Abstract—It is the aim of this paper to focus on some advanced ideas of power adjustments for differentiated services in satellite communications. Dynamic resource allocation explicitly considers the tradeoff between the cost of transmit powers and the error of signal-to-interference-plus-noise ratios. Achieving maximal system capacity and design for reliability are intertwined and mutually reinforcing goals as being incorporated into the framework of minimal-cost-variance control theory. Closely related to the continuing quest for an effective procedure of emission coordination and interference protection [1] and [2] is the problem of terminal reports in presence of interferences and deteriorated link conditions with the hope that these findings would enable future capability concepts of resilient affordable satellite system controllers and ground terminals to achieve assured satellite communications.

1. INTRODUCTION

What makes assured wireless communications such a difficult topic in the context of congested radio environments is that its exercise involves a demand for highly reliable communication systems. On the one side is the availability as a key performance parameter that communication systems must often continue functioning in presence of various unwanted radios that would distort communication and control data being sent. On the other side is the development of new standards that specify scientific approaches to reliability design, assessment, and verification.

Proponents of this view widely accept that communications mode selection and resource allocation decisions made by dynamic radio resource allocation controllers at system controllers (also known as traffic hubs or base stations) characterize the quality of most operations in wireless communications. In this regard, power control has been a powerful mobilizing instrument, with which for instance, to enable power emission coordination and interference protection [1] and [2] as well as secondary-user spectrum access gain maximization [3] and [4].

Minimal-Cost-Variance Power Control for Differentiated Service Satellite Communications

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Nearly all the power control research for wireless communications has revolved around the procedures of optimizing the ensemble average of random performance indices, a view almost completely having ubiquitous support. As may be expected, other power adjustment optimization techniques for all-too-real present replete with complexities and risks may require higher-order statistical moments of the performance index and thus, will result in different power control behavior accordingly. Indeed, dynamic radio resource allocation for assured communications must be approached in practical terms; e.g., How can quality of service guarantees be met given the fact that communication forward and return accesses are likely to operate only once, rather than repeatedly perform the same operation? Can performance reliability be strengthened, and if so, in what aspects? These inquiries find relevance in contexts, in which minimum variance optimization and the likes appear to be the most natural optimization techniques [5], [6] and [7].

There remains another, often overlooked, dynamic radio resource allocation trend over differentiated services in forward and return links that is the necessary underpinning of traffic priorities. Such quality of service guarantees cannot take hold unless different transmission options are supportive of both delay sensitive traffics and best effort traffics. But moving beyond the context of ensemble average of performance indices for power emissions, and even beyond the framework of all data transmission equally important, it is obvious that the research exposition here has to be concerned about dimensions of dynamic radio resource allocation over differentiated service link accesses.

As an aide to the reader, the sections will now be summarized. In Section II, the problem of dynamic radio resource allocation associated with the error tracking of signal-to-interference-plus-noise (SINR) ratios, is formulated. Section III provides distributed power adjustments based on the minimal-cost-variance control theory for discrete stochastic SINR-based systems, wherein the variance of a performance index is minimized while the expected value of the performance index is constrained a-priori. In Section IV, the scope of power control by the satellite system hub threaten by potential disruptions of terminal reports from the remote terminals for return communication accesses and link quality estimates of forward link access, is investigated with Kalman estimation with Brownian disturbances. As matters stand now, some conclusions and future research directions are given in Section V.
2. The Model of Return-Link Power Control

Figure 1 depicts the traditional “hub-spoke” satellite network topology with a satellite system controller, connected through satellite transponder(s) to a large but finite population of \( \hat{N}_r \) remote terminals. Multi-beam coverage allows to implement a frequency re-use scheme, which allocates a given frequency band and polarization to a “group” of non-adjacent beams.

In order to accompany the ever increasing demand for bandwidth and cost per Mega-bit per second reduction, the satellite throughput has to be maximized. In addition, the satellite network is based on transmission formats, such as: forward link of time-division multiplexing and return links of multi-frequency time-division-multiple access. Further, the satellite system controller is responsible for terminal data forward and communication return accesses to each best channel and/or channel group from and to the satellite system controller through the designated satellite transponder to all active remote terminals.

