INTRODUCTION

There is considerable current interest in modifying the patterns of linear array antennas, adaptively or synthetically, to lower the sidelobes at prescribed locations or in pattern sectors. Together with this objective of reduced sidelobes in specified portions of the pattern, it is generally desirable to preserve such pattern characteristics as gain and beam width, or an already low average sidelobe level. Preservation of pattern integrity demands that the perturbations of the complex element weights, required to achieve lowered sidelobe levels in specified regions, be minimized. A trade-off exists, of course, between the two goals of lowered sidelobes and minimization of the weight perturbations. This suggests that a useful performance measure in sidelobe sector nulling is the weighted sum of the average power in a specified sidelobe region and the squared weight perturbations. By varying the weights assigned to the average power in the sidelobe region and the weight perturbations, and minimizing the performance measure, it is then possible to vary the relative emphasis placed on the two principal objectives.

The purpose of this paper is to present a sidelobe sector null synthesis method based on the minimization of the performance measure described above. It is shown that the element weights that minimize the performance measure can be obtained analytically. Curves are plotted showing the variation of the average sector sidelobe power, the sum of the squared weight perturbations, and the gain in the look direction, with the ratio of the weights in the performance measure, and an example is shown of the patterns with reduced sidelobes obtained using this method.

ANALYSIS

We consider a linear array of \( N \) equispaced, isotropic elements with inter-element spacing \( d \) and phase reference at the array center. Let

\[
\mathbf{w}_0 = [w_{o1}, w_{o2}, \ldots, w_{oN}]^T
\]

and

\[
\mathbf{w} = [w_1, w_2, \ldots, w_N]^T
\]

denote the vectors of the original and perturbed complex weights respectively. The sum of the squared weight perturbations is then by

\[
(\mathbf{w} - \mathbf{w}_0)^\dagger(\mathbf{w} - \mathbf{w}_0)
\]

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The perturbed field pattern is given by
\[ p(u) = \sum_{n=1}^{N} w_n e^{j d_n u} \]
with
\[ d_n = \frac{(N-1)/2 - (n-1)}{N}, \quad n = 1, 2, \ldots, N \]
and
\[ u = (2\pi/\lambda)\alpha \sin(\theta) \]

where \( \lambda \) is the wavelength, and \( \theta \) the pattern angle measured from broadside to the array. Let the sidelobe sector in which the pattern is to be minimized be specified by the interval \([u_o - \epsilon, u_o + \epsilon]\). Then the average power in the sidelobe sector is given by
\[
\frac{1}{2\epsilon} \int_{u_o - \epsilon}^{u_o + \epsilon} |p(u)|^2 \, du = \sum_{n=1}^{N} \sum_{m=1}^{M} w_n^* w_m \exp[j(d_n - d_m)u_o] \text{sinc}[(d_n - d_m)\epsilon]
\]

where \( H \) is the Hermitian matrix whose elements are
\[ H_{mn} = \exp[j(d_n - d_m)u_o] \text{sinc}[(d_n - d_m)\epsilon] \]
and \( \text{sinc}(x) = \sin(x)/x \). We now define the performance measure
\[
P = \mu_1 (w - w_o)^\dagger (w - w_o) + \mu_2 \| H w \| \quad (1)
\]
where \( \mu_1 \) and \( \mu_2 \) are the respective weights assigned to the sum of the squared weight perturbations and the average power in the sidelobe sector. The derivative of \( P \) with respect to the weight vector \( w \) is
\[
2 \mu_1 (w - w_o)^\dagger (w - w_o) + 2 \mu_2 H w .
\]

Equating the derivative to zero and solving for \( w \) gives the desired perturbed weight vector
\[
w = \mu_1 (\mu_1 I + \mu_2 H)^{-1} w_o
\]
\[ = (I + \mu_2/\mu_1 H)^{-1} w_o \quad (2)\]

RESULTS

The weights required to minimize the performance measure defined by (1) were calculated from (2) for uniform arrays of 11, 21, and 41 elements with interelement spacing \( \lambda/2 \). The pattern sector for
reduced sidelobe power was taken to be the interval \( \theta = [20^\circ, 30^\circ] \). The ratio, \( \mu_2/\mu_1 \), of the weights assigned to the average sector sidelobe power and the sum of the squared weight perturbations respectively was varied from 0.0001 to 100,000. In Figures 1, 2, and 3, respectively, the average sector sidelobe power, the sum of the squared weight perturbations, and the gain in the look direction \((0^\circ)\), are plotted as functions of \( \mu_2/\mu_1 \). It is apparent from the figures that the behavior of all three functions is quite complicated. The ability to lower sidelobes in a wide pattern sector without significantly affecting the power density in the look direction increases rapidly with the number of array elements. For an 11 element array, a degradation of 1.1 dB in the look direction is associated with a 30 dB reduction in average sidelobe power, while for a 41 element array, only a 0.1 dB reduction in look direction gain is required to achieve a 30 dB reduction in average sector sidelobe power. No significant shift in the direction of the mainbeam was found with the arrays of 41 and 21 elements over the entire range of \( \mu_2/\mu_1 \). With the 11 element array a shift of -0.1° was obtained with \( \mu_2/\mu_1 = 100 \), increasing in magnitude to a shift of -0.7° with \( \mu_2/\mu_1 = 100,000 \).

As an example of the patterns obtained with the null synthesis method described in this paper, Figure 4 shows the unperturbed 41 element pattern and the perturbed pattern obtained with \( \mu_2/\mu_1 = 100 \). Note how closely the perturbed pattern follows the original pattern except in the immediate vicinity of the nulling sector. The maximum increase in the perturbed pattern sidelobe envelope above the unperturbed envelope is 1.3 dB.

![Figure 1. Average sidelobe power in the sector \([20^\circ, 30^\circ]\) as a function of \( \mu_2/\mu_1 \) for arrays of 11, 21, and 41 elements.](image-url)
had another case with -90 dB suppression in sector