INTRODUCTION

The single integral equation method for scattering by dielectrics [1] reduces the memory requirements for an electromagnetic scattering calculation from those of other common methods to those for a perfect conductor. This is especially important for three-dimensional scatterers, for which the number of patches covering the surface needs to be large enough to obtain accurate results. Special considerations apply to particular configurations, such as the scattering by two touching spheres [2], and here we describe the scattering by a rough surface represented by a measured square area, which we assume is embedded in a plane interface.

The exact computation described here can be used to determine the validity of approximate statistical methods [3], [4], which are not necessarily accurate when the wavelength of the light is comparable to the feature size or when the surface is not a perfect conductor. An extension of this formulation can be applied to a system with several interfaces [5] such as a few layers of a multilayer mirror used in soft X-rays optics. Here we consider a homogeneous dielectric that fills a half-space and has a rough surface. Details about the method and the derivations can be found in the references [1], [2], and [5].

The single integral equation for dielectrics is similar to that used for perfect conductors [6], which has the form

\[ J_x(x) = 2\frac{\partial}{\partial x} \times \Pi^{0}(x) + 2\frac{\partial}{\partial x} \times \int_{S} dS' \frac{\partial}{\partial x} \times \nabla G(x - x') J_x(x') \]  

(1)

where \( J_x \) is the unknown surface current density and the line through the integral indicates that the self-patch contribution has been eliminated. This vector has only two independent components, which are the unknown surface fields in this simpler problem.

INTEGRAL EQUATION

Fields scattered by a dielectric homogeneous body can be determined from a single unknown tangential surface vector field \( J_x \) that obeys an integral equation that can be either singular or hypersingular [7], [8]. Here we use the singular equation. We choose \( x \)- and \( y \)-axes in the plane interface parallel to the sides of and centered in the square. Since the auxiliary fields [1] have to satisfy the outgoing wave condition, we have to subtract the homogeneous fields that result from the scattering by the flat average surface. These are the sum of the incident and reflected fields, \( E^{0} \) and \( H^{0} \), in region 1 above the surface and the refracted fields, \( E^{1} \) and \( H^{1} \), in region 2 below the surface.

The single integral equation, which replaces a set of two integral equations [6], is

\[ \left[ \begin{array}{c} \frac{1}{2} + \Lambda_{1}^{I} \\ \frac{1}{2} + \Lambda_{2}^{I} \end{array} \right] \left[ \begin{array}{c} \Pi_{1}^{I} \\ \Pi_{2}^{I} \end{array} \right] \left[ \begin{array}{c} \frac{1}{2} + \Lambda_{1}^{R} \\ \frac{1}{2} + \Lambda_{2}^{R} \end{array} \right] \left[ \begin{array}{c} \Pi_{1}^{R} \\ \Pi_{2}^{R} \end{array} \right] = -\left[ \begin{array}{c} \frac{1}{2} + \Lambda_{1}^{I} \\ \frac{1}{2} + \Lambda_{2}^{I} \end{array} \right] \left( \hat{n} \times E^{0} \right) + \left( \hat{n} \times H^{0} \right) \]  

(2)

where the homogeneous fields on the right side are \( E^{0} = E^{\text{in}} - E^{\text{out}} \) and \( H^{0} = H^{\text{in}} - H^{\text{out}} \). The tangential functionals \( \Lambda_{i}^{I} \) and \( \Pi_{i}^{R} \), with a subindex that refers to the parameters \( i \) and \( \mu \) of the corresponding region, are given by the principal-value integrals.
\[ I'(x) = \hat{n} \times P \int_S \left( \frac{(1 - ikR) \exp(ikR) \hat{x}(x') \cdot \hat{R}}{4\pi R^3} \right) \cdot dS' \]  

(3)

\[ M'(x) = i\omega \hat{n} \times P \int_S \left[ \frac{(1 - ikR) \nabla \cdot \hat{x}(x') \hat{R}}{4\pi R^3} - \hat{x}(x') \right] \exp(ikR) \cdot \]  

(4)

where \( \hat{x} \) is the field point, \( x' \) is the source point, \( \hat{n} \) is the normal to the surface, \( R = |\hat{n} - x'\) and \( R = |\hat{R}| \). The independent unknown surface fields are \( \chi(x,y) \) and \( \chi'_x(x,y) \), and the third component \( \chi_z \) is given by \( \chi_z = - (n_z/n_x) \chi_x - (n_z/n_y) \chi_y \) and we assume that \( n_z \) does not vanish. The surface gradient operator in (4) reduces to

\[ \nabla \cdot \hat{x} = (\chi_{xx} + \chi_{xy}) + [\chi_{yy}(\chi_x + \chi_y) + \chi_{xy} \chi_y]/(1 + \chi_y^2 + \chi_x^2) \]  

(5)

where \( \chi(x,y) \) represents the surface and subindices after a comma indicate partial derivatives, e.g., \( \chi_{xx} = \partial^2 \chi/\partial x^2 \). The meaning of the product of functionals represents, for instance, \( I' M'(x,y) \).

