Parameter Estimation of the Lankarani-Nikravesh Contact Force Model Using a New Modified Linear Method*

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Abstract—Parameter estimation plays a key role in describing a dynamical system behavior accurately. Thus, the inverse problems to identify the parameter values which characterize the dynamical system have attracted much attention from the engineering field in recent years. The Lankarani-Nikravesh (L-N) contact force model, which is proven to be more consistent with the physics of contact, is employed to describe the contact process in this paper. Based on the Taylor series and exponentially weighted recursive least squares (EWRLS) estimation method, a new modified linear method is proposed to identify the dynamical parameters of the L-N contact force model. Some simulation examples are presented to evaluate the convergence sensitivity of the modified method and the existing Haddadi method to parameter initial conditions.

I. INTRODUCTION

A mechanical system consists of several parts which are connected by joints. The availability of a joint depends on the relative motion between the connecting parts, which indicates the existence of the clearance and thus the vibrations and noises [1]. Over the last few years, extensive work has been done to study the effects of the clearance on the overall performance of the mechanical system and many contact force models have been proposed to quantify the contact effects [2-6]. The most famous nonlinear model was proposed by Hertz [7] during the Christmas vacation in 1880 at the age of twenty-three [8]. However, Hertz contact force model does not account for the energy dissipation during the contact-impact process, so it cannot be used during both phases of contact, namely the loading and unloading phases [5, 9]. Based on Hertz model, many contact force models which took the energy dissipation into account were proposed, such as the Hunt-Crossley model [10], Herbert and McWhannell model [11]. Among all the models above, the Lankarani-Nikravesh (L-N) model [12, 13] is one of the most popular and frequently used ones in mechanical systems. Online estimation of the unknown contact environment plays a key role in the overall dynamical analyses of the mechanical systems. However, the existing challenges in the identification of nonlinear systems have severely limited the use of the L-N model for online application. Many researchers have devoted themselves into this study. As for the Hunt-Crossley contact force model, Diolaiti et al. [14] utilized a two-stage technique for online identification. However, it showed that the method was sensitive to parameter initial conditions and parameter changes. Moreover, the error propagation from one stage to the other made the estimation results converge slowly [15, 16]. Compared to Diolaiti et al. [14], Amir Haddadi et al. [15, 16] proposed a single-stage method for online estimation of the Hunt-Crossley model, which linearized the Hunt-Crossley model so that all three parameters could be identified in one stage during a real-time process. However, two conditions needed to be met in the linearization process, which were not always under control [15, 16].

Following the work of Amir Haddadi et al. [15, 16], a new modified single-stage linear method, which is based on the Taylor series and exponentially weighted recursive least squares (EWRLS) estimation method [15-17], is proposed in this paper. The new modified method is compared with the Haddadi method under different environments, e.g. hard and soft ones, respectively. Moreover, the convergence sensitivity to parameter initialization changes is evaluated numerically for both methods. In addition, the effect of measurement noise for the modified method is investigated through numerical simulations.

II. PARAMETER ESTIMATION OF THE L-N CONTACT FORCE MODEL USING THE NEW MODIFIED METHOD

A. The L-N Contact Force Model

The contact force model describing the contact process is of great importance for the dynamical analyses of the entire mechanical system. For the popularity of the L-N contact force model, it is introduced and analyzed in this paper.

Using the general trend of the Hertz model, Lankarani and Nikravesh [12, 13] developed a contact force model which took the energy dissipation into consideration. The normal contact force is separated into elastic and dissipative components and can be expressed as,

\[ F = Kx^2 + Dx \]  \hspace{1cm} (1)

\[ K = \frac{4}{3\pi (\sigma_i + \sigma_j)} \left[ \frac{R_i R_j}{R_i - R_j} \right]^\frac{1}{2} \]  \hspace{1cm} (2)

\[ \sigma_k = \frac{1 - \nu_k^2}{\pi E_k} \quad (k = i, j) \]  \hspace{1cm} (3)

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(a) Penetration versus time (b) Contact force versus time (c) Contact force versus penetration

Figure 1. Two internal colliding spheres modeled by the L-N contact force model

\[ D = \chi x'' = \frac{3K(1 - e^2)}{4\chi^{-1}} \]  

(4)

where the elastic part is represented by the first item, and the energy dissipation is represented by the second item in (1). \( K \) is the contact stiffness coefficient. \( x \) and \( \dot{x} \) are the relative penetration and its corresponding velocity, respectively. \( D \) is the damping coefficient. \( \chi \) is called the hysteresis damping factor and \( x^{-1} \) represents the initial contact velocity.

