RADIATION SCATTERED BY TWO TOUCHING SPHERES

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INTRODUCTION

During the manufacture of powder metal the size distribution of the metal spheres can be determined to some extent by the distribution of light scattered by the spheres while they are streaming by a laser beam. Micrographs show the presence of chains of spheres and of spheres with a number of smaller spheres attached to them. To model the scattered light we assume that the spheres are sparse enough to scatter light independently, an assumption that is obviously invalid for agglomerates of spheres. It is thus of interest to compare the pattern of the radiation scattered by such agglomerates with the pattern produced by isolated spheres.

We use an integral equation to compute the fields scattered by two perfectly conducting touching spheres when a plane monochromatic wave is incident from an arbitrary direction. We have validated the program by symmetry considerations, for instance, for top incidence, and by its reduction to the single sphere, whose results can be compared directly to those obtained from the Mie formulas. Integral equations that solve the Maxwell equations exactly are especially useful in the resonance region, where the size of the spheres is comparable to the wavelength. The method is not practical for spheres much larger than the wavelength because memory limitations do not allow for a sufficient number of patches per wavelength. The use of approximate methods carries the risk of ignoring special features of the touching spheres, such as the region near the point of contact which resembles a sharp wedge.

We choose the z-axis through the centers of the spheres with the origin at the point of contact and the x-axis so that the propagation vector is in the xz-plane, as shown in Fig. 1. We then consider TM and TE polarizations, with \( H_x \) and \( H_z \) along the y-axis, respectively. At each surface point we define a triad of orthonormal vectors formed by the unit normal \( n \) and the tangent vectors \( p \) and \( t \). The spheres have radii \( a_1 \) and \( a_2 \). The incident wave is defined by the wave vector \( \mathbf{k} \) and its time dependence is \( \exp(-i\omega t) \). The magnitude of \( \mathbf{k} \) is \( k \), which in terms of the wavelength \( \lambda \) or the circular frequency \( \omega \) is \( k = 2\pi/\lambda = \omega/c \), where \( c \) is the speed of light.

![Fig. 1. Geometry of the scatterer.](image)

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The surface current that is established on the surface of a perfect conductor satisfies the integral equation [1]

\[ \mathbf{J}(\mathbf{z}) = 2\pi i \mathbf{H}(\mathbf{z}) - 2\pi i \int_{S} \mathbf{J}(\mathbf{z}') \times \mathbf{F}(\mathbf{R}) d\mathbf{S}', \]  

(1)

where \( \mathbf{R} = \mathbf{z} - \mathbf{z}' \), \( R = |\mathbf{R}| \), and \( \mathbf{F}(\mathbf{R}) \) is the derivative of the outgoing free-space Green function \( G(\mathbf{R}) \) divided by \( R/2 \), that is,

\[ G(\mathbf{R}) = \exp(ikR)/(4\pi R), \]
\[ F(\mathbf{R}) = 2G'(\mathbf{R})/R = -(ikR - 1)\exp(ikR)/(2R^3). \]

The dash through the integral sign in (1) means that the self-patch contribution has been taken out. This is an integral equation of the second kind, which is generally better behaved than the integral equation of the first kind that can be derived from the continuity equation for the electric field. The surface current \( \mathbf{J} \) is tangential to the surface and is best expressed in terms of the tangent vectors \( \phi \) and \( \mathbf{t} \) by setting

\[ \mathbf{J}(\mathbf{z}) = \mathbf{J}_{\phi}(\mathbf{z}) \phi(\mathbf{z}) + \mathbf{J}_{\mathbf{t}}(\mathbf{z}) \mathbf{t}(\mathbf{z}), \quad \mathbf{z} \in S. \]

(3)

thereby reducing the storage requirements and building in the tangential nature of \( \mathbf{J} \). The integral equation (1) takes the form

\[ \mathbf{J}(\mathbf{z}) + \int_{S} d\mathbf{S}' \mathbf{F}(\mathbf{R}) \mathbf{A}(\mathbf{z}, \mathbf{z}') \mathbf{J}(\mathbf{z}') = \mathbf{C}(\mathbf{z}), \]

(4)

where \( \mathbf{A} \) is the 2x1 matrix with components \( A_{1} \) and \( A_{2} \), \( \mathbf{C} \) is a 2x1 matrix that depends on the polarization of the incoming wave,

\[ C_{1}(\mathbf{z}) = 2\cos \theta \sin \phi \mathbf{H}(\mathbf{z}), \quad C_{2}(\mathbf{z}) = -2\cos \theta \sin \phi \mathbf{H}(\mathbf{z}), \]

(5)

for the TM mode and

\[ C_{1}(\mathbf{z}) = \left(2/\omega_{0}\right)[\sin \theta \sin \phi \hat{\mathbf{t}}(\mathbf{z}) - \cos \theta \hat{\mathbf{n}}(\mathbf{z})] \mathbf{E}^{in}(\mathbf{z}), \]
\[ C_{2}(\mathbf{z}) = \left(2/\omega_{0}\right)[\sin \theta \cos \phi \hat{\mathbf{n}}(\mathbf{z}) - \cos \theta \hat{\mathbf{t}}(\mathbf{z})] \mathbf{E}^{in}(\mathbf{z}), \]

