MISSILE AUTOPILOT DESIGN USING DYNAMIC INVERSION AND STRUCTURED SINGULAR VALUE SYNTHESIS

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Abstract

A methodology incorporating both dynamic inversion and structured singular value synthesis is employed to design a robust nonlinear autopilot for an air-to-air missile. The nonlinear components include fast and slow inversion loops which approximately linearize the system when closed in the absence of modeling errors. A feedback control law is designed for this closed-loop system in the presence of structured uncertainties. The current controller architecture differs from previous work in the choice of desired body rate dynamics used in the fast inversion. In this case, the desired dynamics correspond to the body rate dynamics at a central flight condition.

1. Introduction

Several recent efforts have concurrently applied the techniques of dynamic inversion and structured singular value (\(\mu\)) synthesis to the control of supermaneuverable aircraft. Reference 1 represents one example of such an effort. From this research there arises the question of whether the same techniques may be applied to the control of air-to-air missiles, which are inherently supermaneuverable to some extent. This question is answered affirmatively in Reference 2, in which the details of a particular robust nonlinear autopilot design are presented. In this paper, the authors propose an alternate controller architecture and evaluate its performance against that of Reference 2.

2. Autopilot Design

Figure 1 depicts the flight control system architecture. The slow dynamic inversion generates fast commands \(p_c\), \(q_c\) and \(r_c\) corresponding to the slow commands \(\alpha_c\), \(\beta_c\) and \(\rho_c\). The fast dynamic inversion computes the necessary elevator, aileron and rudder control inputs, denoted by \(\delta_e\), \(\delta_a\) and \(\delta_r\), respectively. The linear robust controller is included to achieve robust tracking of external commands.

![Flight Control System Architecture](image)

Figure 1: Flight Control System Architecture

Dynamic Inversion

Nonlinear dynamic inversion uses control inputs to cancel undesirable terms in a system's equations of motion using negative feedback of those terms. Moreover, the most basic form of dynamic inversion requires that the system have at least as many control inputs as it has measurements. Accordingly, the authors employ the two time-scale approach to dynamic inversion of Reference 3 to approximately linearize the missile dynamics. A review of this approach is included in Reference 2.

It is assumed that the dynamics associated with the aerodynamic angles (\(\alpha\) and \(\beta\)) operate on a slower time-scale than those of the body rates (\(p\), \(q\) and \(r\)). The body rate dynamics are fast in the sense that the control inputs \(\delta_e\), \(\delta_a\) and \(\delta_r\) have a significant direct effect on their rates. The body rates, in turn, contribute significantly to the derivatives of the aerodynamic angles. Invoking the two time-scale approximation, the body rates become inputs used to control to the aerodynamic angles.
In the slow dynamic inversion, body rate commands $p_c$, $q_c$ and $r_c$ are computed based on external commands $\alpha_c$, $\beta_c$ and $p_{ue}$. In this case, inversion is applied in exactly the same manner as in Reference 2.

The fast dynamic inversion computes control inputs $\delta_a$, $\delta_q$ and $\delta_r$ based on the fast commands $p_c$, $q_c$ and $r_c$. The inversion attempts to replace the actual body rate dynamics with desired dynamics of the form

$$\begin{bmatrix}
\dot{\delta}_a \\
\dot{\delta}_q \\
\dot{\delta}_r
\end{bmatrix} =
\begin{bmatrix}
0 & L_\beta & L_p & L_r \\
M_\alpha & 0 & M_q & 0 \\
0 & N_\beta & N_p & N_r
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
p \\
q \\
r
\end{bmatrix}
+ \begin{bmatrix}
0 & \dot{L}_a \\
0 & \dot{L}_q \\
0 & \dot{L}_r
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_q \\
\delta_r
\end{bmatrix}
$$

where the subscript $d$ refers to desired values. The aerodynamic derivatives in Equation (1) correspond to a linear dynamic model at some condition which is central to the missile's flight envelope.