Hereinafter, the return link Signal-to-Interference-plus-Noise Ratio (SINR) measured at the satellite system controller for terminal \( i \) on the \( n^{th} \) channel during terminal \( i \)’s assignment activation, \( k \) is governed by

\[
\gamma_{i,n}(k) = \frac{p_{i,n}(k)\bar{h}_{i,n}}{\sum_{j=1,j \neq i}^{N_r} \bar{p}_{j,n}(k)\bar{h}_{j,n} + \sigma_{i,n}^2(k)}.
\]

where effective isotropically radiated power, \( p_{i,n}(k) \) for terminal \( i \) at \( n^{th} \) channel. The channel gain from terminal transmitter \( i \) to receiver terminal \( j \) on the \( n^{th} \) channel is denoted by \( \bar{h}_{i,j,n} > 0 \). Further, the additive white Gaussian noise \( \sigma_{i,n}^2(k) \triangleq N_0(k)B_n \) represents the power spectral density \( N_0(k) \) from other radio interferences and thermal noise received at the satellite system controller and \( B_n \) is the physical bandwidth of the \( n^{th} \) channel. Multi-access interference \( \bar{p}_{j,n}(k)\bar{h}_{j,n} \) is contributed by \( \bar{p}_{j,n}(k) \), which is the transmit power of terminal \( j \) received by the satellite system controller for terminal \( i \) on the \( n^{th} \) channel.

On a related note, the satellite system controller uses this link quality estimate (1) and the amount of forward link per-class traffic to make communication mode assignments and then sends the communication return access assignments, as frequently as once every epoch, to the terminals which share the same return-link channel group. The terminals use the assignments starting at the specified times (i.e., activation epoch) and for the specified durations (i.e., assignment persistence).

Due to the large number of active terminals, the satellite transponder receiver owns a large dynamic signal range with large envelop fluctuations. When considering return link assignments, the nonlinear distortions introduced by the satellite transponder’s high power amplifier play an important role. Normally, nonlinear distortions could be avoided by backing off the amplifier. The implication is that the amplifier would have to work in its linear region. However, this practice would not result in a power efficient operation of the amplifier, which is especially important in the case of satellite communications. In order to achieve a good high power amplifier-power efficiency, small output backoff values are required, which often lead to a tradeoff between maximizing output power on the one side and avoiding degradations due to nonlinear distortions on the other side.

Given this background, then, it is necessary to consider a simple power emission management with a power efficient operation of the high power amplifier with low backoff values. The satellite system controller will determine an appropriate emission power adjustment for terminal \( i \) on the \( n^{th} \) channel, \( u_{i,n}(k) \), which is defined as the difference between the effective isotropically radiated power power, \( p_{i,n}(k) \) at activation epoch, \( k \) and the effective isotropically radiated power, \( p_{i,n}(k+1) \) at activation epoch, \( k + 1 \) to limit unnecessary terminal power transmissions over the level required to close the link that supports the user’s data rate and total uplink noise power at the input of the satellite transponder receiver.

Throughout the following development, all active terminals with excess powers are required to back off their effective isotropically radiated powers to share the satellite transponder. Special attention is further paid to the input backoff requirement that is defined as the ratio between the input power that maximizes the output power to the input power that delivers the desired linearity of the satellite power amplifier. One method to impose such an input backoff requirement, \( bo_{i,n}(k) \) denoting the backed off power level at terminal \( i \) is to have its positive impacts on the maximum terminal effective isotropically radiated power, \( p_{i,n}(n_0) \) of terminal \( i \) on the \( n^{th} \) channel. In all, the power budget management having an initial power budget, \( p_{i,n}(n_0) \) is governed by the difference equation

\[
p_{i,n}(k+1) = p_{i,n}(k) - u_{i,n}(k) + bo_{i,n}(k)
\]

\[
p_{i,n}(n_0) = p_{i,n}^0.
\]

For now, the underpinning of dynamic resource allocation is to match radio resources to traffic demands, more efficiently use of radio resources. A terminal is allocated a minimal amount of resources which depends on the overall availability and traffic priority. For instance, the satellite system controller will make sure that every active remote terminals in its satellite network will meet but not exceed the SINR thresholds, which can be supported by the most robust supportable modes. Note that these modes require the least bandwidths and powers. Yet, they are however expected to have the most time slots.

