RELATION BETWEEN CREEPING-WAVE ELECTROMAGNETIC
TRANSIENTS AND COMPLEX-FREQUENCY SCATTERING POLES

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The Singularity Expansion Method (SEM) has underlined the fact that the impulse response of conducting targets is characterized by certain scattering poles in the complex-frequency plane. We have shown that the experimentally observed ringing of electromagnetic (or mechanical) resonances is theoretically obtained as the Fourier transform of the above mentioned poles of the scattering amplitude. We show here how these poles can be connected in families of "layers" in such a way that each one of these layers is the manifestation of a given "creeping wave", previously obtained by means of the Watson-Sommerfeld Method. The SEM (without the mentioned surface wave interpretation), has also predicted the "ringing" in the form of damped sinusoids (using a temporal Laplace transform) of the individual target-resonances caused by the pulses incident on the target. For the example of lowest-order TM-type electromagnetic scattering by a perfectly conducting sphere, we show that the residue sum over a properly selected subset of the SEM-poles along each of the layers (k=1,2,3,...) is what actually synthesizes any given, repeatedly circumnavigating individual surface-wave or pulse. This selective sum which synthesizes the creeping waves, represents the physical content of the SEM poles, while the whole summation over all damped sinusoids (the Prony series), as customarily done in SEM, tends to obscure the underlying physical phenomenon. We further demonstrate mathematically that the Prony sub-series which selectively adds the contributions of the poles along the kth-layer, already accounts for the multiple circumnavigations of the corresponding creeping waves. Finally, we show by means of the stationary phase method that the pulse arrival times correspond to the creeping pulses as they propagate around the sphere with their group velocities. We illustrate these findings with numerical calculations of the first and second order creeping wave pulses around the sphere, exhibiting their correct synthesis, their multiple circumnavigations, and their correct arrival times governed by the group velocity. These graphs clearly exhibit the dominance of the first creeping wave over the higher ones due to the increasingly larger attenuations of the successively higher ones.

Hence, while the damped sinusoid contributed by a single pole to the transient amplitude constitutes the ringing of that pole's resonance, the partial Prony sub-series over the kth layer of poles is

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what synthesizes the \( x \)th creeping wave, which is thus produced as the coherent addition of many ringing processes involving intricate cancellations, already accounting for multiple circumnavigations, and with arrival times governed by their group velocity.

A detailed evaluation of the scattered TE and TM returns from a conducting sphere when a delta pulse \( \delta(z-ct) \) is incident upon it, is shown in Figs. 1 and 2. The evaluation is based on the method of stationary phase applied to the expression for the scattered pressure pulse,

\[
\chi_s(\theta, \tau) = \text{Re} \left\{ \frac{h_n^{(2)}(\chi_{n2})}{h_n^{(1)}(\chi_{n2})} \frac{p_n'(\cos \theta) e^{-i\chi_{n2}\tau}}{\chi_{n2}} \right\} \tag{1}
\]

This evaluation is quite complicated and its details cannot be given here. The final result can be summarized by two sets of plots. One set gives the functions \( Z_s(\theta, z) \), a factor that controls the main part of the distortion undergone by the scattered pulse. This factor is

\[
Z_s(\theta, z) = e^{-\gamma / M}
\]

where

\[
M = \sqrt{(2N+1)^{2n+\beta}} (1-a-1)^{5/4} \tag{3a}
\]

\[
\gamma = [(2N+1)^{2n+\beta}a(1-a)^{3/2}/4\sqrt{3}(1-a)^{-1/2} \tag{3b}
\]

where \( \alpha = (\tau \mid \theta') \) is the relevant variable proportional to \( \tau = (ct-r)/a \). These plots are shown in Fig. 1. More complete sets of results are given by the functions \( Y_s(\theta, \tau) \) which describe the distorted pulse in a more accurate way and contain the functions \( Z_s(\theta, \tau) \) as multiplicative factors. These functions are shown in Fig. 2.

The expression for the \( Y_s \) functions is

\[
Y_s(\theta, \tau) = \frac{h_n^{(2)}(\chi_{n2})}{h_n^{(1)}(\chi_{n2})} \sqrt{\frac{\csc \theta}{\pi(2n+1)}} \sqrt{\frac{\pi q^2}{12}} Z_s(\theta, \tau) \tag{4}
\]

Figures 1 and 2 have upper and lower parts (consisting of three graphs each) which correspond respectively to the first \((s=1)\) and the second \((s=2)\) creeping waves. In either one of these cases the plots exhibit results corresponding to three circumnavigations (i.e., \( N=0 \), none; 1, one; and 2, two) of these creeping waves around the sphere, in addition to the half-circumnavigation implied in the backscattering.
results shown in the figures. These figures further show (a) how the peaks, controlled by the group velocities, are shifted in $\alpha$ from one circumnavigation to the next, (b) how the wave amplitudes decrease with the number of circumnavigations, and, (c) how all the results vary angularly near the backscattering ($\theta = 0^\circ$) direction and finally, (d) how the first approximation to the pulse distortion (Fig. 1, Eq. 2) compares to the more accurate calculation (Fig. 2, Eq. 4). This whole analysis serves to further our understanding of the role of the multiple circumnavigations, of the surface (creeping) waves around the sphere and the SEM-poles that generate them when they are synthesized along properly selected branches or layers in the complex-frequency (SEM)$^3$ domain.

REFERENCES


Fig. 1. $z_g(t; z)\cos x = z_g(t; z)^2$ for the first (top) and second (bottom) creeping waves, each for $u=0,1,2$. 

Fig. 2. $z_g(t; z)\cos x = z_g(t; z)^2$ for the first (top) and second (bottom) creeping waves, each for $u=0,1,2$. (For several)