Array Element Failure Correction for Signals with Wide Angular Distribution

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INTRODUCTION

In recent publications (1,2) it was shown that when beamforming is accomplished digitally, the signals from failed elements of an array antenna can be replaced by signals derived from the remaining array elements, and that this procedure could be carried out even if numerous signals were incident on the array. The reconstruction maintains the complete time dependence of the signals. The algorithm has since been verified experimentally using digital data from a 32 element receiving array.

The earlier studies left two important issues unresolved. First, it was assumed that one had perfect knowledge of the phase constant for any wave whose incident signal is $F_p = A_p e^{j k_p d}$. While such data can be obtained through the use of direction finding algorithms that operate even in the presence of array element failures, there can still be occasions when source location is only approximately known. A second important issue looks to the real utility of the technique to maintain the advantage of a low sidelobe distribution over an angular region, whether sources are present or not, or whether sources become distributed over an entire angular region (like clutter). This last issue is the subject of the present paper.

ELEMENT FAILURE CORRECTION FOR DISCRETE OR DISTRIBUTED SIGNALS

In the general case, an array receives signals from a number of discrete or distributed sources. If the number of incident waves is small, and their phase constants known exactly, then one can use the algorithm to separate each of the incident signals so that they can be weighted, summed, and substituted for the failed elements. In the absence of such accurate knowledge, the best that can be done is

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to attempt the solution at some number of sample points (sample propagation constants) in the neighborhood of where signals are expected.

Assuming a large periodic array and discounting edge element pattern effects, one can exercise the algorithm using the output signals from $P_n$ elements. The algorithm is processed once for each point 's' of the set 'S' of $P_n$ sample points, to arrive at the following remaining signal at element $n$.

\[
\left[\lambda_n e^{j2\pi s} \sum_{s' \in S} A_{s'} e^{j2\pi s'} E_n(s') \right] = \frac{P_n^{-1}}{E_n(s)} \tag{4}
\]

for all $s \in S$

with $E_n(p) = \prod_{s \in S} C_s(p) \tag{5}$

for $C_s(p) = 2\pi \frac{1}{2} \left( \frac{1}{\lambda_n^2 - \lambda_s^2} \right) \sin \left( \frac{\lambda_n^2 - \lambda_s^2}{2} \right) \tag{6}$

In this expression the term in brackets is the remaining set of signals for a particular value $s$, corresponding to a propagation constant $\lambda_s$. The waves represented in the summation include all of those waves with $\lambda_p$ not corresponding to one of the sample points. These are grouped in the set $S$. The expression at right indicates the $P_n$-1'th iteration in the process, and is simply divided by the constant $E_n(s)$. This remainder signal must be obtained for each value of $s$ within the set $S$.

The remainder signal is exact, but does not in general represent a complete separation of any one incident signal unless all of the waves are located at the sample points $\lambda_p$. When each does, then all the terms in the summation at right vanish, and the remaining signal is just the desired $p$'th signal, sampled at element $n$. In this case, the algorithm gives the exact expression for the signal at element $m$, in terms of the signal at the $n$'th element using the relationship below.

\[
P_m = \sum_{s \in S} \left[\lambda_n e^{j2\pi s} \right] e^{j2\pi s} \tag{7}
\]

In the more general case, the bracketed term of equation 7 is replaced by that in equation 4. The expression does not reduce any
longer to a single term since there remain contributions from waves not located at a sample point. Two sources of error are apparent. First, the signals are propagated from element \( n \) to element \( m \) using a propagation constant \( \alpha \) that is not the same as the correct \( \alpha_p \). Second, each contribution \( A_p \exp(jn\alpha_p) \) is used \( P \) times, once for each sample point, and the complete signal corresponding to the \( \alpha_p \) is the sum of all these terms each multiplied by a different exponential term.

Depending upon the number of sample points chosen, the expression 7 using the bracketed term \( 4 \) can still be an excellent approximation to the correct amplitude and phase of the failed signal for every wave near the sampled region. For example, the sidelobe level presented at any source will be the same as the ideal array at \( P \) points within the sampled region.

Figure 1 shows the patterns of a perfect 32 element array and one with four failed elements. The pattern of the ideal array is shown solid on the figure, and is that of a -40dB Taylor illumination, with \( \pi = 10 \). The dashed pattern shows the pattern that results when the array has four failures, at elements 15, 16, 17, and 18. Figure 2 shows the effective sidelobe structure over a wide region from 0 to 30 degrees, and compares the results of performing the correction algorithm at points equispaced in sine space, including the end points, \( \sin 0 \) and \( \sin 0.5 \). This is the response to any one signal moved throughout the 30 degree region. Since the process is linear, this is the response to any one signal of a discrete or continuous spectrum within that range. The solid curve of Figure 2 is that of the ideal array, and duplicates a part of Figure 1. The dotted curve is that for \( P = 2 \), and so matches the ideal curve only at the end points, where \( p \) corresponds to \( \sin 1 \) and \( \sin 2 \). The dash-dotted curve was computed for \( P = 7 \), and so matches the ideal curve at points spaced 0.08 apart across the region. The dashed curve follows the ideal solid curve almost perfectly except at the ends of the sampled region, matching sidelobe levels and null positions throughout the interior region. This curve was computed with \( P = 12 \). Since the region contains 8 sidelobes, it is clear that matching the pattern at 12 positions produces an excellent representation of the ideal pattern over the sampled region.

CONCLUSION

This paper has presented details of a technique for correcting the pattern of an array with element failures and digital beamforming. The technique has been shown to be useful in reducing the general sidelobe ratio over an angular region that can be many sidelobes wide.
REFERENCES


Figure 1: Pattern of 32 element array with Taylor -40dB illumination. Solid curve for ideal array. Dashed curve: array with elements 13, 16, 17, 18 failed

Figure 2: Local array pattern performance.
- Ideal array
- Array with P_0 = 7
- Array with P_0 = 2
- Array with P_0 = 12