As such, the tracking errors with respect to the target SINR, \( \gamma_{i,n}^t \), at the \( n^{th} \) channel for terminal \( i \) at the activation epoch \( k \) is given by

\[
e_{i,n}(k) = \gamma_{i,n}(k) - \gamma_{i,n}^t.
\]
A consequence of (1), (2), and (3) is further obtained as

\[ e_{i,n}(k+1) = e_{i,n}(k) - \frac{h_{i,i,n}}{I_{i,n}(k)} u_{i,n}(k) + \frac{h_{i,i,n}}{I_{i,n}(k)} b_{o,i,n}(k) + p_{i,n}(k+1) h_{i,i,n} \left[ I_{i,n}(k) - I_{i,n}(k+1) \right] \]

(4)

where all the interferences, \( I_{i,n}(k) \) collected at the \( n^{th} \) channel of terminal \( i \) at the activation epoch \( k \) are computed by average power measurements via an appropriate energy detector and denoted by

\[ I_{i,n}(k) = \sum_{j=1,j \neq i}^{N_s} p_{j,n}(k) h_{j,i,n} + \sigma_i^2(k). \]

(5)

From this formalistic perspective, all remote terminals have clear preferences, represented by a quadratic utility function. To fix the idea, it is sufficient for the moment to consider the interpretation that is often used in economics. Here the state and control described by the mathematical expression (4) are assumed to describe deviations of certain economic variables from their target values; e.g., the following cost then describes: the goal of the satellite system controller is to regulate the SINR error deviations around zero; its own variables are also defined by

\[ A_{i,n}(k) \triangleq \begin{bmatrix} 1 & h_{i,i,n} \frac{1}{I_{i,n}(k-1)} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad x_{i,n}(k) \triangleq \begin{bmatrix} e_{i,n}(n) \\ b_{o,i,n}(n) \\ p_{i,n}(n) \\ 1 \end{bmatrix} \]

\[ B_{i,n}(k) \triangleq \begin{bmatrix} h_{i,i,n} \frac{1}{I_{i,n}(k-1)} \\ 0 \\ -1 \\ 0 \end{bmatrix}; \quad x_{i,n}(n_0) \triangleq \begin{bmatrix} e_{i,n}(n_0) \\ b_{o,i,n}(n_0) \\ p_{i,n}(n_0) \end{bmatrix}. \]

Also included is the cost function (6) that is rewritten in terms of the dynamical state variable, \( x_{i,n}(k) \). In this cost function, many different trades in SINR regulation error, power resource conservation, and power emission constraints will be emerged to create the so-called performance index for more effective use of existing radio resources; e.g.,

\[ J_{i,n}(n_0) = \sum_{k=n_0+1}^{N} x_{i,n}(k)^T Q_{i,n}(k-1) x_{i,n}(k) + r_{i,n}(k-1) u_{i,n}^2(k-1) \]

(8)

where

\[ Q_{i,n}(k-1) \triangleq \begin{bmatrix} q_{i,n}(k-1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} s_{i,n}(k-1) \\ 0 & 0 & -\frac{1}{2} s_{i,n}(k-1) & 0 \end{bmatrix} \]

As part of the comprehensive review of various types of traffic classes, which are reported separately in the terminal reports, it is proposed that return link resources which may meet the demands of delay sensitive traffic and best effort traffic be modified. As shown in Figure 2, the intent herein includes the following recommendations: that the effects of terminal communication return access assignments and reports be assigned different priorities; that differentiated services be related to burst modes, modulations, coding and radio resource assignments so that there will be more specific transmission policies sent by the satellite system controller to its active remote terminals.

\[ \text{Figure 2. Differentiated Services Lossy Satellite Network} \]

In the differentiated services architecture as depicted in Figure 2, \( H \) policy for delay sensitive traffic concerns a higher cost and lower loss probability as compared to \( L \) policy with a high probability at the cost of more losses for best effort traffic. In other words, the differentiated service mechanism adopted by terminal \( i \) assumes a cognitive use of the high-quality service class because its guarantee is obtained at the cost of communication resources taken from the low-quality one and so it should be used only when it is strictly needed.
In effect, traffic classes need to be classified as high or low-priority traffics. New binary variables $\delta_{k,n}$ and $\varsigma_{k,n}$ for the channel $n$ at the allocation activation epoch $k$ are likely to be established as decision strategy switches to transmit terminal communication return access parameters, including power emission adjustments and to measure the return link signal-to-noise ratios (SNRs) and the amount of forward link per-class traffic with the most appropriate policy for optimal satellite communications while minimizing the use of the high-cost policy $H$.

