obtained result is thought to be sufficient for pouring by a self-transfer-type automatic pouring system. In Fig.8, the inclination of the surface of the liquid is caused by the acceleration of the container transfer, though sloshing is suppressed in the steady-state; that is, residual vibration can scarcely be found. Furthermore, direct observation confirmed that swirl-type sloshing did not occur. A comparison with the result of $H_{\infty}$ control in the previous study [1], which did not include tracking control to a target system, shows that almost equivalent slosh-suppression performance was obtained. Therefore, by using the two-degrees-of-freedom control system through the Hybrid Shape Approach, tracking control has been achieved and sloshing has been suppressed.

Further, Fig.9 shows that the two-degrees-of-freedom control system by the Hybrid Shape Approach can shift the tracking machine (ladle) to different target points of the moving object as shown in Fig.4. The tracking position shifts to track three target points of the moving object in sequence. Even in such a case, residual vibration of the liquid in the container can scarcely be found, as shown in Fig.9. Consequently, the proposed control system is also effective while shifting between molds in a mold line.

VI. CONCLUSION

This paper has presented a two-degrees-of-freedom control system designed using the Hybrid Shape Approach in order to achieve tracking control of liquid load transfers while also dampering vibration. As a result, it was demonstrated that the proposed control system well satisfied requirements of both tracking control to a moving object and slosh suppression while a liquid container transfer system is tracked to the movements of the target system. Moreover, it has been shown that this system is able to sequentially shift to the various target points of the moving object while tracking. The obtained results are thought to be very useful for various transfer systems with vibration mechanisms, as well as for liquid container transfer systems.

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