The surface gradient in the last term in (2) is best computed analytically. From Maxwell's equations we find \( \nabla \cdot (\hat{n} \times \hat{E}) = \omega c \hat{n} \times \hat{E} \), which we use to derive

\[ \nabla \cdot (\hat{n} \times \hat{H}^{b0}) = \nabla \cdot (\hat{n} \times \hat{H}^{b1}) = \nabla \cdot (\hat{n} \times \hat{H}^{b2}) = \omega c \hat{n} \times \hat{E}^{b1} - \omega c \hat{n} \times \hat{E}^{b2} \]

(6)

The corresponding term in (2) then becomes

\[ M'(\hat{n} \times \hat{H}^{b0}) = \hat{n} \times P \int_S \left[ \frac{(1 - ikR) E_x(x') \hat{R}}{4\pi R^3} - \frac{i\omega \hat{n} \times \hat{E}^{b0}(x')}{4\pi R} \right] \exp(ikR) \cdot \]  

(7)

where the amplitude of the normal component of the homogeneous electric field is

\[ E_n(x) = \hat{n} \cdot \left[ (e_1/e_2) E^{b1}(x) - E^{b2}(x) \right] \]

(8)

The integral equation (2) is then approximated by a system of linear equations by a numerical method such as point-matching [6].

**SPLINE INTERPOLATIONS**

The average plane of the measured heights is first determined by a least-squares fit. Then, to avoid extraneous scattering by the edges of the patch, we match the edge points to the plane by an interpolation. To obtain the normal to the surface and to determine the derivatives of \( \zeta \) needed in (3) we interpolate the surface by means of a two-dimensional cubic spline, which allows us to compute the needed derivatives.

The computed scattering cross section shows a spurious effect due to the grating formed by the grid points where the heights are measured. To avoid this problem, we use the
spline interpolation to specify the heights at points that are shifted with respect to the actual measurements. Since the surface elements are difficult to determine for points that are individually shifted, we form a new grid with irregular intervals. Computations for a perfectly conducting surface have shown that this procedure essentially eliminates the spurious scattering effect.

SURFACE GRADIENT TERMS

The surface gradient of $\vec{\gamma}$ can be eliminated from the functional in (4) by performing an integration by parts, which makes the integral equation hypersingular. Alternatively, we have to replace the partial derivatives of $\chi_x$ and $\chi_y$ by approximations. We have found that it is necessary to use a second-order approximation for the derivatives, which is simple for interior points but somewhat more complicated for edge points. Taking into account the $N$ irregular intervals between the grid lines on each side, we obtain

$$\chi_{adj} = \sum_i D_i^{xx} \chi_{adj}^{xx(i)} = \sum_i D_i^{yy} \chi_{adj}^{yy(i)}$$

(9)

where $\chi_{adj}^{xx(i)} = \chi_{xx(i)}^{x(i)}$, $\chi_{adj}^{yy(i)} = \chi_{yy(i)}^{y(i)}$, and the nonvanishing coefficients are

$$D_i^{xx(i)} = \frac{x_i - x_{i+1}}{(x_i - x_{i+1})(x_{i+1} - x_{i+2})}, \quad D_i^{yy(i)} = \frac{x_i - x_{i+1}}{(x_i - x_{i+1})(x_{i+1} - x_{i+2})}, \quad i = 2,...,N-1, \quad (10)$$

$$D_1^{xx(i)} = \frac{2x_1 - x_2 - x_3}{(x_2 - x_3)(x_3 - x_1)}, \quad D_{N-1}^{xx(i)} = \frac{x_1 - x_2}{(x_1 - x_2)(x_2 - x_3)} \quad (11)$$

$$D_1^{yy(i)} = \frac{2x_1 - x_2 - x_3}{(x_2 - x_3)(x_3 - x_1)}, \quad D_{N-1}^{yy(i)} = \frac{x_1 - x_2}{(x_1 - x_2)(x_2 - x_3)} \quad (12)$$

with similar expressions for $D_i^{xy(i)}$.

SELF-PATCH CONTRIBUTIONS

The integrands in the functionals (3) and (4) become singular on the self-patch, where $\vec{R}$ vanishes. The contributions of the self-patch $S_i$ should be finite, at least in the symmetric limit implied by the symbol $P$ in front of the integrals [9]. We can express the self-patch contributions in terms of the integrals

$$\hat{F}_1 = \hat{n} \left[ P \int_{S_i} d\vec{s} \left( 1 - ik\vec{R} \right) \chi(\vec{R}) \times \hat{n} \vec{R} / R^3 \right],$$

(13)

$$\hat{F}_2 = \hat{n} \times P \int_{S_i} d\vec{s} \left( 1 - ik\vec{R} \right) \chi(\vec{R}) \vec{V}_s \times \hat{n} \vec{R} / R^3,$$

(14)

464
The expansions of the different factors in the integrand have to be carried out in a consistent manner. The surface field can be expanded to first order in the local variables $\tilde{x} = x' - x$ and $\tilde{y} = y' - y$, but we do not keep higher-order derivatives in the expansion of the surface divergence (5). Some of the resulting integrals vanish by symmetry, while others can be evaluated, albeit with some difficulty.

We can now compute the fields scattered by a rough surface,

$$E^w(x) = \tilde{M}_1(x) , \quad H^w(x) = \tilde{L}_1(x) ,$$

once the integral equation for $\tilde{x}$ has been solved. These integrals can either be calculated at a finite distance above the patch, or they can be approximated for large $R$ to determine the far fields.

CONCLUDING REMARKS

The scattering cross section for an arbitrary rough surface represented by a measured patch can be determined from the amplitudes of the computed scattered fields. Since the homogeneous fields have been subtracted out, the results are valid outside the regions where the incident and reflected fields dominate.

The memory requirements are proportional to $N^4$ and to the number of layers, each of which requires one tangential field. Unless one is able to take advantage of virtual memory in the solution of the linear equations, only relatively small areas of the surface can be represented in the calculations, especially when several dielectric layers form the scattering surface. The CPU time is not a major limiting factor.

REFERENCES