When $\delta_{k,n} = 1$, the forward link behavior at allocation epoch $k$ is described by the binary Bernoulli variable $\nu_{k,n}^{H}$ (i.e., high-priority service); otherwise, $\nu_{k,n}^{L}$ (i.e., low-priority service) is considered. And if the Bernoulli variable is equal to 1, the terminal communication return access assignments from the satellite system controller are reached to terminal $i$; otherwise, they get lost. As expected, the same holds for the return link behavior of forward link SNRs and the amount of return link per-class traffic between terminal $i$ and the satellite system controller. Mathematically, the probabilities of successful reception are

$$
Pr\{\nu_{k,n}^{H} = 1\} \triangleq \nu_{n}^{H} > \nu_{n}^{L} \triangleq Pr\{\nu_{k,n}^{L} = 1\}
$$

where the constants $\nu_{n}^{H}$, $\nu_{n}^{L}$ represent the different transmission loss rates from the terminal to the satellite system controller.

In essence, the effect of transmission loss on the transmitted data can be modeled by multiplying the terminal assignment parameters with binary independent and identically distributed Bernoulli variables [8] and [9]. Therefore, the state and measurement equations of the communication return access (7) can be rewritten as follows

$$
\begin{align*}
  x_{i,n}(k + 1) &= A_{i,n}(k)x_{i,n}(k) \\
  &+ [(1 - \delta_{k,n})\nu_{k,n}^{H} + \delta_{k,n}\nu_{k,n}^{H}]B_{i,n}(k)u_{i,n}(k) \\
  y_{i,n}(k) &= [(1 - \varsigma_{k,n})\lambda_{k,n}^{L} + \varsigma_{k,n}\lambda_{k,n}^{H}]x_{i,n}(k)
\end{align*}
$$

and finally, subject to the performance measure for SINR tracking errors, power conservations, and power emissions

$$
J_{i,n}(n_0) = \sum_{k=n_0+1}^{N} z_{i,n}(k)Q_{i,n}(k-1)z_{i,n}(k) + r_{i,n}(k-1)u_{i,n}^{2}(k-1) \tag{13}
$$

where for each allocation epoch $k$, the discrete-time coefficients $A_{i,n}(k) \equiv A_{i,n}(k)$, $B_{i,n}(k) \equiv B_{i,n}(k)$, and $G_{i,n}(k) \triangleq [1 0 0 0]^{T}$ are deterministic and bounded matrix-valued functions. In addition, any unmodeled dynamics, including $p_{i,n}(k + 1)h_{i,n}[I_{i,n}(k) - I_{i,n}(k + 1)]n_{i,n}(k + 1)$ in (4) are now approximated by the system noise characterized by a zero-mean additive white Gaussian noise, $w_{i,n}(k)$ with its variance $E\{w_{i,n}^{2}(k)\} = Q_{i,n}^{0}$ as can be seen in (11).

A further concern now involves dynamic resource allocation in attempting to meet request information rates, low probability of interception and low probability of detection requirements, power conservation needs, or any for any other operational reasons by remote terminals. Data transmission shall therefore be conducted only once, rather than repeatedly perform the same operation, then minimum-cost-variance optimization appears to be the most natural optimization technique for power adjustments. In this technique, the variance of a performance index is minimized while its expected value is constrained a-priori.

As an aide to the reader, this section should be seen as an important signpost representing the theory of minimum variance optimization. In particular, the emphasis on minimizing of the variance of $J_{i,n}(n_0)$ while its mean is forced to obey a constraint has a bearing on the current setting; e.g.,

$$
E\{J_{i,n}^{2}(n_0)|Z_{i,n}(n_0)\} = E^{2}\{J_{i,n}(n_0)|Z_{i,n}(n_0)\} \tag{14}
$$

is minimized, while

$$
E\{J_{i,n}(n_0)|Z_{i,n}(n_0)\} = h_{i,n}(n_0, Z_{i,n}(n_0)) \tag{15}
$$

where $E\{|\cdot|\}$ denotes the conditional expectation operator and the data $Z_{i,n}(n_0) \triangleq \{z_{i,n}(n_0)\}$.

