Chatter detection based on adaptive estimation of the fourth cumulant of the modeling error

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Abstract

This paper proposes a design method for chatter detection based on the fourth order statistics estimation applied to the modeling error. Usually, a test of hypothesis is built on an a priori knowledge of the statistical distribution. The frequency function is often assumed gaussian. In the case of machining process without chatter, the gaussian assumption is not valid. Consequently, the proposal of an adaptive estimation of the fourth cumulant on a sliding window is introduced for machining process monitoring with possible chatters. The modeling error of the cutting force is the difference between the output of nonlinear model of cutting force and a real force estimation taking into account the stiffness of the sensor. A milling process illustrates the effectiveness of the proposed chatter detection strategy.

1 Introduction

Machining process objectives result in the removal of material with respect of the time delivery and of the “total quality” of the finished part. During a machining process, the workpiece-tool system may chatter. In conventional machining, this phenomenon can be due to a control strategy which is ineffective for a given operating mode. Establishing a correct value of the feed per tooth $f_r$ not only increases tool performance but also gives improved efficiency to the machining process. The operating point $(V_c, f_r)$ has to be chosen by the operator. Theoretically, as proposed by Altintas [1], the design of the lobes of stability could allow the determination of a good axial depth of cut $a_u$. Moreover, the necessary a priori knowledge on the machining process is not easily available. One of the main motivations for the proposal of an on line monitoring method resides in the characterization of the process. This strategy has to help the operator to find an operating point by experiment which guarantees a safety functioning.

The paper presents the design steps of a method for the chatter detection. Section II proposes a nonlinear modeling of the cutting forces and the estimation of the real forces based on a mass cancellation approach. The modeling of the dynamometer takes into account only the stiffness. We will see that the damping coefficient is neglected. Section III gives the principle of the chatter detection based on the fourth order statistics of the modeling error. Usually, the tests of hypotheses are built upon the a priori knowledge of the statistical distribution. The proposal of an adaptive estimation of the fourth cumulant on a sliding window is motivated by the machining process monitoring with possible chatters. Usually, an off-line identification of chatter frequencies is carried out by an experimental modal analysis for multidegree-of-freedom systems. In section IV, the chatter detection method is applied on a milling process. The frequency function is often assumed gaussian. In the case of machining process without chatter, the assumption does not hold. The modeling error of the cutting force is generated to build a pertinent indicator for the machining process online monitoring. A milling application illustrates the effectiveness of the proposed chatter detection strategy.

2 Cutting Force Modeling and Estimation

The mechanics of oblique cutting operations are usually evaluated by geometrical and kinematic models. The components of the cutting force, acting on the $j$th cutter, are represented by a radial $F_{rj}$, a tangential $F_{tj}$ and an axial $F_{aj}$ forces (see Fig. 1.a). Altintas et al.[1] introduced a linear force modeling in function of the chip thickness $h_j$, corresponding to the $j$th tooth, which describes a three force components in function of the axial depth of cut $a_u$, of the cutting force coefficients $K_{rj}, K_{tj}, K_{aj}$, and of the edge constants $K_{erj}, K_{etj}, K_{eaj}$. In this work, the nonlinear coefficients $m_{rj}, m_{tj}, m_{aj}$ have been associated to the corresponding chip thicknesses in order to take into account the nonlinearities, as:

$$\begin{align*}
F_{rj} &= K_{erj}a_u h_j^{1-m_{rj}} + K_{erj}a_u \\
F_{tj} &= K_{etj}a_u h_j^{1-m_{tj}} + K_{etj}a_u \\
F_{aj} &= K_{eaj}a_u h_j^{1-m_{aj}} + K_{eaj}a_u
\end{align*} \tag{1}$$

The nonlinear coefficients are considered in the relations (1) for each cutter $j$ and each component $F_{rj}, F_{tj}$.
Figure 1: Cutting force components (a) and model of the workpiecedynamometer system (b).

