Design of Efficient Terahertz Antennas: Carbon Nanotube versus Gold

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Introduction

Carbon nanotube (CNT) has been introduced as a new material for antenna radiators and transmission lines in Terahertz frequencies. The intrinsic conductivity of single-wall carbon nanotube (SWNT) is rather higher than that of copper in DC, however due to its very small radius (on the orders of a few nanometers), its resistance per unit length is very high [1]. These properties limit its utilization for RF applications, such as electrical interconnects and radiating elements. Thus, to reduce the high resistance of SWNT, the bundled structure of CNTs is suggested [2]. In laboratory measurements up to 50 GHz, it was shown that the resistance of bundled CNT is simply equal to component value of discrete circuit element of SWNT divided by its number density [CNTs/µm] [2]. The current fabrication technology produces a number density of 10 [CNTs/µm] in well-aligned arrays [3]. The discrete circuit model of SWNT in theory [4], the series kinetic inductance $L_k$ of 16nH/µm and the resistance of 6.5kΩ/µm are employed in this paper. In addition, the fact that the quality factor of this RL series circuit of SWNT is approximately 100 at frequencies higher than 6.5 THz invokes the application of SWNTs as antenna radiators at terahertz frequencies. This high Q factor at terahertz frequencies is preserved in the bundled structure because SWNTs in the bundle are connected in parallel. Also, one dimensional transport of electrons along SWNT indicates that the concept of skin depth for the conventional conductor is not applicable, so the high conductivity of SWNT is obtained even though the thickness of bundled SWNT is very small compared to the wavelength. On the other hand, the surface resistivity of general conductors increases with frequency due to skin depth effect. Thus, it is expected that the performance of CNT antenna should surpass that of metallic antennas in terms of radiation efficiency at a particular frequency in Terahertz region. To apply high frequency properties of metal (the thin gold film in this paper), the Drude model is utilized [5]. In this paper, an anisotropic resistive sheet model to represent bundled CNTs electromagnetically is presented. Then, a numerical simulation using method of moments (MoM) is developed to calculate radiation efficiencies of resonant strip antennas made up of bundled CNT and thin gold film. The radiation efficiencies are compared as a function of frequencies and number density of bundled CNT to find a crossover frequency where CNT antenna outperforms the thin gold film antenna.

Resistive Sheet Model of Bundled Carbon Nanotube

In a bundled CNT, the electrons in each single SWNT are predominantly passing along the tube axis while there is also some movement in the transverse direction, which indicates that electrons can jump to adjacent SWNTs. Even though this microscopic electron transport is occurring randomly, the reported experiments [2], which have verified the scalability of inductance element of SWNT in the bundled structure, indicate that the overall current along the bundle is axial. Thus, to examine the electromagnetic properties of bundled CNT aligned in a planar structure, a macroscopic model is needed.
Since the thickness of a layer bundled CNT is much thinner than the wavelength up to optical frequencies, such material can be modeled by a resistive sheet. If \( \hat{n} \) is the unit vector normal to the resistive sheet drawn at the upward (positive) side of the sheet and \([\ldots]\) denotes the discontinuity across the sheet, the boundary conditions are shown in (1). Fig. 1 shows the proposed geometry of a strip dipole antenna constructed from strands of CNTs. Each SWNT in the bundle is equivalent to series inductance (\( L_k \)) and resistance (\( R_{CNT} \)), which are connected in parallel. Since SWNT is aligned along x axis, only the x-directed electric field can excite the current on the surface.

\[
\hat{n} \times \hat{n} \times \mathbf{E} = -\overline{R} \cdot \mathbf{J}; \quad \mathbf{J} = [\hat{n} \times \mathbf{E}]^\tau \quad (1)
\]

\[
[R_x] = \frac{(R_{CNT} + j\omega L_k)}{N} \quad (2)
\]

where

\[
\overline{R} = \begin{bmatrix} R_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3)
\]

Figure. 1: Geometry of a strip antenna made up of bundled carbon nanotube.

In the rectangular segment indicated by \( \Delta \) and \( \delta \) in Fig. 1, the surface resistivity along x axis is derived as shown in (2) by dividing voltage across \( \Delta \) with the current flowing through \( \delta \). \( N \) represents the number of SWNTs in unit width. The surface resistivity (\( R_x \)) of bundled CNT is proportional to the resistance and kinetic inductance of SWNT and is inversely proportional to number density of SWNTs. This one directional current flowing causes the anisotropic property and results in \( \overline{R} \), the complex surface resistivity dyad as shown in (3). The boundary condition (1) is implemented in a MoM code developed for antenna simulations.

