Organizing Integral Equation for Complex Structure for Object Oriented Programming

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Introduction

With rapid advances in computational electromagnetics, numerous new models, formulations, and algorithms are constantly emerging. As a result, the algorithms in this area become increasingly complex. Often time, codes are rewritten to incorporate complex ideas, and it can be a wasteful and repetitive process. Integral equation solvers are in general more complex compared to differential equation solvers. Hence, in order to arrive at a maintainable, sophisticated integral equation solver code, object oriented programming (OOP) paradigm has to be adopted. The EIGER project [1] was one such effort to develop more maintainable integral equation solver codes for electromagnetics. We propose here another view of OOP for integral equation solvers, emphasizing on the operator aspect of them. Hence, we could enjoy the advantages of a typical OOP code: reusability, flexibility, reliability, maintainability, extendibility, and readability. This code can model complex structures by the method of moments with fast multiple algorithm (FMA) as the accelerator. In the future, it can be easily extended to incorporate new features.

Formulation

A complex structure usually involves homogeneous, inhomogeneous or anisotropic dielectric, closed and open PEC surface, thin dielectric sheet (TDS), impedance boundary condition (IBC) and etc. In order to handle this kind of problems, first the entire environment is divided into regions and each object is associated with one or two regions. We express the scattered field on each object in terms of all the induced currents in this region and then combine those field current equation to obtain a set of MOM equations. Since this way of organization is physics-driven, it is flexible to form different equations, such as combinations of EFIE, MFIE, CFIE with PMCHWT [2] or Muller and VIE [3]. Each region is characterized by a unique Green’s function so that it is convenient to incorporate FMA.

A typical scattering problem is shown in Fig 1. S1 is a homogeneous dielectric object with some other objects embedded in it, S2 is a PEC surface, and V3 is an inhomogeneous dielectric object. For any region, usually there are six kinds of currents,

\[ J_{sd} \] - surface dielectric electric current
\[ M_{sd} \] - surface dielectric magnetic current
\[ J_{sp} \] - surface PEC electric current
\[ J_{st} \] - surface TDS electric current
\[ J_v \] - volume electric current

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Figure 1: A typical complex structure with various objects

\( \mathbf{M}_v \) - volume magnetic current

The electric field produced by these currents can be expressed in region 0 as

\[
\mathbf{E}_0(\mathbf{r}) = \mathcal{L}^{e}\mathbf{J}_{sd} + \mathcal{K}^{e}\mathbf{M}_{sd} + \mathcal{L}^{e}\mathbf{J}_{sp} + \mathcal{L}^{e}\mathbf{J}_{st} + \mathcal{L}^{e}\mathbf{J}_{v} + \mathcal{K}^{e}\mathbf{M}_{v} \tag{1}
\]

Similarly, the magnetic field in region 0 is

\[
\mathbf{H}_0(\mathbf{r}) = \mathcal{K}^{h}\mathbf{J}_{sd} + \mathcal{L}^{h}\mathbf{M}_{sd} + \mathcal{K}^{h}\mathbf{J}_{sp} + \mathcal{K}^{h}\mathbf{J}_{st} + \mathcal{K}^{h}\mathbf{J}_{v} + \mathcal{L}^{h}\mathbf{M}_{v} \tag{2}
\]

For homogeneous dielectric, the field in region \( i \) can be written as

\[
\mathbf{E}_i(\mathbf{r}) = \mathcal{L}^{e}\mathbf{J}_{sd} + \mathcal{K}^{e}\mathbf{M}_{sd} \tag{3}
\]

\[
\mathbf{H}_i(\mathbf{r}) = \mathcal{K}^{h}\mathbf{J}_{sd} + \mathcal{L}^{h}\mathbf{M}_{sd} \tag{4}
\]

where the operators are defined as

\[
\mathcal{L}^{e}\mathbf{X}(\mathbf{r}) = \int_D dv' \left( \mathbf{I} + \frac{\nabla \nabla}{k_0^2} \right) g(\mathbf{r}, \mathbf{r}') \cdot \mathbf{X}(\mathbf{r}')
\]

\[
\mathcal{K}^{e}\mathbf{X}(\mathbf{r}) = \nabla \times \int_D dv' \mathbf{X}(\mathbf{r}') g(\mathbf{r}, \mathbf{r}')
\]

\[
\mathcal{L}^{h}\mathbf{X}(\mathbf{r}) = \frac{1}{\eta^2} \mathcal{L}^{e}\mathbf{X}(\mathbf{r})
\]

\[
\mathcal{K}^{h}\mathbf{X}(\mathbf{r}) = -\mathcal{K}^{e}\mathbf{X}(\mathbf{r})
\]

