Optimizations of Source Distribution in Wireless Power Transmission for Implantable Devices

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Introduction
Implantable medical devices will play an important role in modern medicine for preventive and post-surgery monitoring, drug delivery, local stimulation, and biomimetic prosthesis. To reduce the risk of wire snapping, and replacement and corrosion of embedded batteries, wireless delivery of energy to these devices is desirable. Current studies in wireless power transmission into biological tissue tend to operate below 10 MHz because of the common belief that lower operating frequency yields higher power transfer efficiency. Our previous work [1], however, showed that the optimal frequency maximizing the power transfer efficiency is in the low GHz frequency range. The corresponding wavelength inside tissue is a few centimeters which is in the same order as the dimension of the transmit coil. This suggests that we should consider using an array of transmit coils to focus the electromagnetic energy and hence boost up the received power further.

This paper examines the gain of using an array of transmit coil for wireless power transfer over dispersive tissue. As the received power is proportional to the square of the induced emf, we optimized the source distributions to maximize the induced emf considering the following various constraints and compared those results. First, since the amount of transmit current might be constrained in the actual implementation, we found the source distribution which maximizes the induced emf for a given transmit current. Secondly, from the engineering point of view, the source distribution which maximizes the power transfer efficiency was solved. Lastly, from the clinical safety specification, we evaluated the source distribution to maximize the induced emf for a given peak-value of Specific Absorption Rate (SAR) constraints. After performing the optimizations for various frequencies ranging from 1 MHz to 5 GHz, we found that all three optimizations could potentially boost up the received power by about 2 orders of magnitude at GHz-range compared to those in MHz-range. In the GHz-range, optimization of the source distribution yields at least an order of magnitude better performance than the non-optimized counterpart. Furthermore, the optimal efficiency derived from the three optimization strategies are approximately the same.

Problem Formulation
We modeled a tissue with an air-muscle half-space medium. Although a current source can have any shape, we restricted our current source to be the collection of N by N vertical magnetic dipoles (VMDs) distributed in the square of length l, located on the xy-plane and centered at the origin (Figure 1). Once we solve the field distribution for a single VMD using the Sommerfeld integrals, we could find the optimum source distribution which maximizes the emf induced at the receive dipole located at \( \mathbf{r}_f = -d_f \mathbf{z} \) and pointing in the direction \( \mathbf{n} = \mathbf{z} \) under certain constraints. Suppose \( \mathbf{H}_n(\mathbf{r}) \) and \( \mathbf{E}_n(\mathbf{r}) \) are the electromagnetic fields at \( \mathbf{r} \) emanated from the unit current magnetic moment located at the \( n \)-th current loop out of total \( N^2 \) loops. Then, the total fields are given by:

\[
\mathbf{H}(\mathbf{r}) = \sum_{n=1}^{N^2} s_n \mathbf{H}_n(\mathbf{r}) \quad \mathbf{E}(\mathbf{r}) = \sum_{n=1}^{N^2} s_n \mathbf{E}_n(\mathbf{r})
\]  

(1)

where \( s_n \) is the transmit current of \( n \)-th current loop. The total tissue loss equals to:

\[
P_{\text{loss}} = \frac{\omega \varepsilon_0}{2} \int_{\text{tissue}} \text{Im} \epsilon_r(\mathbf{r}) |\mathbf{E}(\mathbf{r})|^2 d\mathbf{r} = \frac{\omega \varepsilon_0}{2} \sum_{n,m}^{N^2} s_n s_m \int_{\text{tissue}} \text{Im} \epsilon_r(\mathbf{r}) \mathbf{E}_n(\mathbf{r}) \mathbf{E}_m^*(\mathbf{r}) d\mathbf{r}
\]

(2)

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Figure 1: (a) The half-space tissue model. The source is placed on the $xy$-plane and muscle is separated from the source by $d$. The receive dipole is located at $r_f = -d \mathbf{\hat{z}}$. (b) The current sheet made of an array of infinitesimal VMD.

where $\epsilon_0$ is the permittivity of free space, and the induced emf at the receive dipole is given by:

$$emf = \omega \mu_0 A_r |\mathbf{H}(-d \mathbf{\hat{z}}) \cdot \mathbf{\hat{n}}|$$

where $\mu_0$ is the permeability of free space, $A_r$ is the area of the receive dipole. We assumed that the receive dipole is small such that the magnetic field incident on the dipole can be regarded to be constant.