Applying dynamic inversion results in the following expressions for the control inputs.

$$\begin{bmatrix}
\dot{\delta}_a \\
\dot{\delta}_q \\
\dot{\delta}_r
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & L_\alpha & L_\beta & L_p & L_r \\
M_\alpha & 0 & 0 & M_q & 0 \\
0 & N_\alpha & N_\beta & N_p & N_r
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
p \\
q \\
r
\end{bmatrix}
+ \begin{bmatrix}
0 & \dot{L}_a \\
0 & \dot{L}_q \\
0 & \dot{L}_r
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_q \\
\delta_r
\end{bmatrix}
$$

Structured Singular Value Synthesis

The dynamic inversion control laws effectively equalize the missile dynamics throughout its flight envelope. While the equalized dynamics are linear, they do not exhibit the decoupled first-order command response which characterized the desired fast dynamics in Reference 2. A robust controller must therefore be designed around the linearized plant in order to achieve satisfactory response to body rate commands. This controller need not be gain scheduled, since this is accounted for implicitly in the dynamic inversion. Structured singular value ($\mu$) synthesis is used to design a controller which achieves an approximately decoupled first-order response to commands in $p$, $q$ and $r$ in the presence of multiplicative input uncertainties and measurement errors. Figure 2 illustrates the design plant for $\mu$-synthesis. The weighting functions $W_p$ and $W_q$ are constant, with magnitudes of values of 0.5 and 0.1, respectively.

$$W_p = \frac{s + 30}{s + 0.03}$$

An additional weighting function is included to pre-filter $\alpha$ and $\beta$, which are treated as exogenous inputs to the $\mu$-synthesis interconnection structure. The weight ($W_{ex}$) is described by

$$W_{ex} = \frac{s + 30}{2000s + 60}$$

The $\mu$-synthesis controller is designed to approximate the first-order decoupled body rate dynamics of Reference 2. These dynamics are given by

$$\begin{align*}
\dot{p}_i &= \omega_p(p_c - p) \\
\dot{q}_i &= \omega_q(q_c - q) \\
\dot{r}_i &= \omega_r(r_c - r)
\end{align*}$$

where the bandwidths $\omega_p$, $\omega_q$ and $\omega_r$ are equal to 30 rad/sec.

The $\mu$-synthesis algorithm yields a high-order controller which must be approximated by a system of lower order. The reduced-order controller is computed using a common technique based on balanced realization and optimal Hankel norm approximation.
3. Results

In most respects, the results obtained using the controller architecture described in this paper were comparable to those of Reference 2. Figure 3 shows the robust stability structured singular value with the reduced-order $\mu$-controller in the closed-loop. Robust stability was achieved, but the magnitude of the allowable multiplicative plant uncertainty was reduced from that of Reference 2. Figure 4 depicts the nonlinear step response of the closed-loop system to a stability-axis roll rate command of $p_{\alpha}$ = 300 deg/sec at various angles of attack. The results, computed at $\alpha = 1^\circ$, $10^\circ$, $20^\circ$ and $30^\circ$, indicate that the closed-loop body rate dynamics are not sensitive to variations in the slow states. Figure 5 shows the $\beta$ response for the $\alpha = 1^\circ$, $10^\circ$ and $20^\circ$ cases.

4. Conclusions

The robust nonlinear controller architecture proposed in this paper illustrates how the desired dynamics of nonlinear inversion can affect the role of the linear robust controller. In Reference 2, nonlinear inversion control results in a decoupled first-order response to commands and a $\mu$-synthesis controller is included merely for robustness. In the current system, however, nonlinear inversion control is used to linearize the missile dynamics and provide approximately equal open-loop dynamics throughout the flight envelope. The linear robust controller must not only achieve satisfactory command response in the presence of uncertainty, but also reject the effects of coupling with the slow aerodynamic angle states. This increased dependence on the $\mu$-controller may partially explain why the system is robust to lower levels of uncertainty than those of Reference 2, which makes more effective use of the system's nonlinear control elements.

5. References

