Calculation of Losses in a Superconductive Resonator using FDTD

*Gary T. Roan
Naval Research Laboratory
Washington, DC 20375

Kawthar A. Zaki
University of Maryland
College Park, MD 20742

Abstract

The parallel plate resonator method is a popular means to determine the surface resistance of superconductors at microwave frequencies [1]. In this paper, the Finite Difference Time Domain (FDTD) method is used to analyze the resonant frequency and the dielectric and conductor losses within a microwave enclosure containing a parallel plate resonator. The resonator’s losses are examined for the purpose of identifying experimental conditions that lead to inaccurate surface resistance measurements.

Summary

The parallel plate resonator technique for the measurement of the surface resistance of superconductors is described in [1]. In this method, a parallel plate resonator is formed from two samples of a superconductive material separated by a thin dielectric spacer. The parallel plate resonator is placed in a slightly larger microwave enclosure for testing. No electrical connections are made to the resonator, but excitation of the transverse electromagnetic modes between the parallel plates are accomplished by placing small electrical probes near to the open edges of the resonator. The electromagnetic fields within the test enclosure are dominated by the transverse electromagnetic modes between the plates of the resonator as a result of the close spacing between the plates. In the ideal case, the Q of the enclosure is dominated by the super-conductive losses of the parallel plate resonator. The exact placement of the resonator within the larger enclosure is not critical. The method has become popular principally because it does not require detailed knowledge of the modal distributions within the resonator and because sample preparation is relatively simple. Other methods such as the cavity end wall replacement method often suffer from large experimental error. In the end wall replacement method, particular cavity modes are measured with all normal conducting materials and with one end wall replaced by superconductive material. However, the loss in the super-conductor must be found by subtracting two measurements in which differences between the normal cavity and the cavity with one superconductive end wall are relatively small.

As described in [1], the three most significant losses that determine the Q of the parallel plate resonator are: (1) the resistive loss due to the surface resistance $R_s$ of the super-conductor within the parallel plate resonator, (2) the dielectric loss inside the dielectric spacer, and (3) the radiation loss at the edges of the open resonator. The unloaded Q of the enclosure is given by [1]:

$$Q^{-1} = \frac{R_s \beta}{s} + \tan \delta + \alpha s$$  \hspace{1cm} \text{Eqn. (1)}

U.S. Government Work Not Protected by U.S. Copyright
where $s$ is the resonator spacing, $\tan \delta$ is the dielectric loss tangent, and $\alpha$ and $\beta$ are proportionality constants that depend on the radiation loss and the modal structure of the resonant cavity, respectively. Taber [1] shows that $\beta$ is independent of the mode number for the open parallel plate resonator and is equal to $1/n_0\mu_0$. In Taber's experimental procedure, the surface resistance of the super-conductor $R_s$ is found by measuring the unloaded $Q$ for a small resonator spacing $s$ (typically 0.00254 cm or less) and assuming the losses are dominated by the surface resistance of the super-conductor $R_s$. In such a case, $R_s$ is equal to $s/\omega_0\delta$.

In this paper, the field distributions and cavity losses within the microwave enclosure and parallel plate resonator are examined using the Finite Difference Time Domain (FDTD) method [2]. Using FDTD, the electric and magnetic field components are calculated throughout the enclosure in the time domain on a non-uniform mesh. The mesh consists of a number of basic three-dimensional Yee cells on which the six components of the electric and magnetic fields are situated. The three components of the electric (E-) fields are oriented tangential to and in the middle of the edges of a Yee cell, while the magnetic (H-) fields are oriented normal to and in the center of the faces of the Yee cell. Due to the very small spacing used between the superconductive surfaces of the parallel plate resonator (about 0.00254 cm), a different cell dimension is used perpendicular to the plane of the plates ($z$-dimension) than was used in the tangential directions ($x$- and $y$-dimensions).

In the FDTD method, a time dependent electric source is introduced into the cavity by setting the electric field components at one or more cells equal to a function of time. The particular time dependence of the source used is not critical, but the bandwidth of the applied pulse must be sufficient to excite the resonator modes of interest. For the calculations here, a Gaussian time dependence was used. The time dependent fields are updated in the usual leap-frog manner where the H-fields throughout the mesh are updated during the first half-time step, then the E-fields are updated throughout the mesh during the second half-time step. The input pulse was applied until the source decayed to some small nominal value, then the source is turned off and the fields at the source locations are updated for the remainder of the run in the usual manner.