The fundamental concern of $h_{i,n}(n_0, Z_{i,n}(n_0))$ is with practical considerations, including desired response, permissible deviations from the desired response, complexity of the controller, etc. How to proceed, given this general assessment? It turns out the choice of $h_{i,n}(n_0, Z_{i,n}(n_0))$ is not entirely arbitrary. It must be selected such that it is always greater than

$$
\inf_{u_{i,n}(n_0), \ldots, u_{i,n}(N-1)} E\{J_{i,n}(n_0)|Z_{i,n}(n_0)\} \tag{16}
$$

At this moment, it may be shown that for the special class of linear-quadratic problem, the mean value constraint is intuitively given by

$$
\begin{align*}
  J_{i,n}(n_0, Z_{i,n}(n_0)) &= m_{i,n}(n_0) \\
  &+ z_{i,n}(n_0)M_{i,n}(n_0)z_{i,n}(n_0) \tag{17}
\end{align*}
$$

where $m_{i,n}(n_0) \in \mathbb{R}^{+}$ and $M_{i,n}(n_0)$ is a symmetric and non-negative $4 \times 4$ real-valued matrix. Moreover, both $m_{i,n}(n_0)$ and $M_{i,n}(n_0)$ should be selected such that

$$
J_{i,n}(n_0, Z_{i,n}(n_0)) > \alpha_{i,n}(n_0, Z_{i,n}(n_0)) \tag{18}
$$
where $\alpha_{i,n}(n_0, Z_{i,n}(n_0))$ is as given by (16).

As depicted in Figure 3, a recursion equation for the optimal variance cost calls for the standard procedure for this type of problems; first, the constraint equation is appended to the expression to be minimized by means of a Lagrange multiplier, $\mu_{i,n}(n_0)$, and then the resulting equation is imbedded into the more general class of problems where $n_0$ is a variable rather than a fixed initial time. Clearly, the solution of the more general problem leads trivially to the solution of the problem posed herein. Consequently, it is desired to find $\mu_{i,n}(k)$ and the power increment policy be of the form

$$\gamma_{i,n}(k) = \gamma_{i,n}(k, Z_{i,n}(k)), \quad n_0 \leq k \leq N - 1, \text{ such that}$$

$$E \left\{ J^2_{i,n}(k) | Z_{i,n}(k) \right\} - E^2 \{ J_{i,n}(k) | Z_{i,n}(k) \} + 4\mu_{i,n}(k) [E \{ J_{i,n}(k) | Z_{i,n}(k) \} - h_{i,n}(k, Z_{i,n}(k))]$$

is minimized, where $\mu_{i,n}(k) \in \mathbb{R}^+$ is a Lagrange multiplier, and where the four pre-multiplying $\mu_{i,n}(k)$ has been introduced just for convenience. Note that $Z_{i,n}(k)$ contains all the information available to power adjustments at time $k$ and the form chosen for $\gamma_{i,n}(k)$ together with a boundedness requirement contribute to the definition of the class of admissible controls.

Before proceeding with the development of the recursion equation however, let $\gamma^*_i(k) \triangleq \{ \gamma_{i,n}(k), \gamma_{i,n}(k + 1), \ldots, \gamma_{i,n}(N - 1) \}, k = n_0, \ldots, N$, and let

$$VC_{i,n}(k, Z_{i,n}(k)) | \gamma^*_i(k) =$$

$$E \left\{ J^2_{i,n}(k) | Z_{i,n}(k) \right\} - E^2 \{ J_{i,n}(k) | Z_{i,n}(k) \} + 4\mu_{i,n}(k) [E \{ J_{i,n}(k) | Z_{i,n}(k) \} - h_{i,n}(k, Z_{i,n}(k))]$$

where $VC_{i,n}$ signifies “variance cost.”