The following simulation is conducted between the bushing and pin, of which the radii are 10 mm and 9.5 mm, respectively. The elastic modulus is 207 Gpa, the Poisson’s ratio is 0.3, the initial contact velocity is 1m/s and the restitution coefficient is 0.9.

In Fig. 1, it can be easily observed that the penetration and contact force vary in a continuous manner during the entire contact process. The relation between the contact force and the penetration presents nonlinearity and the area surrounded by the hysteresis plot indicates the energy dissipation.

\[ F(t) = Kx''(t) + \chi x''(t)\dot{x}(t) + \varepsilon \]  

(5)

where \( \varepsilon \) is the measured noise during the process.

To linearize the above equation, the natural logarithm is taken on both sides,

\[ \ln(F(t)) = \ln(Kx''(t) + \chi x''(t)\dot{x}(t) + \varepsilon) = \ln(K) + n \ln(x(t)) + \ln(1 + \frac{\chi\dot{x}(t)}{K} + \frac{\varepsilon}{Kx''(t)}) \]  

(6)

For natural logarithm,

\[ \ln(1 + \alpha) \approx \alpha, \text{ if } |\alpha| < 1 \]  

(7)

By assuming \( \frac{\chi\dot{x}(t)}{K} + \frac{\varepsilon}{Kx''(t)} \approx 0 \), that is \( \frac{\chi\dot{x}(t)}{K} \ll 1 \) and \( \frac{\varepsilon}{Kx''(t)} \ll 1 \), (6) can be simplified as,

\[ \ln(F(t)) \equiv \ln(K) + n \ln(x(t)) + \chi\dot{x}(t) \]  

(8)

However, it should be noted that the speed of operation depends on the characteristics of the desired task and the identification process must be stopped when the penetration is smaller than the threshold penetration [15, 16], which means the assumptions above are always not under control especially for high speed situations.

C. The New Modified Method

In this paper, we modify the method by using the Taylor series. For the L-N model, we also take natural logarithm on both sides of (5), so we can get (6). Taylor series is chosen since

\[ \ln(1 + X) \equiv \ln(1 + X_o) + \frac{1}{1 + X_o} (X - X_o) \]  

(9)

where \( X_o \) is the expansion point.

For the term \( \ln(1 + \frac{\chi\dot{x}(t)}{K} + \frac{\varepsilon}{Kx''(t)}) \) in (6), we can define

\[ X = \frac{\chi\dot{x}(t)}{K} + \frac{\varepsilon}{Kx''(t)} \]  

(10)

And the expansion point can be expressed as,

\[ X_o = \frac{\chi\dot{x}_0}{K} + \frac{\varepsilon}{Kx''_0} \]  

(11)

Thus,

\[ \ln(1 + \frac{\chi\dot{x}(t)}{K} + \frac{\varepsilon}{Kx''(t)}) \equiv \ln(1 + \frac{\chi\dot{x}_0}{K} + \frac{\varepsilon}{Kx''_0}) \]  

(12)
Using the least squares family of estimation method, (13) can be linearly parameterized as,
\[ y_k = \phi_k^T \theta_k + \xi_k \quad (x_k > 0) \]  
(14)
where \( \phi_k^T \) is the regress vector, \( k \) is the iteration number, \( \theta_k \) is the vector composed of unknown dynamical parameters, \( \xi_k \) represents all the noise and modeling error in (13).

Thus, (13) can be reformulated as,
\[ \ln(F(t)) \]
\[ \approx \ln(K) + n \ln(x(t)) + \ln(1 + \frac{\chi V_0}{K}) + \frac{1}{1 + \frac{\chi V_0}{K}} (\frac{\chi \xi(t)}{K + \chi V_0} - \frac{\chi V_0}{K + \chi V_0} + \xi) \]  
(15)

Thus,
\[ \phi_k^T = [1, \dot{x}_k, \ln(x_k)] \]  
(16)
\[ \theta = [\ln(K + \chi V_0) - \frac{\chi V_0}{K + \chi V_0}, \frac{\chi}{K + \chi V_0}, n]^T \]  
(17)
\[ y_k = \ln(F_k) \]  
(18)