**Optimization Problem**

Since we are going to calculate the tissue loss and SAR numerically in the discrete domain, we have to choose the grid size small enough to avoid significant numerical error. As the wavelength at 5 GHz inside tissue is about $\lambda = 8$ mm, we chose $\lambda/8 = 1$ mm to be our grid size which will enable us to calculate the power loss within a few percent of error. We maximized the induced emf with various constraints as ① given transmit current, ② given total tissue loss, and ③ given peak 1 g-average SAR value. Defining $(N^2 \times 1)$-sized arrays $\mathbf{h}$ and $\mathbf{s}$, and $(N^2 \times N^2)$-sized matrix $\mathbf{K}$ having components as:

$$h|_n = [H^*_n(r_f) \cdot \mathbf{\hat{n}}] \quad s|_n = [s_n],$$

$$\mathbf{K}|_{n_1,n_2} = \left[ \int_{tissue} \text{Im} \epsilon_r(r) \mathbf{E}_{n_1}(r) \mathbf{E}^*_{n_2}(r)dr \right] \approx \frac{1}{M} \sum_{m=1}^{M} \text{Im} \epsilon_r(r_m) \mathbf{E}_{n_1}(r_m) \mathbf{E}^*_{n_2}(r_m)$$

where $r_m$ refers to $m$-th grid point in the tissue and $M$ denotes the number of total grid points in the tissue. Then, the above three optimization problems can be formulated as:

Maximize ① $\frac{\omega^2 \mu_0^2 s^\dagger h h^\dagger s}{s^\dagger s}$

subject to $\frac{\omega^2 \mu_0^2 s^\dagger h h^\dagger s}{\omega \epsilon_0 s^\dagger K s}$

subject to $\omega \epsilon_0 \text{Im} \epsilon_r(r_m) |\mathbf{E}(r_m)|^2 \leq \theta_{th}$ for all $1 \leq m \leq M$

where $(\cdot)^\dagger$ denotes the conjugate transpose operation and $\theta_{th}$ is the product of the grid volume and SAR constraint per gram of tissue. The constraint of ③ is simplified in the formula by restricting the peak value of SAR in the discrete grid domain instead of the peak 1g-average SAR value.

Using the Cauchy-Schwartz inequality and simple linear algebra, one can show that $s_{opt} = h$ and $s_{opt} = K^{-1}h$ are the solutions for ① and ②, respectively. On the other hand, there is no closed-form solution for ③. Instead, we can modify ③ to a convex problem and numerically solve the problem [2]. First, define the $(M \times N^2)$-sized matrix $\mathbf{E}_x$ and $\mathbf{E}_y$ having
components as $E_{x|m_n} = \left[\sqrt{\text{Im} \epsilon_r(r_m)}E_n(r_m) \cdot \hat{x}\right]$ and $E_{y|m_n} = \left[\sqrt{\text{Im} \epsilon_r(r_m)}E_n(r_m) \cdot \hat{y}\right]$, so that $E_{x(y)|m_n}$ refers to the $x(y)$-component of E-field at $r_m$ emanated from the $n$-th source, multiplied by $\sqrt{\text{Im} \epsilon_r(r_m)}$. We don’t have to define $E_z$ since the $z$-component of E-field due to VMD source is always zero. Then, the constraint can be expressed as $\text{Im} \epsilon_r(r_m)|E(r_m)|^2 = |E_x(s(m)|^2 + |E_y(s(m)|^2 \leq \theta'_{th} = \theta_{th}/\omega \epsilon_0$ for all $1 \leq m \leq M$. Since the sum of two squared norm is inherently not convex, we modify the constraints to be convex as $\max_{1 \leq m \leq 2M} (|E(s(m)|)) \leq \sqrt{\theta'_{th}/2}$ where $(2M \times N^2)$-sized $E$ is the vertically concatenated matrix of $E_x$ and $E_y$. And also, since the objective to maximize $s^\dagger h h^\dagger s$ under this max-norm constraints is equivalent to maximize a concave function $\text{Re}(h^\dagger h)$ under the same constraints, ③ can be converted to a convex optimization problem as:

$$\text{Maximize } \omega^2 \mu_0^2 \text{Re}(h^\dagger h) \text{ subject to } \max_{1 \leq m \leq M} (|E(s(m)|)) \leq \sqrt{\theta'_{th}/2} \quad (5)$$

Now, we could use a conventional convex optimization program (for example, cvx [3] in Stanford) to numerically solve this.