\[ h_j = (\varepsilon(\phi_0) - \varepsilon(\phi_0 - \phi_{pj}))\sin(\phi_j) \]

where \( \phi_0 \) is the instantaneous spindle position and \( x \) is the table position. Fig. 1.a displays the components of the cutting force \( F_{cm} = [ F_{cx}, F_{cy}, F_{cz} ]^T \). These are the sum of the forces \( F_{cj}, F_{ij} \) and \( F_{aj} \) corresponding to the tooth at the angular position \( \phi_j \):

\[
\begin{align*}
F_{cx} &= \sum_{j=1}^{z} F_{cj} \cos(\phi_j) + F_{aj} \sin(\phi_j) \\
F_{cy} &= \sum_{j=1}^{z} F_{ij} \sin(\phi_j) - F_{aj} \cos(\phi_j) \\
F_{cz} &= \sum_{j=1}^{z} F_{aj}
\end{align*}
\]  

Force measurements denoted \( F_m = [ F_{x_m}, F_{y_m}, F_{z_m} ]^T \) are shown in Fig. 2. \( F_{x_m}, F_{y_m} \) and \( F_{z_m} \) are conditioned by a two poles low pass Butterworth analog filter. The dynamics of the Kistler® dynamometer is modeled by a pure stiffness \( K \) (see Fig. 1.b). \( M \) is the equivalent mass for the workpiece and the interface with the sensor. According to Newton's law of motion, the system acceleration \( \ddot{X} = [ \ddot{x} \; \ddot{y} \; \ddot{z} ]^T \) is:

\[
M\ddot{X} + KX + F = 0
\]

The resulting parameters have been validated on several measurement sequences. Fig. 2 displays the cutting force estimation \( \hat{F}(\phi_0) \), represented by the black dotted curves \( (F_x, F_y, F_z) \).

Figure 2: \( F_m \) in dotted gray, \( F \) in dotted black and \( F_c \) in black (a) \( x \) axis, (b) \( y \) axis and (c) \( z \) axis according to the angular position \( \phi_0 \).

It can be stated that the force \( F \) applied to the structure is different from \( F_m \) measured by the dynamometer, by an amount dependent on the acceleration level \( \ddot{X} \). Then, the real cutting force corresponding to the given operating mode \( F = [ F_x, F_y, F_z ]^T \) could be estimated by:

\[ \hat{F} = F_m - \hat{M}\ddot{X} \]
forces and accelerations according to the relation (3):

\[ e_M = F_c - \hat{F} \]  

(4)

For the given operating mode, the cutting force coefficients, the edge constants and the associated nonlinear coefficients are evaluated for each tooth-work material pair. The estimated coefficients are given in Table 1. In Fig. 2, the resultants \((F_{cx}, F_{cy}, F_c)\) of the cutting force model \(F_c\) are represented by the solid black curves. In this work, the modeling error is introduced to reveal discrepancies on which chatter detection is based.

| Table 1: Cutting coefficients for each tooth-work material pair. |
|-------------|-------|-------|-------|-------|-------|
| \(j\)      | \(1\) | \(2\) | \(3\) | \(4\) | \(5\) |
| \(K_{cr}[N/mm^2]\) | 855.8 | 1245 | 2215 | 2679 | 1791 |
| \(K_{ct}[N/mm^2]\) | 1319 | 348.8 | 1938 | 759.2 | 617.1 |
| \(K_{cu}[N/mm^2]\) | 538.9 | 19.4 | 1066 | 472.4 | 522.6 |
| \(m_{cr}\) | 0.47 | 0.25 | 0.11 | 0.34 |
| \(m_{ct}\) | 0.10 | 0.45 | 0.50 | 0.64 |
| \(m_{cu}\) | 0.32 | 0.8 | 0.66 | 0.75 |
| \(K_{cr}[N/mm]\) | 0 | 181.5 | 12.29 | 0 |
| \(K_{ct}[N/mm]\) | 2.74 | 104.3 | 0.05 | 0 |
| \(K_{cu}[N/mm]\) | 0 | 164.1 | 0.06 | 0 |

3 Chatter Detection Principle

Usually, the modal analysis provides the corresponding frequencies of the structural vibration of the system. So, the monitoring of a particular frequency is an intuitive approach to avoid chattering. Moreover, the modal parameter extraction is not easy to carry out for the machining process [7].

