Introduction

Vector potentials are frequently used in electromagnetics as a mechanism to simplify analytical difficulties when determining electromagnetic fields that result from current distributions [1]. When real current distributions are present, the vector potential “A” is commonly used and oriented in the direction of the current (e.g., the z-direction for a dipole aligned along the z-axis.) The vector potential “F” is commonly used when fictitious magnetic currents are presumed (e.g., equivalent magnetic currents on the aperture of a horn antenna.) Again, “F” is oriented in the same direction as the magnetic current. Less often used is the fact that a transverse source (real or fictitious) can simultaneously excite both vector potentials [2]. Such would be the case for an equivalent filament of magnetic current about the perimeter of a patch antenna exciting both vector potentials in a direction orthogonal to the patch. This paper examines the generation of these coupled vector potentials for a patch antenna geometry.

Formulation of the Problem

Figure 1 – Patch antenna geometry and expressions for infinitesimally distributed magnetic currents in the $\widetilde{a}_{\phi}$ or the $\widetilde{a}_{\rho}$ directions.

As a case study, we consider patch geometry as shown in Figure 1. The study will presume magnetic currents expressible in cylindrical coordinates - suitable
for a patch antenna of circular, pie, as well as other canonical shapes. As such, currents will be in the \( \vec{a}_\phi \) or the \( \vec{a}_\rho \) directions. Implicitly understood, is that procedure will give the Green’s functions for the geometry of interest. The potentials are then found after multiplying the magnetic currents and integrating over the primed coordinates as explicitly given in equations 1.0 and 2.0. The superscript “2” on the vector potentials indicates that they are valid for the region \( d \leq z \leq \infty \) while the superscript “1” indicates the region \( 0 \leq z \leq d \). Note that this paper closely follows the methodology and definitions given in [3] for problems of this nature.

\[
\begin{align*}
\begin{bmatrix}
F_z^1(\vec{r}) \\
A_z^1(\vec{r})
\end{bmatrix} &= \begin{bmatrix}
\bar{a}_z \psi^{\prime 1}(\vec{r}) \\
\bar{a}_z \psi^{\prime \ast 1}(\vec{r})
\end{bmatrix} = \bar{a}_z \int_s \begin{bmatrix}
G_{z\phi}^1(\vec{r}';\vec{r}) \\
G_{z\rho}^1(\vec{r}';\vec{r})
\end{bmatrix} \begin{bmatrix}
M_{\phi}(\vec{r}') \\
M_{\rho}(\vec{r}')
\end{bmatrix} d\vec{s}' \\
F_z^2(\vec{r}) &= \begin{bmatrix}
\bar{a}_z \psi^{\prime 2}(\vec{r}) \\
\bar{a}_z \psi^{\prime \ast 2}(\vec{r})
\end{bmatrix} = \bar{a}_z \int_s \begin{bmatrix}
G_{z\phi}^2(\vec{r}';\vec{r}) \\
G_{z\rho}^2(\vec{r}';\vec{r})
\end{bmatrix} \begin{bmatrix}
M_{\phi}(\vec{r}') \\
M_{\rho}(\vec{r}')
\end{bmatrix} d\vec{s}'
\end{align*}
\]

1.0

The electric and magnetic fields are related to these vector potentials through:

\[
E_\rho = \frac{1}{j \omega \epsilon} \frac{\partial^2}{\partial \rho \partial z} \psi^a(\rho, \phi, z) - \frac{1}{\rho} \frac{\partial}{\partial \phi} \psi^f(\rho, \phi, z) \quad 3.0
\]

\[
E_\phi = \frac{1}{j \omega \epsilon \rho} \frac{\partial^2}{\partial \phi \partial z} \psi^a(\rho, \phi, z) + \frac{\partial}{\partial \rho} \psi^f(\rho, \phi, z) \quad 4.0
\]

\[
E_z = \frac{1}{j \omega \epsilon} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \psi^a(\rho, \phi, z) \quad 5.0
\]

\[
H_\rho = \frac{1}{j \omega \mu_0} \frac{\partial^2}{\partial \rho \partial z} \psi^f(\rho, \phi, z) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \psi^a(\rho, \phi, z) \quad 6.0
\]

\[
H_\phi = \frac{1}{j \omega \mu_0 \rho} \frac{\partial^2}{\partial \phi \partial z} \psi^f(\rho, \phi, z) - \frac{\partial}{\partial \rho} \psi^a(\rho, \phi, z) \quad 7.0
\]

\[
H_z = \frac{1}{j \omega \mu_0} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \psi^f(\rho, \phi, z) \quad 8.0
\]