All the solutions for optimizations were scaled afterwards to make sure that the peak 1g-average SAR value is 1.6 mW/cm$^3$. If our purpose is to maximize emf and therefore maximize the received power $P_r$ in the tissue with constraints on SAR, the optimization ③ would be the most direct and certain method to achieve it. However, unlike ① and ②, ③ is a numerical convex optimization problem which takes long time to solve it in real-time and requires lots more knowledge about the dielectric properties of the tissue which might not be feasible in practice.

**Numerical Example**

In this work, we used variables of $(d, d_f, l, N, M) = (6 \text{ mm}, 4 \text{ cm}, 8 \text{ cm}, 11, 71 \times 71 \times 30)$ to numerically evaluate the results. If we use an array of uniform sources, there is no focused $|emf|^2$ visible and SAR distributes particularly high below the envelop of source coils near the air-muscle interface and decreases sharply within the tissue. Using the optimal source distribution, however, Figure 2 shows obviously enhanced $|emf|^2$ around the focal point and relatively evenly distributed SAR across the tissue at 1.8 GHz frequency. This advantageous effect of source optimization becomes much less obvious in MHz frequency range.

To see the results more quantitatively, Figure 3 shows the various optimization results as well as the result from uniform source versus frequency. We adjusted each result so that the peak 1g-average SAR in the tissue is always 1.6 mW/cm$^3$. To explain this in more detail, first we normalize the source distribution so that the norm of each source distribution becomes 1, i.e. $\frac{1}{N} \sum_{n=1}^{N} |x_n|^2 = 1$. Then, the field distribution and the average SAR for each 1 cm$^3$ volume around each grid point is calculated. The source distribution is re-adjusted to make the peak 1g-average SAR become 1.6 mW/cm$^3$, and the field distribution is also scaled by the same factor. Figure 3(c) plots the transmit current $I_t$ which is the normalization
Figure 3: Frequency sweep.

factor. Figure 3(a) plots $|emf|^2$ versus frequency after normalization. The $|emf|^2$ in GHz-range frequency is almost 50 times higher than that in 1 MHz frequency. The optimizations are effective in the GHz-range because the wavelength in this range is comparable to the dimension of the source. Defining the efficiency as $\eta := \frac{1}{\pi |emf|^2} P_{\text{loss}}$, Figure 3(b) plots the efficiency versus frequency. Interestingly, all three optimization strategies show similar $|emf|^2$ and efficiency. This implies that we don’t have to insist to use time-consuming emf maximization with the local SAR constraint. Rather, we can choose to maximize $|emf|^2$ with the constraint of $P_{\text{loss}}$ or transmit current to save computation and more importantly, to lax the need to estimate the dielectric properties of the medium.

Another observation is that in the MHz-range frequency, transmit current to reach SAR constraint is above 10 A for all cases, which is practically infeasible. In other words, at MHz-range frequency, before we violate the SAR constraints, it would be the current supply capability which limits the power transfer into the tissue. On the other hand, at GHz-range frequency, since only few tens of mA is required to meet the SAR constraint, we are not limited by the current supply capability and can fully exploit the SAR constraint to maximize the induced emf.

Conclusions

In this work, we investigated the source distributions which maximize the induced emf at the implant inside tissue under various constraints. Higher induced emf at the receive dipole is desirable since it will transfer more power into the device in the tissue if we assume the load impedance unchanged. From the results presented, we claimed that the advantage of GHz-range power carrier over MHz-range carrier is twofold. First, we can fully exploit the SAR constraints and maximize the $|emf|^2$ at the focal point where the receiver locates. On the other hand, as the current capability that can be supplied by modern devices is usually no more than 1 A, the current supply capability works as a bottleneck to power transfer for devices operating in the MHz-range. Second, we could achieve about 50 times higher $|emf|^2$ in GHz-range than that in the MHz-range.

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References