We will have to test Equations (1) and (2) over different surfaces and volume to form the integral equations. By so doing, we have

\[
\mathbf{E}_{0d}(\mathbf{r}) = \mathcal{L}^{e,\mathbf{J}_{sd}} + \mathcal{K}^{e,\mathbf{M}_{sd}} + \mathcal{L}^{e,\mathbf{J}_{sp}} + \mathcal{L}^{e,\mathbf{J}_{st}} + \mathcal{L}^{e,\mathbf{J}_{v}} + \mathcal{K}^{e,\mathbf{M}_{v}} \tag{5}
\]

\[
\mathbf{E}_{0p}(\mathbf{r}) = \mathcal{L}^{e,\mathbf{J}_{sd}} + \mathcal{K}^{e,\mathbf{M}_{sd}} + \mathcal{L}^{e,\mathbf{J}_{sp}} + \mathcal{L}^{e,\mathbf{J}_{st}} + \mathcal{L}^{e,\mathbf{J}_{v}} + \mathcal{K}^{e,\mathbf{M}_{v}} \tag{6}
\]

\[
\mathbf{E}_{0v}(\mathbf{r}) = \mathcal{L}^{e,\mathbf{J}_{sd}} + \mathcal{K}^{e,\mathbf{M}_{sd}} + \mathcal{L}^{e,\mathbf{J}_{sp}} + \mathcal{L}^{e,\mathbf{J}_{st}} + \mathcal{L}^{e,\mathbf{J}_{v}} + \mathcal{K}^{e,\mathbf{M}_{v}} \tag{7}
\]

\[
\mathbf{H}_{0d}(\mathbf{r}) = \mathcal{K}^{h,\mathbf{J}_{sd}} + \mathcal{L}^{h,\mathbf{M}_{sd}} + \mathcal{K}^{h,\mathbf{J}_{sp}} + \mathcal{K}^{h,\mathbf{J}_{st}} + \mathcal{K}^{h,\mathbf{J}_{v}} + \mathcal{L}^{h,\mathbf{M}_{v}} \tag{8}
\]

\[
\mathbf{H}_{0p}(\mathbf{r}) = \mathcal{K}^{h,\mathbf{J}_{sd}} + \mathcal{L}^{h,\mathbf{M}_{sd}} + \mathcal{K}^{h,\mathbf{J}_{sp}} + \mathcal{K}^{h,\mathbf{J}_{st}} + \mathcal{K}^{h,\mathbf{J}_{v}} + \mathcal{L}^{h,\mathbf{M}_{v}} \tag{9}
\]

\[
\mathbf{H}_{0v}(\mathbf{r}) = \mathcal{K}^{h,\mathbf{J}_{sd}} + \mathcal{L}^{h,\mathbf{M}_{sd}} + \mathcal{K}^{h,\mathbf{J}_{sp}} + \mathcal{K}^{h,\mathbf{J}_{st}} + \mathcal{K}^{h,\mathbf{J}_{v}} + \mathcal{L}^{h,\mathbf{M}_{v}} \tag{10}
\]

The superscript denotes the nature of the testing basis and the subscript denotes the basis function.
Two notes are in order: (i) The above equations can also be tested with \( \hat{n} \times E_0 \) or \( \hat{n} \times H_0 \) in some place. In this case we denote the operator as \( K_{sd}^{h,sd} \) instead of \( K_{sd}^{h,sd} \). (ii) We have absorbed \( J_s \) into \( J_{sp} \) since they are of the same genre, and \( L_{sp}^{e} \) and \( L_{st}^{e} \) just differ by a multiplicative constant.

We can also test (3) and (4) to obtain

\[
E_{sd}^{0} = L_{sd}^{ei, sd} J_{sd} + K_{sd}^{ei, sd} M_{sd} \quad (11)
\]

\[
H_{sd}^{0} = K_{sd}^{hi, sd} J_{sd} + K_{sd}^{hi, sd} M_{sd} \quad (12)
\]

The equation for EFIE, MFIE, CFIE and PMCHWT formulations can be obtained by combining Equations (5) to (12). In order to handle more complex structures, the following models can be incorporated into the current code and some implementation tips are listed below.