Solutions for currents residing at the air-dielectric interface then utilize the boundary condition:
Clearly, equation 9.0 couples the $z$-components of the electric and magnetic fields. Accordingly, the $z$-directed vector potentials are also coupled. Using the Fourier-Bessel transform pair given in equations 10.0 and 11.0, one can arrive at the dyadic Green’s functions given in equation 12.0. Note that this equation first finds solutions in the spectral domain and then uses the inverse transform to convert back to the spatial domain in this equation.

\[ -\hat{a}_z \times (\vec{E}_r(d^+) - \vec{E}_r(d^-)) = \vec{M}_s, \]

\[ n=+\infty \]

\[ \psi(\rho, \phi, z) = \sum_{n=-\infty}^{n=+\infty} e^{jn\phi} \int_0^\infty J_n(\beta \rho) \bar{\psi}_n(\beta, z) \beta \, d\beta \]

\[ n=+\infty \]

\[ \bar{\psi}_n(\beta, z) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty J_n(\beta \rho) \psi(\rho, \phi, z) e^{-jn\phi} \rho \, d\rho \, d\phi \]

\[ \left[ \overline{G}^{f2} (\vec{r}, \vec{r}') \right] = \frac{\overline{G}}{4\pi} \sum_{n=-\infty}^{n=+\infty} \int [H_n^{(2)}(\beta \rho) \cos (k_1 d) e^{-jk_1(z-d)}] \Phi_n(\phi-\phi') \]

\[ n=+\infty \]

\[ \frac{j n}{\beta \rho} J_n(\beta \rho') \frac{k_1}{T_e(\beta)} \overline{\alpha}_\rho \]

\[ - j \frac{n}{\beta \rho} k_1 J_n(\beta \rho') \frac{1}{T_e(\beta)} \overline{\alpha}_\phi \]

\[ J_n(\beta \rho') \frac{k_1}{Z_o T_m(\beta)} \overline{\alpha}_\rho \]

\[ + \frac{1}{Z_o T_m(\beta)} \overline{\alpha}_\phi \]

These dyads are now in a form suitable for integration by asymptotic techniques such as saddle point integration. Proceeding with saddle point integration and integrating the dyads over the appropriate current distributions, one obtains:

\[ \left[ \overline{F}_z(r, \theta, \phi) \right] \sim \frac{1}{2\pi} \frac{e^{-jkz}}{r} \cos\left[ k_1(\theta)d \right] \cos(\theta) \left[ \sum_{n=-\infty}^{n=+\infty} \int e^{jk_1(\theta)d \cos(\theta)} \right] \]

\[ \left[ \overline{A}_z(r, \theta, \phi) \right] \sim \frac{1}{2\pi} \frac{e^{-jkz}}{r} \sin(\theta) \left[ \sum_{n=-\infty}^{n=+\infty} \int e^{jk_1(\theta)d \sin(\theta)} \right] \]

\[ -\overline{a}_z \]

\[ \frac{j k_1(\theta)}{T_e(\theta)} \cos(\phi-\phi') \]

\[ \frac{j k_1(\theta)}{T_e(\theta)} \sin(\phi-\phi') \]

\[ \frac{j k_e r}{Z_o T_m(\theta)} \sin(\phi-\phi') \]

\[ \frac{j k_e r}{Z_o T_m(\theta)} \cos(\phi-\phi') \]

\[ M_{\rho'}(\rho', \phi') \]

\[ M_{\phi'}(\rho', \phi') \]

\[ \rho' \, d\rho' \, d\phi' \]

The spherical coordinate representation of the radiated fields is now easily found using:
Conclusion

Solutions detailing the radiated fields of patch antennas often determine the electric field components. These components are then combined with appropriate coordinate transforms to realize the spherical coordinate representation suitable for plotting radiation patterns. This paper presents a slight variation on this process that eliminates such a tedious calculation and realizes the electric fields directly from the asymptotic evaluation of the z-directed vector potentials “A” and “F”.

References