Generally, the detection algorithms are, recursively, computed on a centered signal or on an error signal when the process model is available [3]. In this work, the first step of the chatter detection scheme deals with the generation and the conditioning of the modeling error. Fig. 3 shows the input variables sampled at a monitoring period \(T_m\). \(X\) is the current table position, \(\phi_0\) the spindle angular position, \(\ddot{X}\) is the acceleration measured on the workpiece and \(F_m\) is the force measured by the dynamometer. The modeling error is calculated at each sampling period, according to the relation (4). The variables \(X\) and \(\phi_0\) are given by two incremental encoders. \(X\) and \(F_m\) are filtered by analog Butterworth filters introducing a group delay. When the cutters are going out of the workpiece, the resultants of the cutting force \(F_c\) calculated from \(X\) and \(\phi_0\) present abrupt variations with respect to 0. The estimation of the real cutting force \(\hat{F}\) calculated from \(\ddot{X}\) and \(F_m\) induces aberrant values due to the group delay. The modeling error \(e_M\) is filtered to reject these particular values which giving the residual \(r\).

The design of the chatter detector cannot be based on a deterministic modeling of the vibration modes. Because the characterization of the workpiece-tool system is not easy to achieve, in a general case. According to the tool, its wear and the workpiece mass, the vibration frequencies are varying.

In this work, the frequency function of the residual \(r\) is analysed. The vibration frequencies are not a priori known. Moreover, it is observed from experiments that the vibrations introduce in the measurement sequence some values that modify the distribution associated to the modeling error. The 4th order moments \(\mu_4(r)\) and central moments \(\mu_1(r)\) of the residual are defined, respectively, as:

\[ m_1(r) = E[r^1] \quad \text{and} \quad \mu_1(r) = E[(r^2 - \mu_2(r))^2] \]  

(5)

where \(E[.\] is the mathematical expectation. The statistical moments can be derived from the study of the residual. In order to avoid the gaussian assumption, the frequency function \(p_r(u)\) of the random variable \(r\) can be represented by a family of Pearson's laws [11] as:

\[ p_r(u) = p_0(u - a_1)^{p_1}(a_2 - u)^{p_2} \]  

(6)

where \(a_1, a_2, p_1, p_2\) are constant parameters and \(p_0\) is calculated by \(\int_{-\infty}^{\infty} p_r(u)du = 1\).

The Pearson's coefficients \(\beta_1\) and \(\beta_2\) defined in the relation (7) determine the shape changes of the frequency function. They are expressed as a ratio of the moments \(\mu_3\) and \(\mu_4\) on \(\mu_2\), given by the relation (5).

\[ \beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad \beta_2 = \frac{\mu_4^2}{\mu_2^4} \]  

(7)

The Pearson's criteria \(R\) and \(K\) give the type of the frequency function and are calculated by:

\[ R = \frac{6(\beta_2 - 3\beta_1 - 1)}{3\beta_1^2 - 2\beta_1 + 6} \]

\[ K = \frac{\beta_1}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \]  

(8)
The characterization of the population \( r \) issued from \( eM \) is carried out by the calculation of the mean, the variance, the skewness and the kurtosis coefficients. For all the statistical samples of \( P \), the value of \( K \) remains inside the interval \([0, I]\). According to Pearson et al. [11], the frequency function is represented by a Pearson's law of the fourth type. In this case, the parameters of the relation (6) are given by:

\[
\begin{align*}
\alpha_2 & = a_2 = c - i a \\
\beta_2 & = \beta_2 = \frac{R}{2} \left[ 1 - \sqrt{\frac{K}{1 - K}} \right] - 1
\end{align*}
\]

where \( i^2 = -1 \) and

\[
\begin{align*}
\alpha & = \frac{m_3(r)}{|m_3(r)|} \sqrt{\mu_2(r)(R + 1)(K - 1)} \\
c & = m_3(r) - a \sqrt{\frac{K}{1 - K}}
\end{align*}
\]

and the mode is given by:

\[
d = m_1(r) + \frac{2a}{(R - 2)} \sqrt{\frac{K}{1 - K}}
\]

