Spherical Metasurface for Radiation Pattern Control

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Abstract—We present a method to reshape the radiation pattern of an antenna into an arbitrarily defined pattern through the use of a spherical impedance metasurface that completely encloses a source. We derive analytical equations to determine the sheet impedance distribution of the spherical metasurface to realize a $4\pi$ steradian desired electric field image pattern. To demonstrate the performance, we reshape the radiation pattern of a small dipole current source into a near-field pattern taking the shape of a nautical anchor.

I. INTRODUCTION

Electromagnetic metamaterials have given engineers greater control over the propagation of electromagnetic fields. This ability has led to a variety of metamaterial approaches used to alter the phase-front profile of a plane wave to a desired distribution [1]–[3]. We expanded this application to apply to cylindrical waves [4], by surrounding a two-dimension radiating line current with a cylindrical metasurface.

In this article, we use a spherical metasurface to transform the radiated electromagnetic field from a finite source into a full $4\pi$ steradian desired image pattern. The source is fully contained within the spherical metasurface, which allows for control of electromagnetic radiation over all angular directions. Similar to the concepts in [3] and [4], the transformation mechanism is achieved through electric field boundary condition interactions at the metasurface.

Finally, we present full-wave simulation results showing the utility of the procedure by reshaping the radiated electric field pattern of a small dipole current source to an image of a nautical anchor at a near-field radial distance.

II. ANALYSIS

The spherical metasurface, with geometry shown in Fig. 1, provides the ability to direct $\hat{\theta}$ and $\hat{\phi}$-polarized electromagnetic waves into a desired image at a specified distance. The first step is to decompose the desired image pattern into orthogonal free-space modes, taking the form

$$E_d(r, \theta, \phi) = C \sum_{n=1}^{\infty} \sum_{m=-n}^{n} R_n(r) \Theta_{mn}(\theta) \Phi_{mn}(\phi).$$

(1)

$R_n(r)$ represents the radial propagation behavior of each mode and is used to reverse propagate the desired image pattern to the metasurface boundary. $\Theta_{mn}(\theta)$ and $\Phi_{mn}(\phi)$ represent the variation of each mode with respect to the elevation and azimuthal angles, and $C$ is a complex scalar. The orthogonality of these modes allows for each to be propagated independently and form an aggregate image at a specified distance.

Since the desired pattern is spherical, the elevation and azimuthal variations can be expressed using a Fourier-Legendre series. The elevation variation for each mode is represented by the associated Legendre polynomial of $\cos(\theta)$, shown by

$$\Theta_{mn}(\theta) = P_n^m(\cos(\theta))$$

(2)

The azimuthal variation of each mode is given as,

$$\Phi_{mn}(\phi) = a_{mn}\cos(m\phi) + b_{mn}\sin(m\phi)$$

(3)

where $a_{mn}$ and $b_{mn}$ are Fourier-Legendre coefficients of the desired image pattern. This expression is similar to a Fourier series except that the coefficients depend on the elevation angle variation in addition to the variation in the azimuth. These coefficients can be calculated from a function representing the desired image pattern using integral equations in [5].

The radially dependent term of all outward-propagating spherical waves are described by spherical Hankel functions of the second kind of the same order as the mode number $n$. With this in mind, the radial behavior of each mode is

$$R_n(r) = \frac{h_n^{(2)}(kr)}{h_n^{(2)}(kr_1)}.$$

(4)

Dividing by $h_n^{(2)}(kr_1)$ normalizes the radial behavior amplitude such that the desired image pattern, or its corresponding Fourier-Legendre series, is observed at the image distance. This expression, along with those for the elevation and azimuthal variations, can be substituted into (1) to provide a mathematical description of the desired electric field pattern.

The complex scalar $C$ is used to scale the Fourier-Legendre series amplitude and provide a phase shift. Therefore, we use

$$C = e^{j\alpha} M_0 E_0,$$

where $e^{j\alpha}$ provides the phase change, $M_0$ is a scalar multiplier, and $E_0$ is the maximum magnitude of the electric field at the image distance radiated by the source.