1. **IBC**: The impedance boundary condition (IBC) can be implemented two ways. One way is to assume that an IBC surface has both \( J_s \) and \( M_s \) and they are related by an impedance value. Another way is to assume that \( E_{\text{total}} = E_i + E_s \neq 0 \) on IBC, and let \( E_{\text{total}} = J_s/Z_s \) to get \( E_i + E_s = J_s/Z_s \). The first method is more rigorous than the second one but it needs a closed surface.

2. **CRM**: When contact region model (CRM) [4] is required, we have to make sure that the mesh is consistent in the contact region, and that the matrix elements are accurately evaluated for the mutual interaction term.

3. **TDS**: When thin dielectric sheet (TDS) is needed for an open surface, it is best to allow for half basis near the edge of and open surface.

4. **Junction Basis**: When an open surface meets a closed or open surface, a new junction basis is needed at the edge of the open surface that meets another surface.

5. **Wire Basis**: If wire basis is added, we also need the addition of a surface-wire junction basis.

Now we manipulate Equations (5) to (12) to make it suitable to use FMA. Generally, FMA accelerates the matrix-vector-product (MVP) by preforming three steps: aggregation, translation and disaggregation. Each operator share the same translator but need a distinct set of radiation and receiving pattern. In another word, each MVP can only handle one kind of operator. Thus, we need to rewrite the MOM equation and separate the operators to use FMA.

\[
\begin{bmatrix}
E_{0}^{sd} \\
E_{0}^{sp} \\
E_{0}^{0}
\end{bmatrix} =
\begin{bmatrix}
L_{sd}^{e_s} & L_{sp}^{e_s} & L_{0}^{e_s} \\
L_{sd}^{e_s} & L_{sp}^{e_s} & L_{0}^{e_s} \\
L_{sd}^{e_s} & L_{sp}^{e_s} & L_{0}^{e_s}
\end{bmatrix}
\begin{bmatrix}
J_{sd} \\
J_{sp} \\
J_{0}
\end{bmatrix} +
\begin{bmatrix}
K_{sd}^{e_s} & K_{sp}^{e_s} & K_{0}^{e_s} \\
K_{sd}^{e_s} & K_{sp}^{e_s} & K_{0}^{e_s} \\
K_{sd}^{e_s} & K_{sp}^{e_s} & K_{0}^{e_s}
\end{bmatrix}
\begin{bmatrix}
M_{sd} \\
M_{sp} \\
M_{0}
\end{bmatrix} = 0 \quad (13)
\]

\[
\begin{bmatrix}
H_{0}^{sd} \\
H_{0}^{sp} \\
H_{0}^{0}
\end{bmatrix} =
\begin{bmatrix}
K_{sd}^{h_s} & K_{sp}^{h_s} & K_{0}^{h_s} \\
K_{sd}^{h_s} & K_{sp}^{h_s} & K_{0}^{h_s} \\
K_{sd}^{h_s} & K_{sp}^{h_s} & K_{0}^{h_s}
\end{bmatrix}
\begin{bmatrix}
J_{sd} \\
J_{sp} \\
J_{0}
\end{bmatrix} +
\begin{bmatrix}
L_{sd}^{h_s} & L_{sp}^{h_s} & L_{0}^{h_s} \\
L_{sd}^{h_s} & L_{sp}^{h_s} & L_{0}^{h_s} \\
L_{sd}^{h_s} & L_{sp}^{h_s} & L_{0}^{h_s}
\end{bmatrix}
\begin{bmatrix}
M_{sd} \\
M_{sp} \\
M_{0}
\end{bmatrix} = 0 \quad (14)
\]

In each region, MVP would be performed at most four times to get all the field needed to form the equations.
Numerical Results

Following the formulation given above, the code can be organized in an object-oriented style. More specifically, regions and the containing objects are described by different classes. Their characteristics are described by the member variables in the class. Operators are also modelled as a class with member functions to perform jobs such as matrix-vector product and matrix element generation. The basis is another kind of class with RWG basis and pairs of tetrahedron basis being its child class. In the future, other basis such as curvilinear basis would be easily added as long as we keep the interfaces of these child classes the same. The numerical result of a typical example is given in Fig 2. It uses the combination of EFIE, PMCHWT and VIE and invokes all the five different kinds of current currents. The result shows good agreement with the result of a well tested code “fastant”.

![Figure 2: The radar cross section of a typical scattering problem containing different objects.](image)

Conclusion

Using object oriented ideas to organize a typical MOM allows us to enjoy the benefits of OOP: it is easier to maintain, extend and reuse. We have set the basic framework of a OOP based MOM that would incorporate more features in the future.

References


