1 Introduction

The dual-surface integral equations have been presented previously [1],[2] as a technique for eliminating the spurious resonances associated with the electric- and magnetic-field integral equations. A body of revolution computer program (DSBOR) for perfect electric conductors has been formulated using the dual-surface magnetic-field integral equation (DMFIE) [3]. This paper presents the development of DSBOR and its capability for solving electrically large bodies using the fast Fourier transform (FFT) and conjugate gradient (CG) methods.

2 Theoretical Formulation

The DMFIE can be derived by starting with the "interior" MFIE

$$-\vec{H}_{inc}(\vec{r}) = \frac{1}{4\pi} \int_S \vec{J}(\vec{r}) \times \nabla' \psi(\vec{r}, \vec{r}') dS'$$  \hspace{1cm} (1)

where $\vec{H}_{inc}(\vec{r})$ is the incident magnetic-field, $S$ is the surface of the perfectly conducting scatterer, $\vec{J}(\vec{r})$ is the surface current, and $\psi(\vec{r}, \vec{r}')$ is the free space Green's function. The current $\vec{J}(\vec{r}')$ is uniquely determined at every frequency if (1) is satisfied for all $\vec{r}$ inside $S$. By adding on cross (1), at points $\vec{r}$ on an inside surface $S_i$ parallel to $S$, to the original MFIE, one obtains the DMFIE:

$$\hat{n} \times \vec{H}_o(\vec{r}) = \frac{\vec{J}(\vec{r})}{2} - \hat{n} \times \frac{1}{4\pi} \int_S \vec{J} \times \nabla' \psi_0 dS',$$  \hspace{1cm} (2)

where $\vec{H}_o$ and $\psi_0$ are defined as

$$\vec{H}_o(\vec{r}) \equiv \vec{H}_{inc}(\vec{r}) + \alpha \vec{H}_{inc}(\vec{r} - \epsilon \hat{n}),$$

$$\psi_0(\vec{r}, \vec{r}') \equiv \psi(\vec{r}, \vec{r}') + \alpha \psi(\vec{r} - \epsilon \hat{n}, \vec{r}').$$

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The $f_2$ in (2) is the principal-value surface integral excluding the singular point, $r'' = r$, and $\alpha$ is a complex constant. The DMFIE (2), although identical in form and comparable in complexity to the original MFIE, provides a unique solution for the surface current $\mathbf{J}$ at all frequencies as long as the constant $\alpha$ has an imaginary part and the positive real constant $\delta$ is less than $\lambda/2$. For the body of revolution solution, $\alpha$ was chosen to be $\gamma(\sqrt{-1})$ to weight the fields on $S$ and $S_1$ equally. To keep the surface $S_1$ about an equal distance between the two critical values, $\delta = 0$ and $\lambda/2$, $\delta$ was selected to equal $\lambda/4$ [2].

Figure 1 plots the normalized radar cross section (RCS), $\sigma/\pi r^2$, in the backscattering direction for a perfect electric conducting sphere as computed using the MFIE, the Mie series, and the DMFIE. The spurious resonances in the MFIE solution are eliminated by the DMFIE which agrees well with the Mie series.

3 Numerical Solution

Our numerical solution to the DMFIE solution parallels the body of revolution solution in Mautz and Harrington [4]. However, instead of using triangular basis and testing functions, we use the simpler pulse basis and impulse testing functions.

The similarity of the DMFIE to the original MFIE allows it to be solved numerically by a small modification to any existing MFIE computer code. By adding the values of the incident field and the free-space Green's function multiplied by $\alpha$ at the points $r'' = \delta$ to the respective values at $r''$ in the original MFIE computer program one obtains the DMFIE program.

The body of revolution can be described by the surface tangential vector $\hat{t}$ and the azimuthal vector $\phi$. The $\phi$ dependence of the current is expanded into a Fourier series. The expansion of the current into a Fourier series allows the decoupling of the $\phi$ and $t'$ integrals in the integral equation, and allows the $\phi'$ integration to be performed for each term in the Fourier series. For each term in the Fourier series one divides the body into $N$ $t$-segments leading to $2N$ unknowns because each segment has two components of current. If the Fourier series requires $M$ modes for convergence of the current, it is necessary to solve $M$ systems of linear equations each with $2N$ unknowns.

For axial incidence of the plane wave ($\theta_{inc} = 0$ or 180 degrees), only the $\pm 1$ Fourier modes are required. For an incident angle of 0 degrees, the bistatic scattering of a 300$\lambda$ by 300$\lambda$ right circular cylinder (6000 unknowns) was computed using Gaussian elimination to solve the set of linear equations for five segments per wavelength. This required 570 minutes of CPU time on a Cray C90 supercomputer and is shown in Figure 2. The size of the body was limited by the available queue time on the Cray C90, not the available RAM.

Off axis, the maximum number of Fourier modes $M$ is equal to approximately $1.18 a \sin \theta_{inc} + 2$ where $a$ is the maximum radius of the body. The number of unknowns $N$ is proportional to $a/\lambda$, the electrical arc length of

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the BOR. Assuming the width and length of the BOR are proportional, the matrix fill time and matrix solution time using Gaussian elimination are proportional to \((d/\lambda)^3\). For broadside incidence the matrix solution time for each \(M\) is \((d/\lambda)^3\) and so the total matrix solution time is proportional to \((d/\lambda)^4\). The \(\varphi'\) integration has the form of a Fourier transform and can be computed efficiently using a FFT [5]. This reduces the matrix fill time for broadside incidence from a \((d/\lambda)^4\) to a \((d/\lambda)^3 \log_d(d/\lambda)\) functional dependence. The use of the CG iterative method allows one to reduce this \((d/\lambda)^4\) solution time to a \((d/\lambda)^3\) solution time. This means that the total computer time required for our BOR solution is asymptotically, for large \(d/\lambda\), proportional to \((d/\lambda)^4\) for axial incidence and \((d/\lambda)^3 \log_d(d/\lambda)\) for broadside incidence. The FFT requires an increased amount of computer storage, but this is the trade-off one must tolerate to save computer time. The bistatic scattering of a \(20\lambda \times 20\lambda\) right circular cylinder was computed at broadside incidence \((\theta_{inc} = 90\ degrees)\). The size of the cylinder was limited by the available RAM on the Cray C90. The solution (Figure 2) required 10 minutes of CPU time for Gaussian elimination with no FFT, 3 minutes for Gaussian elimination with the FFT and approximately 2 minutes for the CG method with the FFT.

References


Figure 1: Backscattering Cross Section Versus $ka$ of a PEC Sphere

Figure 2: H-Plane Bistatic RCS of a Right Circular Cylinder