Fig. 1. Geometry of the spherical impedance metasurface with the interior visible. $r_0$ is the radial distance to the metasurface, $r_1$ is the radial distance to the image pattern, $\theta$ is the elevation angle, and $\phi$ is the azimuthal angle.
As with a Fourier series, higher modes of \( n \) are required to accurately represent a complex desired image pattern with a Fourier-Legendre series. These higher modes can be created with a spherical metasurface of any radius, but they may quickly attenuate during outward propagation if the corresponding evanescent region extends further than the metasurface. Rough boundaries for the evanescent region for each mode \( n \) are given by \( 0 \leq r \leq n/k \) [6]. This boundary is approximate, but it does suggest that an appropriate radius of the metasurface should be \( r_0 > N/k \), where \( N \) is the highest order harmonic in the Fourier-Legendre series.

Once the Fourier-Legendre series is calculated, each mode is reverse propagated to the metasurface by simply setting \( r = r_0 \). The electric field required to be scattered from the metasurface \((E_s)\) is then calculated by subtracting the incident source electric field \((E_i)\) from the desired electric field \((E_d)\) at the metasurface boundary, shown by

\[
E_s(r_0, \theta, \phi) = E_d(r_0, \theta, \phi) - E_i(r_0, \theta, \phi).
\]

The method of moments for surfaces of arbitrary shapes [7] is then used to solve for the surface currents required to generate the necessary scattered electric field from the spherical metasurface. The spherical metasurface must consist of flat triangular patches having dimensions smaller than \( \lambda/10 \) to implement the surface impedance distribution using metamaterial unit cells. After the method of moments, the triangles can be consolidated into square unit cells to complete a metamaterial implementation.

The mutual impedance matrix, \( \mathbf{Z} \), can be calculated since it is purely dependent on the geometry of the metasurface. With \( \mathbf{Z} \) known, the complex surface current magnitudes over each edge of the patch model are determined by using the calculated values of \( E_s(r_0, \theta, \phi) \) with the process in [7].

The current density vector at the center of each triangular face is determined by averaging the current magnitudes over each edge. The current density, \( J \), and desired electric field over each patch face (indexed by \( i \)) are converted to spherical vector representation and used to calculate the required surface impedance of the spherical metasurface.

\[
Z_{\text{sheet}}(i) = \frac{E_{\text{desired}}(i)}{-J(i)}
\]

Since the electric field produced by a dipole is in the opposite direction as the dipole moment (the surface current density of the unit cell in this case) a negative sign is included to produce the correct surface impedance. This ratio, calculated separately for each vector direction, results in the surface impedance in the correct surface impedance. This ratio, calculated separately for each vector direction, results in the surface impedance in the correct surface impedance. This ratio, calculated separately for each vector direction, results in the surface impedance in the correct surface impedance.

III. NEAR-FIELD PATTERN EXAMPLE

To demonstrate the capability of a spherical metasurface to transform the radiated electric field pattern of a source, the radiation pattern of a current dipole is reshaped in both elevation and azimuth to display the image of a nautical anchor at a near-field distance. The desired pattern was imposed on the dominant \( \theta \)-polarized electric field. The mutual impedance matrix \( \mathbf{Z} \) for the spherical metasurface was calculated with and exported from FEKO, a commercial method of moments simulation software, for use in the design procedure.

The metasurface was set to have a radius of \( r_0 = 2\lambda \), an image radius of \( r_1 = 3\lambda \), and we chose to limit the Fourier-Legendre series to \( n = 9 \). \( M_0 = 1.7 \) and \( \alpha = 0.54\pi \) were chosen to result in surface impedance values that could be reasonably implemented using PCB methods.

The spherical metasurface was modeled in Ansys HFSS using planar patches of ideal impedance boundaries. Fig. 2(a) shows the ideal Fourier-Legendre image pattern and Fig. 2(b) shows the simulated image pattern; both cases are normalized to the maximum value of the source electric field at the image distance, \( E_0 \). The normalized root-mean-square error between the simulation results and the ideal Fourier-Legendre pattern is 0.059, showing that the simulation results are in good agreement with the desired pattern.

REFERENCES