Volume-Surface Integral Equations with Hybrid Curl-Conforming and Divergence-Conforming Basis Functions

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Introduction

The Volume-Surface Integral Equation (VSIE) has been successfully used in computational electromagnetics for complicated structures, especially for coated objects. Since the VSIE only requires a mesh on the object, it can be used to solve scattering problems of electrically large complicated objects with high accuracy when combined with the Fast Multipole Method (FMM). The VSIE method is characterized by the use of different integral equations on perfect electric conductor (PEC) surfaces and in penetrable objects. The equations are discretized with the extended Galerkin projection method. Usually, scattering problems involving PEC surfaces are solved using Surface Integral Equations (SIE), where the RWG basis functions are commonly used to represent the unknown surface current density. Scattering problems involving dielectric objects can be solved using either the SIE (piecewise homogeneous object only) or the Volume Integral Equation (VIE). When it comes to the selection of a basis for the VIE, both the divergence-conforming and curl-conforming basis functions can be used with no particular advantage provided by one over the other [1]. Divergence-conforming basis functions are usually used with electric flux density as unknowns, because the normal continuity of electric flux density can be enforced. Curl-conforming basis functions are used with electric field as unknowns since the tangential continuity of electric field across the mesh element is automatically enforced. Research on coated objects using VSIE with RWG basis on surface and divergence-conforming basis functions in dielectric volume can be found in [6].

In this paper, the VSIE solved by a hybrid curl-conforming and divergence-conforming basis functions is presented. In this method, new EFIEs are formed first. Then curl-conforming edge basis functions are used for the VIE with the electric field as the unknowns and divergence-conforming edge basis functions are used for the SIE on PEC surfaces with the surface electric current as the unknowns. Furthermore, we set the coefficient for the volume edge basis that are in contact with the conducting surface to zero. This is done because they are the bases which contribute tangential electric field on the PEC surface. It reduces the total number of unknowns, which will be very helpful when the object is a thin dielectric slab with PEC mounted on the surface such as occurs in considering microstrip circuits. To validate this method, a C++ program is implemented and simulation results are compared with the analytical solution for spherical scatterers. Moreover, the performance of this method is compared with the approach that is based on using only divergence-conforming basis functions. Throughout the paper, $e^{j\omega t}$ time convention is assumed and it is suppressed in all equations. The objects’ permeability are assumed as constant $\mu_0$. 

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EFIEs and Impedance Matrix

The forms of the integral equations are critical in the VSIE method not only for their validity but also for the convenience of the basis functions’ selection. Different forms of Electric Field Integral Equations (EFIE) have been studied by many researchers [1][3-5][7,8]. The new EFIEs are based on those studies and are carefully formed by considering the characteristics of basis functions.

On a PEC surface, the tangential component of the scattered electric field and the incident electric field cancels each other, which can be written as

\[-E^{inc}(r)|_{tan} = [j\omega\mu_0 \int_S \left( 1 + \frac{1}{k^2} \nabla \nabla \cdot \right) G_0(r, r') J_s(r')dr' + E_{vol}^{sca}]|_{tan}\]  

(1)

where \( G_0(r, r') = e^{-jk|r-r'|}/4\pi|r-r'| \) is the Green’s function in free space, \( J_s(r') \) is the induced surface current by the incident wave. The integration term in equation (1) is the scattered electric field due to the induced surface current, while \( E_{vol}^{sca} \) is the scattered field on the PEC surface due to the equivalent volume current \( J_{eq}(r) = j\omega\epsilon_0[\epsilon(r) - 1]E(r) \). The EFIE in the dielectric volume can be summarized as

\[ E_{vol}^{sca} - E = -E^{inc} \]  

A new EFIE presented in [1] contains only weakly singular kernel when using curl-conforming basis functions. But it is limited to the dielectric objects. For complicated objects, scattering from PEC surface should also be considered. Thus, a modified EFIE is formed as

\[-E^{inc}(r) = \nabla \times \int_V \nabla G_0(r, r') \times \tau(r')E(r')dr' - \epsilon_r(r)E(r) + E_{vol}^{sca}\]  

(2)

where \( \tau(r') = \epsilon_r(r') - 1 \) and \( E_{vol}^{sca} \) is the scattered field in dielectric volume due to the PEC surface current \( J_s \). After formation of equation (1) and (2), the formulation of \( E_{vol}^{sca} \) and \( E_{vol}^{sca} \) needs to be decided. Although the curl-form of scattered field representation \( E_{vol}^{sca}(r) = \frac{1}{j\omega\epsilon_0} \nabla \times \int \nabla G_0(r, r') \times J(r')dr' \) can be an option from the perspective of symmetry, our research has shown that it is neither a good choice for \( E_{vol}^{sca} \) nor for \( E_{vol}^{sca} \). And we found that the following formulations are more appropriate for the new EFIE from the perspective of numerical implementation.