Among all the variations of the recursive least squares (RLS) method, EWRLS is an estimation method which is widely used for tracking gradual changes in system parameters. The EWRLS equations can be written as [15-17],
\[ L_{k,s1} = \frac{P_k \phi_{s1}}{\lambda + \phi_{s1}^T P_k \phi_{s1}} \]  
(19)
\[ P_{k+1} = \frac{1}{\lambda} [P_k - L_{k,s1} \phi_{s1}^T P_k] \]  
(20)
\[ \hat{\theta}_{k+1} = \hat{\theta}_k + L_{k,s1} [\ln F_{k+1} - \phi_{s1}^T \hat{\theta}_k] \]  
(21)
where \( P \) is the covariance matrix, \( \lambda \) is the forgetting factor which effectively puts emphasis on the last \( \lambda/(1 - \lambda) \) data points. The lower the forgetting factor is, the stronger the system trace ability is, but the more sensitive the system becomes to noise. Thus, convergence instability occurs. While for a larger forgetting factor, the trace ability is weaker and the convergence estimation error is smaller. For contacts with varying parameters, a lower forgetting factor is better, whereas for contacts with constant parameters, a value closer to unity should be selected [16]. To get balance between convergence stability, trace ability and the estimation error, the variable forgetting factor \( \lambda = 1 - 0.01 \exp(-\beta(t - 2)) \), with \( 5 \leq \beta \leq 8 \) is adopted. For \( \beta = 5 \), the estimation performance is the best [16]. Thus, \( \lambda = 1 - 0.01 \exp(-5(t - 2)) \) is chosen in this paper.

The estimated parameters at every step time can be derived according to
\[ \hat{\theta}_k = \hat{\theta}_k (3) \]  
(22)
\[ \hat{K}_k = e^{\hat{\phi}_1 (1)} - \hat{\phi}_2 (2) v_0 \]  
(23)
\[ \hat{K}_k = e^{\hat{\phi}_1 (1)} \hat{\theta}_k (2) v_0 - \hat{\phi}_2 (2) v_0 \]  
(24)

III. SIMULATION EXAMPLES

In this section, the comparisons of performance between the modified method and the Haddadi method are presented under different conditions, e.g., hard and soft environments, with 2% root-mean-square (RMS) white noise considered.

The penetration path is chosen as [16],
\[ \delta = \sin(4t) + 1.8 \sin(11t) + 1.8 \sin(15t) + \delta_0 (\text{mm}) \]  
(25)
where \( \delta_0 \) is the initial penetration which aims to ensure the overall estimation process is during contact, and it is usually set to be 10 mm [16].

A. Comparisons between the Modified Method and the Haddadi Method

Both methods are employed to identify the hard and soft environments with different dynamical parameters as \( [K \chi n] = [1500 \ 50 \ 1.9] \), \( [K \chi n] = [60 \ 30 \ 1.2] \), respectively. From Figs. 2 and 3, it is easy to observe that the convergence rate of the modified method is much faster than that of the Haddadi method especially for parameter \( \chi \), which indicates the availability of the modified method.

B. Effect of the Initial Conditions

In this part, the effect of the initial conditions is investigated for hard and soft environments under different dynamical parameters \( [K \chi n] = [1500 \ 50 \ 1.9] \), \( [K \chi n] = [60 \ 30 \ 1.2] \) by using both methods, respectively. Figs. 4-7 show the estimation results for hard and soft environment with different initial conditions. It can be observed from the results that the dynamical parameters of the L-N model can converge faster under different initial conditions for the modified method, which shows the advantage of the modified method for parameter estimation under a wider range of initial conditions.
Figure 2. Parameter estimation of the L-N contact force model for hard environment with both methods.

Figure 3. Parameter estimation of the L-N contact force model for soft environment with both methods.

Figure 4. Parameter estimation for hard environment with different initial conditions using the Haddadi method.
Figure 5. Parameter estimation for hard environment with different initial conditions using the modified method.

Figure 6. Parameter estimation for soft environment with different initial conditions using the Haddadi method.

Figure 7. Parameter estimation for soft environment with different initial conditions using the modified method.
C. Effect of Measurement Noise

The dynamical parameter estimation results above for the L-N contact force model show acceptable robustness to measurement noise of the modified method. Fig. 8 shows the results of identification for soft environment using the modified method when a force measurement noise with 4% RMS is considered. The results show that the convergence rate is slower compared with the condition in which the level of noise is lower, however, accurate values can be reached in the end.

IV. CONCLUSION

In this paper, a new modified linear estimation method which is based on the Taylor series and EWRLS estimation method is proposed. Compared to the Haddadi method, the new method shows great advantage in convergence rate and applicable scope. Different initial conditions are employed to identify the dynamical parameters of the L-N model under hard and soft environments using both methods, respectively. The agreement of the estimated values with the actual ones shows the great advantage of the new modified method in identifying the dynamical parameters under a wider range of initial conditions. Moreover, the effect of measurement noise is investigated through numerical simulations, which indicates the robustness of the modified method to noise. Future work will aim at the applications of the new modified method in the mechanical systems.

REFERENCES