Fig. 4 shows the histogram and the frequency function identified for an experiment without chatter (see Fig. 4.a) and with chatter (see Fig. 4.b). They are calculated from all the data of the population resulting from the experiment. The effect of a chatter vibrations is evaluated through the cutting force model and is added to the experimental measurements. The parameters of \( p_r \) are summarized in Table 2. The statistical mode \( d \) is always different from the mean value \( m_1(r) \). From the relation (7), it can be established that the Pearson's coefficient \( \beta_2 \) varies significantly, while \( \beta_1 \) is negligible. The variations of the parameters \( \alpha_1, \beta_1, a_1, \beta_2, \) and \( p_1 \) depend on the centered moments of the 4th order and of the square of the centered moment of the 2nd order. The moment \( \mu_4(r) \) reveals the flattening of the frequency function linked to the residual dispersion. This moment is sensitive to the chatter vibrations. The centered moment of the 2nd order is sensitive to a change in a cutting coefficient or a vibration frequency. The centered moment of the 3rd order gives the skew of the frequency function. It is sensitive to a change in the cutting coefficients due to the wear for example.

Usually, the goal of statistical tests is to reveal a change of the parameters for a given distribution \( p_r \), or a change of the distribution [3]. In this work, the detection principle is based on the higher-order statistics. It consists in designing a recursive indicator for monitoring the presence of chatter in the residual. The \( n \)th order moments are calculated from the first characteristic function \( \Phi_r(u) \) by:

\[
m_n(r) = (-i)^n \frac{d^n \Phi_r(u)}{du^n} \bigg|_{u=0}
\]

The designed detector must be sensitive to the change of the \( n \)th order moments of the residual. It is a problem addressed herein. The frequency function \( p_r(u) \) may be determined [8] from the first characteristic function \( \Phi_r(u) \) by the inverse Fourier transform \( p_r(u) = F^{-1} \{ \Phi_r(v) \} \), where \( \Phi_r(v) = E[e^{iuv}] \). The coefficients of the Taylor's series expansion of the second characteristic function \( \Psi_r(u) = \log[\Phi_r(u)] \) are the cumulants of the \( n \)th order denoted \( K_i \):

\[
\begin{align*}
\Phi_r(u) & = \Phi_r(0) + \frac{u}{1!} \Phi_r'(0) + \ldots + \frac{u^n}{n!} \Phi_r^{(n)}(0) + \ldots \\
\Psi_r(u) & = K_1ju + \frac{1}{2!} K_2(jv)^2 + \frac{1}{3!} K_3(jv)^3 + \ldots
\end{align*}
\]
where
\[ K_i(r) = (-i)^d \frac{d^i \Psi_r(t)}{dt^i} \bigg|_{t=0}. \]
The cumulants can be expressed in function of the lower order moments. For the centered variables \( m_4(r) = 0 \), the relations [9] are:
\[
\begin{align*}
K_1(r) &= 0 \\
K_2(r) &= m_2(r) \\
K_3(r) &= m_3(r) \\
K_4(r) &= m_4(r) - 3m_2^2(r)
\end{align*}
\]
Mc. Cullagh introduced the estimator of \( K_4(r) \) as the difference between the estimators of the fourth moment and the square of the second moment [6].
\[ \hat{K}_4(r) = \hat{m}_4(r) - 3\hat{m}_2^2(r) \]
For the centered variable \( r \), the changes of the statistical distributions are due to significant variations of the moments \( m_4 \) and \( m_2 \). These moments are sensitive to the tool wear and to the modeling error. From authors experience, \( K_4(r) \) is less sensitive to these phenomena. Furthermore, it exists a significant discrepancy of \( K_4(r) \) between the population of \( r \) without fault \( K_4(r) \approx 0.5 \) and the population with chatter \( K_4(r) \approx 2 \).

