The Viscoelasticity of Polymer Solutions is The Main Factor Influencing Sweep Efficiency

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Abstract—In this article, a finite volume method for the numerical solution of viscoelastic flows is presented. The flow of a differential upper-convected Maxwell (UCM) model fluid through an abrupt expansion has been chosen as a prototype example. The conservation and constitutive equations are solved using the finite volume method (FVM) in a staggered grid with an upwind scheme for the viscoelastic stresses and a hybrid scheme for the velocities. Numerical results show the viscoelasticity of polymer solutions is the main factor influencing sweep efficiency.

Keywords—Finite volume method; UCM; Viscoelasticity; Sweep efficiency

I. INTRODUCTION

For the past several years, numerical simulation of viscoelastic flows has been a powerful tool for understanding the fluid behavior in a variety of processes of both industrial and scientific interest. Polymeric fluids, owing to their viscoelastic character, are of particular interest in the numerical simulation community because of their wide applications in materials processing and their different behavior from that of Newtonian fluids in ways which are often complex and striking. Although there have been many successful numerical predictions of elastic fluid flows, the We which stands for the elastic is low. In this paper, the flow of a UCM model fluid through a 4:1 sudden expansion is studied using a stable finite volume scheme [1]. The solution method succeeds to provide accurate numerical solutions, and elasticity levels up to We=3.2.

In the process of water flooding alone, the residual oil remaining within porous media is difficult to be displaced or recovered. In comparison, polymer flooding is more effective. Experimental results [2,3] indicated that the viscoelasticity of polymer solutions can enhance the displacement efficiency, but there are few theoretical studies on this subject.

II. MATHEMATICAL MODEL

A. The model of sudden expansion channel

The micro-pores of an actual reservoir are in general complicated. These pores are often simplified in numerical simulation. The problem geometry is shown in Fig. 1. It concerns the flow of a UCM model fluid through a planar 4:1 sudden expansion. Then, flow behavior of viscoelastic polymer solutions is studied in this simplified physical model. Note that the dimension in the figure is dimensionless.

![Fig.1 The model of sudden expansion channel](image_url)

B. Governing equations

The isothermal flow through expansions for incompressible fluids, such as polymer solutions and melts, is governed by the equation of continuity and motion, which can be expressed as

\[ \nabla \cdot \mathbf{v} = 0 \]  \hspace{1cm} (1)

\[ \rho \nabla \cdot \nabla \mathbf{v} = -\nabla p + \nabla \cdot \tau \]  \hspace{1cm} (2)

The constitutive equation that relates the stresses \( \tau \) to the deformation history is the UCM model, which in its differential form is written as

\[ \tau + \lambda \frac{\partial \tau}{\partial t} = \mu \dot{\gamma} \]  \hspace{1cm} (3)

where \( \lambda \) is the relaxation time, \( \tau \) stands for Oldroyd’s upper convected derivative of the stress tensor \( \tau \).

The above equations are non-dimensionalized by introducing the non-dimensional variables:

\[ x_D = \frac{x}{L}, \quad y_D = \frac{y}{L}, \quad u_D = \frac{u}{U}, \quad v_D = \frac{v}{U}, \quad \tau_D = \frac{L}{\eta U} \tau, \]

\[ p_D = \frac{L}{\eta U} p \]

where the characteristic velocity (U) and characteristic length (L) are taken as the average velocity in the downstream half channel and the width of the downstream half channel, respectively. \( u \) is the velocity component in the x direction, \( v \) is the velocity component in the y direction.

For a two-dimensional system of rectangular co-ordinates
(x, y) with velocity components (u, v), the conservation equation (1) for continuity can be written as
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\] (4)

The momentum Eq. (2) is given by
\[
Re\left[\frac{\partial}{\partial x}(u u) + \frac{\partial}{\partial y}(u v)\right] = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}
\]
\[
Re\left[\frac{\partial}{\partial x}(v v) + \frac{\partial}{\partial y}(v u)\right] = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}
\] (5)

and the constitutive equations for the UCM model can be written as
\[
\frac{\partial}{\partial x}(Weu \tau_{xx}) + \frac{\partial}{\partial y}(Wev \tau_{xx}) = 2\frac{\partial u}{\partial x} -(1-2We\frac{\partial u}{\partial y}) \tau_{xx} + 2We \frac{\partial u}{\partial y} \tau_{xy}
\] (7)
\[
\frac{\partial}{\partial x}(Weu \tau_{yy}) + \frac{\partial}{\partial y}(Wev \tau_{yy}) = 2\frac{\partial v}{\partial y} -(1-2We\frac{\partial v}{\partial y}) \tau_{yy} + 2We \frac{\partial v}{\partial y} \tau_{xy}
\] (8)
\[
\frac{\partial}{\partial x}(Weu \tau_{xy}) + \frac{\partial}{\partial y}(Wev \tau_{xy}) = (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}) - \tau_{xy} + We \frac{\partial v}{\partial x} \tau_{xx} + We \frac{\partial u}{\partial y} \tau_{yy}
\] (9)