In order to prevent the mathematical details from obscuring the concepts to be analyzed, some of the steps leading to the recursion equation for the variance cost have been relegated to [5]. At this stage, the assumption of linear control laws leads naturally to optimal quadratic costs, that is, for linear control laws it is always possible to write,

$$VC^*_{i,n}(k + 1, Z_{i,n}(k + 1)) =$$

$$v^*_{i,n}(k + 1) + z^T_{i,n}(k + 1)V^*_{i,n}(k + 1)z_{i,n}(k + 1)$$

where $v^*_{i,n}(k + 1) \in \mathbb{R}^+$ and $V^*_{i,n}(k + 1)$ are symmetric and nonnegative $4 \times 4$ real-valued matrices and whereas $n_0 \leq k \leq N - 1$. Thus, for $\beta_{i,n}(k) \triangleq A_{i,n}(k)z_{i,n}(k) + [(1 - \delta_{k,n})\nu_{k,n}^H + \delta_{k,n}v_{k,n}^H]B_{i,n}(k) \gamma_{i,n}(k)$, it follows that

$$E \{ VC^*_{i,n}(k + 1, Z_{i,n}(k + 1)) | Z_{i,n}(k) \} =$$

$$v^*_{i,n}(k + 1) + \beta^T_{i,n}(k)V^*_{i,n}(k + 1)\beta_{i,n}(k) + Tr \{ V^*_{i,n}(k + 1)Q^i_{W,n}(k) \} + \beta^T_{i,n}(k)\beta_{i,n}(k) + Tr \{ V^*_{i,n}(k + 1)(Q^i_{W,n} + 1) \beta_{i,n}(k) \}$$

Aside from the relevance of $S_{i,n}(k) \triangleq Q_{i,n}(k) + M_{i,n}(k + 1)$ for $n_0 \leq k \leq N - 1$, to the terminal conditions given by $m_{i,n}(N) = 0, M_{i,n}(N) = 0, v^*_{i,n}(N) = 0$, and $V^*_{i,n}(N) = 0$, some mathematical manipulations further yield

$$VC^*_{i,n}(k, Z_{i,n}(k)) =$$

$$\min_{\gamma_{i,n}(k), \mu_{i,n}(k)} \left\{ 4\beta^T_{i,n}(k)S_{i,n}(k)Q^i_{W,n}S_{i,n}(k)\beta_{i,n}(k) + \right.$$

$$+ E \{ (w_{i,n}(k)S_{i,n}(k)w_{i,n}(k)w_{i,n}(k))V^i_{i,n}(k) \}$$

$$+ v^*_{i,n}(k + 1) + \beta^T_{i,n}(k)V^i_{i,n}(k + 1)\beta_{i,n}(k) + Tr \{ V^*_{i,n}(k + 1)Q^i_{W,n}(k) \} +$$

$$+ 4\mu_{i,n}(k)[m_{i,n}(k + 1) + v^T_{i,n}(k)\beta_{i,n}(k) + Tr \{ S_{i,n}(k)Q^i_{W,n}(k) \} - m_{i,n}(k) - z^T_{i,n}(k)M_{i,n}(k)z_{i,n}(k) \} \right\}.$$ (23)

Performing the minimization with respect to $\gamma_{i,n}(k)$ for each remote terminal $i$ at the $n_{th}$ channel, the optimal minimal-cost-variance controller $\gamma^*_{i,n}(k)$ for power adjustments is given by

$$\gamma^*_{i,n}(k) = K^*_{i,n}(k)z_{i,n}(k)$$

where

$$K^*_{i,n}(k) = \frac{(B^*_{i,n})^T(k)\Lambda_{i,n}(k)A^*_{i,n}(k)}{(B^*_{i,n})^T(k)\Lambda_{i,n}(k)B^*_{i,n}(k) + 4\mu_{i,n}(k)\gamma_{i,n}(k)}$$

$$B^*_{i,n}(k) = [(1 - \delta_{k,n})\nu_{k,n}^H + \delta_{k,n}v_{k,n}^H]B_{i,n}(k)$$

$$\Lambda_{i,n}(k) = S_{i,n}(k)Q^i_{W,n}(k)S_{i,n}(k) + 1/4 V^*_{i,n}(k + 1) +$$

$$+ \mu_{i,n}(k)S_{i,n}(k), \quad n_0 \leq k \leq N - 1.$$ (24)