**Strip Dipole Antenna Simulation**

To evaluate the performance of the antenna made up of bundled CNTs and thin gold film, the length (\( L \)) of strip dipole antenna in Fig. 1 is found to get a resonance half-wavelength over the frequency range of 1 THz to 35 THz. The length of CNT antenna is less than \( \lambda / 2 \) and depends on number density of SWNT (\( N \)) and the kinetic inductance of its equivalent circuit. The width (\( w \)) is simply chosen to be \( L / 6 \).

Figure. 2: Normalized antenna length (2L/\( \lambda \)) as a function of frequency using different CNT density (\( N: \) CNTs/\( \mu m \)).

Figure. 3: Input impedances of the strip antennas shown in Fig. 2.
Fig. 2 and 3 are obtained from MoM simulations for CNT and thin gold film antennas. Fig. 2 shows the normalized antenna length ($2L/\lambda$) at the first resonance for a different CNT number density and also for the thin gold film antenna. Fig. 3 shows the input impedance of the antennas shown in Fig. 2 at their resonant frequencies. Each point in the figures is calculated from an antenna design which has a specific length ($L$) and a material property. As shown in Fig. 2, when CNT density is low, the resonant frequency happens at frequencies where $2L/\lambda$ is less than one. This is due to the fact that series kinetic inductance lowers the resonant frequency. Otherwise, as the number density increases, the series inductance is lowered to a point that effective inductance becomes very small. Fig. 3 shows that the CNT antenna input resistance decreases with frequency. Otherwise, the metallic antenna input impedance is almost constant or slightly increases with frequency. Finally, the radiation and antenna efficiency at the resonant frequencies for CNT and gold antennas are calculated. Fig. 4 shows the antenna efficiency of CNT and gold antennas without considering input mismatch of antennas to 50 $\Omega$ feeding transmission line while Fig. 5 shows the radiation efficiency which accounts for the input reflection to 50 $\Omega$ line.

Figure. 4: Antenna efficiency of strip dipole antenna made up of bundled CNTs (N: CNTs/ $\mu$m) and thin gold film.

Figure. 5: Radiation efficiency (with 50 $\Omega$ matching) of strip dipole antenna made up of bundled CNTs (N: CNTs/ $\mu$m) and thin gold film.
The radiation efficiency of CNT antenna significantly drops compared to the antenna efficiency due to a very small input resistance at Terahertz frequencies. As shown in both figures, the CNT antenna with a number density lower than $10^3$ [CNTs/$\mu$m] does not show efficient radiation efficiency. Also, the gold thin film antenna designed by the Drude model shows a significantly lower efficiency compared to that of DC gold in Terahertz frequencies. To evaluate bundled CNTs as Terahertz antenna applications, the efficiency of CNT antenna should be compared with that of a conventional thin gold film. Fig. 3 and 4 indicate that a number density of CNT antenna should be approximately $3\times10^4$ [CNTs/$\mu$m] to surpass a thin gold film antenna. The radiation efficiency in Fig. 5(b) shows that when the number density (N) is $2\times10^4$ [CNTs/$\mu$m], the radiation efficiency of CNT antenna outperforms thin gold film up to 4 THz. However at frequencies higher than 4 THz, the big mismatch caused by a small input resistance of CNT antenna makes its radiation efficiency lower than that of thin gold film antenna.

**Conclusion**

By using scalability of equivalent circuit of SWNT in its bundle structure, bundled CNTs are modeled by the macroscopic resistive sheet. The numerical simulation implements the boundary condition of the surface resistivity. The results show that the bundled CNT antennas where the density of SWNT lower than $10^5$ [CNTs/$\mu$m] do not show effective radiation efficiency. To be an efficient radiator, the number density of the CNT antenna should be higher than $3\times10^4$ [CNTs/$\mu$m], which indicates the necessity of more densely aligned CNTs in the fabrication process. Thus, bundled CNTs as an antenna radiator in Terahertz region should have a density of approximately thousands of times higher than the currently realizable density, $10$ [CNTs/$\mu$m] inside the bundled structure.

**References**