The dimensionless parameters Weissenberg number (We) and Reynolds number (Re) above defined by[4-5]

III. NUMERICAL ALGORITHM

The constitutive Eq. (3) is solved together with the conservation Eqs. (1) and (2) using the FVM. Here some details about our own implementation of the method is given.

When employing the FVM, the governing equations are written in the following general form[6]
\[
\nabla \cdot (\rho \mathbf{v} \phi) = \nabla \cdot (\rho \nabla \phi) + S
\] (10)

A. Discretization of continuity equations
The discretized continuity equation reflects mass conservation for each cell:
\[
F_u - F_w + F_n - F_s = 0
\] (11)

B. Discretization of momentum equations
\[
\tau'_{xx} = \tau_{xx} - 2\eta \frac{\partial u}{\partial x}
\] (12)
\[
\tau'_{yy} = \tau_{yy} - 2\eta \frac{\partial v}{\partial y}
\] (13)
\[
\tau'_{xy} = \tau_{xy} - \eta \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)
\] (14)

Substituting Eqs. (12)-(14) in the momentum equations and assuming constant viscosity, Eqs. (5) and (6) can be expressed symbolically in a general form:
\[
a_{uu}\tau_{xx} = a_{uw}\tau_{yy} + a_{uv}\mu_{xx} + a_{ux}\mu_{yy} + S_u
\] (15)

C. Discretization of constitutive equations

The adopted viscoelastic model also has the general transport equation from Eq. (10) without diffusion term (\(\Gamma = 0\)). To ensure numerical stability[7], generally, a first-order upwind difference (UD) is used for spatial discretization. Thus, the discretized constitutive equation can be written
\[
a_{uu}\tau_{pp} = a_{up}\tau_{pp} + a_{up}\tau_{pp} + a_{uv}\mu_{pp} + a_{uv}\mu_{pp} + S_{pp}
\] (16)

IV. RESULTS AND DISCUSSION

As discussed above, a numerical simulation method is used and the stream function contour and velocity contour with different We can be obtained. As an example, the stream function and velocity contours with We equates 0 to 3.2 are shown in Fig.2 and Fig.3, respectively.

![Stream function contours](image1)

(a) We=0, Re=10^{-5}

(b) We=0.6, Re=10^{-5}

(c) We=1.2, Re=10^{-5}

(d) We=3.2, Re=10^{-5}

Fig.2 Stream function contours

It is noted that viscoelasticity results in reducing the extent and intensity of the recirculation zones in the reservoir corners of the sudden expansion. The reason is that when the fluid is entering a channel of larger height, it will try to relax...
its stresses along the streamlines. For incompressible viscoelastic fluids this causes expansion in the transverse flow direction.

\[ \text{Fig. 3 Velocity contours} \]

Fig. 3 shows that the area enclosed by the velocity line (equating to 0.015625 value) is larger as the \( We \) increases. The sweep area increases as \( We \) increases, so the sweep efficiency is enhanced. These numerical results are consistent with the results from experimental results.

V. CONCLUSIONS

In this paper, the flow of a UCM model fluid through a 4:1 sudden planar contraction has been studied using a stable finite volume method. The solution method succeeds in obtaining accurate values for all variables.

The results are accurate and offer an improvement over previous numerical solutions. Although the present study has been applied to a UCM fluid in a relatively simple geometry, it can be further extended to other more realistic constitutive equations, such as the UCM or Giesekus–Leonov models, etc. and to other geometries encountered in polymer processing.

Numerical results show that the viscoelasticity of polymer solutions is the main factor influencing sweep efficiency. With increasing elasticity, the flowing area in the corner is enlarged significantly, thus the area with immobile zones becomes smaller. Flow velocity is larger than that for a Newtonian fluid, the sweep area and displacement efficiency increase as the elasticity increases. The viscoelastic behavior of the displacing polymer fluids can in general improve the displacement efficiency in pores compared to using Newtonian fluids. This conclusion should be useful in selecting polymer fluids and designing polymer flooding operations.

REFERENCES