Using this minimal-cost-variance controller (24) parameterized in $\delta_{k,n}$ for power adjustments and performing the minimization in terms of $\mu_{i,n}(k)$ the mean constraint is obtained as follows

$$M_{i,n}(k) = (K^*_{i,n})^T(k)r_{i,n}(k)K^*_{i,n}(k) + (A^*_{i,n})^T(k)S_{i,n}(k)A^*_{i,n}(k)$$

$$m_{i,n}(k) = m_{i,n}(k + 1) + Tr \{ S_{i,n}(k)Q^i_{W,n}(k) \}$$

and the variance

$$V^*_{i,n}(k) =$$

$$(A^*_{i,n})^T(k)[4S_{i,n}(k)Q^i_{W,n}S_{i,n}(k) + V^*_{i,n}(k + 1)]A^*_{i,n}(k)$$

$$v^*_{i,n}(k) = v^T_{i,n}(k + 1) + Tr \{ V^*_{i,n}(k + 1)Q^i_{W,n}(k) \} + E \{ (w_{i,n}(k)S_{i,n}(k)w_{i,n}(k))w_{i,n}(k) \}$$

$$+ E \{ (w_{i,n}(k)S_{i,n}(k)w_{i,n}(k))w_{i,n}(k) \} - Tr^2 \{ S_{i,n}(k)Q^i_{W,n}(k) \}$$

(28)
where $A_{i,n}^*(k) \triangleq A_{i,n}(k) + \bar{B}_{i,n}(k)K_{i,n}^*(k)$ and $n_0 \leq k \leq N - 1$.

Yet, as has been evident all along, there are crucial gaps between the minimal mean and variance cost control problem. In a minimum-mean-cost problem, the solution of the recursion equations helps explain the minimum nature of the expected value of a performance index. Part of a minimum-variance-cost problem is based on the selection of $\mu_{i,n}(k)$, $n_0 \leq k \leq N - 1$ that solution of the recursion equations implies a mean value of the performance index as well as its corresponding minimum variance and optimal control law. Another part is that properly altering $\mu_{i,n}(k)$, $n_0 \leq k \leq N - 1$ validates several such sets of expected values, minimum variances, and optimal control laws. Putting this observation in its most contemporary assertion the claim is that the minimum mean problem is a particular case of the problem herein solved, namely, it is the solution of the recursion equations in the limit as $\mu_{i,n}(k)$ approaches infinity, $n_0 \leq k \leq N - 1$.

### 4. Intermittent Terminal Reporting

Undoubtedly, a central issue of communication return access from remote terminals to the satellite system controller for terminal report messages, including forward link SNRs and return traffic demands, underlies intermittent communications and limited available information as required by dynamic resource allocation. In essence, the effect of communication return access delays and loss of information in the dynamic resource allocation loop by the satellite system controller cannot be neglected. The recent scholarly efforts [12] to extend the discrete-time Kalman estimation in the setting of intermittent observations to modelling of the arrival of the observations as a Bernoulli independently random process exhibit the expected estimation error covariance depending on the tradeoff between loss probability and the system dynamics.

At present, radio interferences and propagation delays have become cause for concern that forward link SNRs and the amount of return link per-class traffic, in addition to channel blockage delays, is creating a situation in which the arrival of link updates and reports from remote terminal $i$ at the activation epoch $k$ and channel $n$ may be defined as a binary random variable $\zeta_{i,n}(k)$ with probability distribution $p(\zeta_{i,n}(k)) = \lambda_{i,n}(k)$, and with $\zeta_{i,n}(k)$ independent of $\zeta_{i,n}(l)$ if $k \neq l$. As might be expected, disturbances pertaining to the terminal report messages described in (12) may be co-integrated [10] and [11]; i.e., time series of the sample functions share a common stochastic drift. Henceforth, the more modest contribution of the research exposition here is the explicit acknowledgement of the terminal report messages (12) possibly corrupted by integrated white noise; i.e., Brownian motion with the mean $E[v_{i,n}(0)] = 0$ and covariance $E[v_{i,n}(0)v_{i,n}^T(0)] = Q^{v_{i,n}}_{i,n}$

$$y_{i,n}(k) = [(1 - \varsigma_{k,n})\lambda_{k,n}^L + \varsigma_{k,n}\lambda_{k,n}^H]z_{i,n}(k) + v_{i,n}(k)$$ (29)

and

$$v_{i,n}(k) = v_{i,n}(k - 1) + \eta_{i,n}(k - 1).$$ (30)