Let us denote by \( R_L \) the statistical sample of size \( L \). It is constructed from the residual signal \( r \) as:
\[ R_L = [ r(k), r(k-1), ..., r(k-L+1) ]^T \quad (12) \]
The design of an estimator \( \hat{K}_4(r) \) asks for an important study of its properties. A simple iterative estimation can be done by:
\[ \hat{K}_4(r) = \frac{1}{L} \sum_{i=1}^{L} r^4(i) - 3\left( \frac{1}{L} \sum_{i=1}^{L} r^2(i) \right)^2 \quad (13) \]
A change of the frequency distribution after the appearance of a faulty behavior is not trivial to define in a real time implementation. But, the fourth cumulant can be recursively rewritten.

### 4 Chatter Detection Application

Amblard and Brossier [2] proposed an estimator to obtain a recurrence from (15). The system equation (14) gives an adaptive estimation of the fourth-order cumulant:
\[
\begin{align*}
\hat{B}(k) &= \hat{B}(k-1) + \mu(k)(r^2(k) - \hat{B}(k-1)) \\
\hat{A}(k) &= \hat{A}(k-1) + \gamma(k)(r^4(k) - \hat{A}(k-1)) \\
\hat{K}_4(k) &= \hat{A}(k) - 3\hat{B}^2(k)
\end{align*}
\]
Let us assume \( \mu(k) = \mu \) and \( \gamma(k) = \gamma \) where the parameters \( \mu \) and \( \gamma \) have to be determined. The system (14) is rewritten (15) as:
\[
\begin{align*}
\hat{B}(k) &= \hat{B}(k-1) + \mu(r^2(k) - \hat{B}(k-1)) \\
\hat{K}_4(k) &= \hat{K}_4(k-1) + \gamma(r^4(k) - 3r^2(k)\hat{B}(k-1) - \hat{K}_4(k-1))
\end{align*}
\]
For \( \mu = 1 \), the recurrence is simplified as:
\[
\begin{align*}
\hat{K}_4(k) &= \hat{K}_4(k-1) + \gamma(r^4(k) - 3r^2(k)\hat{B}(k-1) - \hat{K}_4(k-1))
\end{align*}
\]
For the design assumptions (independence of the samples and uniformly distributed), this estimator is unbiased asymptotically. The convergence analysis was studied in [2]. In the former relation (13), the bias is introduced by the size \( L \) of the sampling window. In the latter (16), the term \( r^4(k) - 3r^2(k)\hat{B}(k-1) \) is sensitive to the large variations of \( r^4(k) \). The two estimators described by the relations (13) and (16) are not convenient to be used in a monitored control strategy. This gives the motivation to design a third estimator \( \hat{K}_4(r) \) for the detection. Let us consider a fourth-order cumulant \( \hat{K}_{4,L}(r) \) designed from the estimator described in (13), with a weighting coefficient \( \nu \), on the window of size \( L \). As recommended by Benveniste et al. [4], an adaptive form of \( \hat{K}_{4,L}(r) \) is expressed by:
\[
\begin{align*}
\hat{B}_L(k) &= \hat{B}_L(k-1) + \frac{1}{L}(r^2(k) - r^2(k-L)) \\
\hat{A}_L(k) &= \hat{A}_L(k-1) + \frac{1}{L}(r^4(k) - r^4(k-L)) \\
\hat{K}_{4,L}(k) &= (1-\nu)\hat{K}_{4,L}(k-1) + \nu(\hat{A}_L(k) - 3\hat{B}_L(k)\hat{B}_L(k-1))
\end{align*}
\]
The estimator for the detection based on the relation (17) is tuned by the size \( L \) of the window and the weighting coefficient \( \nu \).