The resonant frequency is determined by recording the time series of the field components at one or more locations within the parallel plate resonator, and then calculating the Fourier transform of time series. Generally, the amplitude (and phase) of the frequency components of the Fourier transform is a function of the locations of the field sample points and of the source, and the bandwidth of the applied source. However, by sampling the fields at a number of locations and using an input pulse of adequate bandwidth, the chance of missing a particular resonance is minimized.

In this paper, we investigated the influence of the normal conductors and the dielectric on the resonant frequency and the enclosure $Q$. The losses within the enclosure consist of normal conductor and superconductor surface resistance losses, and dielectric losses. The conductor losses within the enclosure were determined for a given resonator spacing by integrating the square of the tangential H-field over the surface of the conductive surfaces, and multiplying by $R_s$. The dielectric losses were calculated by multiplying the square of the E-field by $\varepsilon_0\delta\tan \delta$, where $\varepsilon_0$ is the dielectric permeability, $\omega_0$ is the resonant frequency and $\tan \delta$ is the dielectric loss tangent. Using FDTD, the cavity $Q$ was calculated using

$$Q = \frac{\int \int \int \varepsilon \varepsilon_0^2 \, dv}{\int \int \int R_s H_z^2 \, ds + \varepsilon_0 \int \int \int \varepsilon \tan \delta \, dv}$$

Eqn. (2)

where in the numerical calculations the volume integrals are replaced by summations throughout the volume of the enclosure and by summations over the surface of the conductors.

The summation of the losses over a surface or volume was carried out in the time domain since it requires much less computation and computer memory than transforming the required time domain fields to the frequency domain and then summing. When performed in the time domain, the volume summation of $\varepsilon \varepsilon_0^2$ or the surface summation of $H^2$ are recorded at each time step for the duration of the run, then the
time series of the surface and volume summations are Fourier transformed to determine the amplitude of
the loss as a function of the frequency. For a particular resonant frequency \( \omega_0 \), the amplitude of the loss at a
frequency \( 2\omega_0 \) is used in the calculation of the resonator \( Q \).

Figure 1 plots the value of \( s/\sqrt{Q} \) calculated using FDTD versus the resonator spacing \( s \) for the
fundamental mode of a 1.27x0.635 cm parallel plate resonator. The parallel plate resonator consists of two
1.27x0.635 cm superconductive thin films \( (R_s=0.74\times10^{-5} \text{ ohms at 10 GHz}) \) separated by a sapphire
dielectric spacer \( (\varepsilon_r=2.2, \tan\delta=2\times10^{-3}) \) of thickness \( s \). The resonator is centered in a 1.905x1.25x(0.1+s) cm
thick, normally-conducting enclosure \( (R_N=0.015 \text{ ohms at 10 GHz}) \). The calculated data in Fig. 1 is shown
with and without a 0.05 cm thick LaAlO\(_3\) substrates \( (\varepsilon_r=22, \tan\delta=10^{-3}) \) for the superconductive films. The
values of \( s/\sqrt{Q} \) are shown for three different sapphire spacer thicknesses of 0.00254 cm, 0.0102 cm and
0.0204 cm. The curves shown are quadratic curve fits to the three data points. Fig. 1 shows that the dielectric substrate significantly affects the shape the \( s/\sqrt{Q} \) versus \( s \) curves. In addition, the resonant
frequency and is also significantly influenced by the dielectric substrate. The corresponding unloaded resonant frequencies were 3.811 GHz, 3.601 GHz and 3.434 GHz, respectively, with the LaAlO\(_3\), substrate, or 3.956 GHz, 4.064 GHz and 4.194 GHz without the LaAlO\(_3\), substrate.