\[ E_{vol}^{sca}(r) = \nabla \int_V \nabla G_0(r, r') \cdot \tau(r')E(r')dr' + k_0^2 \int_V G_0(r, r')\tau(r')E(r')dr' \]  

(3)

\[ E_{vol}^{sca}(r) = \frac{j}{\omega\epsilon_0} \nabla \int_S G_0(r, r')\nabla \cdot J(r')dr' - j\omega\mu_0 \int_S G_0(r, r')J(r')dr' \]  

(4)

To solve the VSIE for the unknown density functions, the current density on PEC surfaces is expanded by RWG basis \( f_n^S \) as \( J_s = \sum_{n=1}^{NS} a_n f_n^S \), and the total electric field in dielectric medium by lowest order of edge basis \( f_n^V \) as \( E = \sum_{n=1}^{NV} a_n f_n^V \). Then extended Galerkin’s projection method is used to discretize the EFIEs on PEC surface by \( Z_{mn}^{S*} = \frac{1}{j\omega_0} \langle f_m^S, E_n^{sca} > \) and in dielectric volume by \( Z_{mn}^{V*} = < f_m^V, E_n^{sca} > - E_n > \), where \( <,> \) is defined as reaction inner product [4]. \( E_n^{sca} \) and \( E_n \) are the scattered and the total electric field contributed only by the nth basis, respectively. The impedance matrix is in the form

\[ Z = \begin{bmatrix} Z_{SS} & Z_{SV} \\ Z_{VS} & Z_{VV} \end{bmatrix} \]  

(5)
where the superscript $S$ (surface) and $V$ (volume) are used to represent that the source or test basis function is on a surface or in a volume, respectively. For example, $Z^{SV}$ are the impedance elements of those tested on a surface with the source in a volume. The $Z^{SS}$ can be found in [5] for conducting surfaces.

One of the challenging aspects to evaluate the impedance matrix $Z$ is that singular integral kernels exist when the source and testing cells (triangles or tetrahedrons) are overlapping. One way to treat such scenarios is to use singular extraction to extract the singular terms which can be computed analytically. The other way is to use Duffy’s transformation to regularize the integrals by cancelling the singularity and then computing the integration numerically. In the latter method, high order singularities should be avoided in the integrals. When Duffy’s transformation is used in numerical integrations, $1/R$ order of singularity (such as $G_0$) is appropriate for surface integration, while $1/R^2$ order of singularity (such as $\nabla G_0$) is limited for volume integration. Higher order of singularity may require a greater number of quadrature points or result in less accuracy. In calculating impedance $Z$ matrix, vector identities $\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$ and $\nabla \cdot (aA) = \nabla a \cdot A + a \nabla \cdot A$ are used to move one of the derivative operations to the test functions. After substitution of the vector identities, Guass’ theorem is used to transfer the extra integration to an enclosed surface or line integral. Furthermore, after simplification, the enclosed line integration in $Z^{SV}$ is zero for the RWG basis and one volume integration term in $Z^{VS}$ is also zero due to $\nabla \cdot f^V_m(r) = 0$ for the zeroth order edge basis.

**Results and Discussions**

The test case is a conducting sphere with radius $r = 3.0m$, coated by a dielectric shell with thickness $d = 0.5m$. The dielectric constant of the shell is $\epsilon_r = 4 - j$. The dielectric shell is meshed with 1,875 tetrahedron, while the conducting sphere is modeled by 528 triangles. The average edge length is $l = 0.7m$, and the number of edge basis functions is 5,358. The operating frequency is $f = 15MHz$. An incident plane wave illuminates the object from the angle $\theta^{inc} = 0$ and $\phi^{inc} = 0$. The scattering angle $\phi^{sca}$ is fixed to 0 and $\theta^{sca}$ varies from 0 to 180 degrees. As shown in Figure 1, there is good agreement between the numerical results of this method and the analytical MIE series results. We then compared this result with the all divergence-conforming basis VSIE method using the same mesh file. It is observed that the accuracy of the two methods is comparable (the hybrid basis method is a little bit better in the test case). But 8,438 basis functions are used in the all divergence-conforming basis method. This is 1.57 times the unknowns used in the new method. The converge rates of these two methods are almost the same in this case when an iterative solver is used to solve the linear equations.

In summary, by choosing the right form of scattered field representation due to the PEC surface current and equivalent volume current to form the EFIEs for SIE and VIE, curl-conforming and divergence-conforming basis can be used together to solve scattering problems of complicated objects. From the numerical results, the accuracy of the new method is comparable to the all divergence-conforming method using the same mesh, while the number of unknowns is greatly reduced for the new approach of this paper.
Figure 1: RCS of a coated conducting sphere with radius $r = 3.0m$; thickness of the dielectric material $d = 0.5m$; dielectric constant $\varepsilon_r = 4 - j$; frequency $f = 15MHz$. HVSIE shown in the legend is the new hybrid VSIE method, while VSIE is the all divergence-conforming basis method. The solid lines (MIE) are the analytical solutions.

References