A comparison of the three indicators \( \hat{K}_4(r) \), \( \hat{K}_4(k) \) and \( \hat{K}_{4,L}(k) \) is carried out on a face milling application monitoring. The face milling roughing out operation is defined by the Machine Operation Material Tool (MOMT) set [5] described in Table 3. For these conditions, chatter vibrations appear with an amplitude growing to 0.02 mm on the \( x \) axis. Fig. 6 shows the three cumulants for an experiment in presence of an increasing chatter starting at the table position 146 mm.
(black curves) and for an experiment without chatter (gray curves). Fig. 6.c shows the adaptive cumulant 
(estimations). Moreover, it is sensitive to the tool revolution. For the threshold $S = 0.001$, a lot of false detections appears. A nonlinear modeling of the cutting forces and the estimation of the real forces based on a mass cancellation approach allows the generation of the modeling error. An adaptive estimation, on a sliding window, of the fourth cumulant of the modeling error was designed to reveal the chatter. The optimal choice of $L$ is an open problem. In this approach, the a priori knowledge about the process is reduced to the machining process parameters.

Table 3: MOMT set for a face milling roughing out operation

<table>
<thead>
<tr>
<th>Material</th>
<th>Operation</th>
<th>Machine</th>
<th>Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>C35</td>
<td>$a_0 = 1$ mm</td>
<td>$V_c = 160$</td>
<td>$D = 63$ mm</td>
</tr>
<tr>
<td></td>
<td>$a_0 = 14$ mm</td>
<td>$[m/min]$</td>
<td>$Z = 5$</td>
</tr>
<tr>
<td></td>
<td>$f_o = 0.2$</td>
<td>$N = 800$</td>
<td>$\chi_r = 44, 5^\circ$</td>
</tr>
<tr>
<td></td>
<td>$f_z &gt; 0.15$</td>
<td>$[rpm]$</td>
<td>M660</td>
</tr>
<tr>
<td></td>
<td>$f_z &lt; 0.45$</td>
<td>$P = 10$</td>
<td>SNMK1205</td>
</tr>
<tr>
<td></td>
<td>$[mm/tooth]$</td>
<td>$[KW]$</td>
<td>AZR-31 TTM</td>
</tr>
</tbody>
</table>

Figure 6: Estimation of the residual $r$ with chatter in black and without in grey (a) Iterative cumulant $K_4(r)$ (b) Recursive cumulant $K_4(k)$ (c) Adaptive cumulant on a sliding window $K_4(k)$.

$K_4(r)$ for a sliding window of size $L = 150$ corresponding to a tool revolution. For the threshold $S = 1$, a lot of false detections appears. In Fig. 6.b, the recursive cumulant $K_4(k)$ with a weighting coefficient $\gamma = 0.001$ limits false detections by smoothing the variations. Moreover, it is sensitive to the heavy tail values. Fig. 6.c shows the adaptive cumulant $K_{4,L}$ evaluated with a weighting coefficient $\nu = 0.001$ on a sliding window of size $L$. The chatter vibration is detected at the detection time $t_d = 2.01s$ corresponding to the position 194 mm. The sensitivity of $K_{4,L}$ is more interesting than the ones of $K_4(r)$ and $K_4(k)$.

5 Conclusion

The design steps of a method for the chatter detection and its application on a milling process have been presented. For machining process in presence (or not) of chatter, the frequency function was shown non gaussian. The tests of sequential probability ratio test on the mean or variance changes are not available. A nonlinear modeling of the cutting forces and the estimation of the real forces based on a mass cancellation approach allows the generation of the modeling error. An adaptive estimation, on a sliding window, of the fourth cumulant of the modeling error was designed to reveal the chatter. The optimal choice of $L$ is an open problem. In this approach, the a priori knowledge about the process is reduced to the machining process parameters.

References