(6)

for the TE mode, and \( \mathbf{A} \) is the 2x2 matrix given by

\[ A_{1}(\mathbf{z}, \mathbf{z}') = -\xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{t}(\mathbf{z}') + \xi(\mathbf{z}') \mathbf{t}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}) - \xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{t}(\mathbf{z}) - \xi(\mathbf{z}') \mathbf{t}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}'), \]
\[ A_{2}(\mathbf{z}, \mathbf{z}') = -\xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{n}(\mathbf{z}') + \xi(\mathbf{z}') \mathbf{n}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}) - \xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{n}(\mathbf{z}) - \xi(\mathbf{z}') \mathbf{n}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}'), \]
\[ A_{3}(\mathbf{z}, \mathbf{z}') = -\xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{t}(\mathbf{z}') + \xi(\mathbf{z}') \mathbf{t}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}) - \xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{t}(\mathbf{z}) - \xi(\mathbf{z}') \mathbf{t}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}'), \]
\[ A_{4}(\mathbf{z}, \mathbf{z}') = -\xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{n}(\mathbf{z}') + \xi(\mathbf{z}') \mathbf{n}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}) - \xi(\mathbf{z}) \mathbf{H}(\mathbf{z}) \cdot \mathbf{n}(\mathbf{z}) - \xi(\mathbf{z}') \mathbf{n}(\mathbf{z}) \cdot \mathbf{H}(\mathbf{z}'). \]

(7)

where \( \theta \) and \( \phi \) are the polar and azimuthal angles at the center of the spheres and \( \xi \) and \( \xi' \) are the components of \( \mathbf{n} \) along \( \mathbf{t} \) and \( \mathbf{n} \), respectively.
\[ \xi = \mp a_1 \sin \theta, \quad \zeta = a_1 (1 + \cos \theta) - |z|. \]  

(8)

The integral equation (4) is reduced to a system of linear algebraic equations by the point-matching method. Once the components of the current density are obtained by solving these equations, the far fields in the direction \( \mathbf{k} - \mathbf{r} \) are computed by integration from

\[ \mathbf{H}_s(z) = \mathbf{i} \mathbf{g}(\mathbf{r}) \mathbf{e}_y d\mathbf{s}' J_z(\mathbf{k}') \exp(- ik'z'), \]  

(9)

where \( \mathbf{k}' = k \mathbf{r}' \). The corresponding intensity is proportional to the scattering or radar cross section (RCS).

THE AVERAGING PROCEDURE

The fields are actually measured by using concentric diodes centered on the forward direction of the incident beam. We also assume that the agglomerated particles present themselves in all possible direction to the beam, whence we have to average over the possible directions of incidence and sum over the surface of the circular diodes. In the program, the discretized matrix \( \mathbf{A} \) is computed and factorized only once, since it does not depend on the direction of incidence. When the radiation is coming in along the \( z \)-axis, the values of the outgoing polar angles are determined by the geometry of the detector and the values of the azimuthal angle are distributed uniformly in the range of \( 2\pi \). Otherwise, the components of \( \mathbf{k} \) are determined by rotating the vector by an angle \( \theta_j \) about the \( y \)-axis.

PRELIMINARY RESULTS

We have used the method described above to compute the angular distribution of the far-field intensity for a few examples. We consider a plane wave of laser light of wavelength \( \lambda = 0.6328 \mu m \). In Fig. 2 we show a polar diagram of the angular distribution of scattered light or RCS of a single sphere, \( 1 \mu m \) in radius. In Figs. 3 and 4 we show the RCS of two touching \( 1 \mu m \) spheres for incidence along the \( z \)- and \( x \)-axis, respectively. The scales are not the same and the maximum intensities, \( I \), are indicated in the figures. In Fig. 5 we show similar results for a \( 0.2 \mu m \) sphere attached to a \( 1 \mu m \) sphere for incidence along the \( z \)-axis. The surface current density near the point where the spheres touch tends to be large, but increasing the number of strips on the sphere has little effect on the RCS. Here we divide the polar caps into spherical triangles, and the accuracy of the integration over \( \phi \) is helped by the fact that the integrand is periodic in \( \phi \).

CONCLUDING REMARKS

These preliminary results indicate that a small sphere attached to a large sphere changes little the RCS and that there is a significant change for two equal spheres. More precise conclusions will be drawn from computations of the light measured by the concentric diodes. Since at the frequencies of visible light metals are not
perfect conductors, we also need to address the problem of calculating the radiation scattered by dielectric spheres with complex dielectric constants. Storage requirements can be minimized by using the method of the single integral equation \[3\].

REFERENCES