If the total field energy is concentrated in the parallel plate resonator and the dielectric losses are
small, then the surface resistance of the super-conductor \( R_s = s/\sqrt{Q} \), where the \( Q \) is measured for the entire
enclosure. However, in the case discussed above, the LaAlO\(_3\), substrate effects the proportion of energy
located outside of the parallel plate resonator, and therefore affects the \( Q \) of the entire enclosure. Fields
located outside of the parallel plate resonator are dissipated by the dielectric substrate and by the surface
resistance of the normal conductor that forms the surrounding enclosure. Tables I and II list the \( Q \)s
calculated due to dielectric losses only, super-conductor losses only, and normal conductor losses only for
the resonator with and without the LaAlO\(_3\), substrate, respectively. The reciprocal of the effective \( Q \) for the
entire enclosure is equal to the sum of the reciprocal \( Q \)s due to each of the individual loss mechanisms
acting alone. The value of the surface resistance calculated from the effective \( Q \) and the assumed surface
resistance at the resonant frequency are also shown in Tables I and II. The discrepancy between \( R_s \)
calculated from \( s/\sqrt{Q} \) and the assumed \( R_s \) is smaller in Table II when the LaAlO\(_3\), substrate is excluded.
Table I shows that the effective \( Q \) is dominated by the losses in the super-conductor only when the
resonator spacing \( s \) is less than about 0.00254 cm if the LaAlO\(_3\), substrate is present. This leads to a large
error due to the normal conductor losses in the walls of the enclosure when \( s \) is equal to 0.0102 cm or
larger. In such a case, it should be possible to minimize the normal conductor losses and reduce the error
by coating both sides of the LaAlO\(_3\), substrate with a superconductive film.

In summary, calculations of conductor losses with FDTD have shown that the LaAlO\(_3\), substrate,
with a dielectric constant of about 22, can significantly affect the percentage of field energy outside the
parallel plate resonator. These leads to significant field dissipation by the normal conductors inside of the
enclosure, in which case the assumption that losses within the parallel plate resonator dominate the
effective \( Q \) is invalid and the surface resistance of the superconductive film will not be accurately given by
\( s/\sqrt{Q} \). The addition of a superconductive film to both sides of a LaAlO\(_3\), substrate should improve the
experimental accuracy. Experimental data is forthcoming to support these conclusions.

References:


Figure 1. FDTD Calculation of the Surface Resistance of a Superconductive thin film versus the parallel plate spacing $s$ with and with a LaAIO$_3$ substrate.

Table 1. FDTD results for $Q$, Resonant Frequency and Surface Resistance of a Parallel Plate Resonator with a LaAIO$_3$ substrate.

<table>
<thead>
<tr>
<th>Spacer thickness in cm:</th>
<th>0.0203</th>
<th>0.0102</th>
<th>0.00254</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency in GHz</td>
<td>3.434</td>
<td>3.601</td>
<td>3.811</td>
</tr>
<tr>
<td>$Q$ due to losses in walls of enclosure:</td>
<td>4379</td>
<td>5686</td>
<td>&gt;10000</td>
</tr>
<tr>
<td>$Q$ due to super-conductor losses:</td>
<td>46975</td>
<td>20820</td>
<td>3638</td>
</tr>
<tr>
<td>$Q$ due to dielectric losses:</td>
<td>3303412</td>
<td>28513597</td>
<td>23281149</td>
</tr>
<tr>
<td>$Q$ effective</td>
<td>4005</td>
<td>4465</td>
<td>3638</td>
</tr>
<tr>
<td>$Rs$ calculated from $Q$ effective</td>
<td>0.000688</td>
<td>0.000323</td>
<td>0.000105</td>
</tr>
<tr>
<td>$Rs$ actual</td>
<td>8.73E-05</td>
<td>9.6E-05</td>
<td>0.000107</td>
</tr>
</tbody>
</table>

Table 2. FDTD results for $Q$, Resonant Frequency and Surface Resistance of a Parallel Plate Resonator without a LaAIO$_3$ substrate.

<table>
<thead>
<tr>
<th>Spacer thickness in cm:</th>
<th>0.0203</th>
<th>0.0102</th>
<th>0.00254</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency in GHz</td>
<td>4.194</td>
<td>4.064</td>
<td>3.956</td>
</tr>
<tr>
<td>$Q$ due to losses in walls of enclosure:</td>
<td>15334</td>
<td>26538</td>
<td>119771</td>
</tr>
<tr>
<td>$Q$ due to super-conductor losses:</td>
<td>29707</td>
<td>14437</td>
<td>3514</td>
</tr>
<tr>
<td>$Q$ due to dielectric losses:</td>
<td>21417012</td>
<td>20577909</td>
<td>19713305</td>
</tr>
<tr>
<td>$Q$ effective</td>
<td>10109</td>
<td>9346</td>
<td>3514</td>
</tr>
<tr>
<td>$Rs$ calculated from $Q$ effective</td>
<td>0.000533</td>
<td>0.000174</td>
<td>0.000113</td>
</tr>
<tr>
<td>$Rs$ actual</td>
<td>0.00013</td>
<td>0.00022</td>
<td>0.000116</td>
</tr>
</tbody>
</table>