For illustration purposes, the quantization errors could be modeled by the uncorrelated zero-mean additive white Gaussian measurement noise process, $\eta_{i,n}(k)$ with its covariance $E[\eta_{i,n}(k)\eta_{i,n}^T(l)] = Q^{\eta_{i,n}}_{i,n}(k)\delta_{kl}$ due to finite possible quantization levels associated with forward link SNRs and the amount of return link per-class traffic measurements as depicted in (30). Furthermore, the integrated white noise or Brownian motion, $v_{i,n}(k)$ is expected given the fact that the noisy measurements are passed through the filters matched to the waveform-shaping filter. Then, the samples are taken of every symbol interval as notionally illustrated in Figure 4.

![Figure 4](image_url)

**Figure 4.** Block Diagram of Satellite Hub Receiver

As earlier suggested, it is important that the covariance of the observations by the satellite system controller at time $k$ is $Q^{v_{i,n}}_{i,n}(k)$ if $\zeta_{i,n}(k) = 1$, and $\mu^2_{i,n}I_{4\times4}$ otherwise. In effect, the restraint of letting $\mu_{i,n}$ go to infinity should be imposed when the real observation does not arrive; i.e.,

$$p(v_{i,n}(k)|\zeta_{i,n}(k)) = \begin{cases} N(0, Q^{v_{i,n}}_{i,n}); & \zeta_{i,n}(k) = 1 \\ N(0, \mu^2_{i,n}I_{4\times4}); & \zeta_{i,n}(k) = 0 \end{cases}$$ (31)

From the vantage point of the present, the rather unusual state estimation and filtering problem is to find the estimates of $z_{i,n}(k)$ by means of measurements corrupted by a Brownian motion (30). This new setting is conducive to fully bringing to light the fact that the estimation error of the optimal filter has a divergent variance. As expected, a general trend is to consider the augmented state approach for which one can recast the original problem of estimating the state of (11)

$$z_{i,n}(k) = A_{i,n}^*(k-1)z_{i,n}(k-1) + G_{i,n}^*w_{i,n}(k-1)$$ (32)

as a classical Kalman filtering where the system is subject to white noises. Now, let $z_{i,n}^E(k) = \begin{bmatrix} z_{i,n}(k) \\ v_{i,n}(k) \end{bmatrix}^T$ and $w_{i,n}^E(k) = \begin{bmatrix} w_{i,n}(k) \\ \eta_{i,n}(k) \end{bmatrix}^T$. Then the equations (29), (30), and (32) can be rewritten as

$$y_{i,n}(k) = C_{i,n}^Ez_{i,n}(k)$$ (34)

where the discrete system coefficients are defined by

$$A_{i,n}^E(k-1) = \begin{bmatrix} A_{i,n}(k-1) & 0_{4\times4} \\ 0_{4\times4} & I_{4\times4} \end{bmatrix}; G_{i,n}^E = \begin{bmatrix} G_{i,n}^z_{i,n} & 0_{4\times4} \\ 0_{4\times4} & I_{4\times4} \end{bmatrix}$$

$$C_{i,n}^E(k) = \begin{bmatrix} (1 - \varsigma_{k,n})\lambda_{k,n}^L + \varsigma_{k,n}\lambda_{k,n}^H \\ 0_{4\times4} \end{bmatrix}I_{4\times4}$$

$$E[w_{i,n}^E(k)(v_{i,n}^E(l))] = \begin{bmatrix} Q_{i,n}^w(k) & 0_{4\times4} \\ 0_{4\times4} & Q_{i,n}^w(k) \end{bmatrix} \delta_{kl}.$$
The final attempt broadens the intermittent state estimation inquiry by addressing the technical challenge of how to estimate the state of the dynamics given by (33) and (34). In this regard, the existing results [12] and [13] are further adapted and thus, providing essential features of an useful tool for optimal estimator design within the new setting; e.g.,

\[
\hat{z}_{i,n}(k) = A_{i,n}(k-1)\hat{z}_{i,n}(k-1) + L_{i,n}(k)[y_{i,n}(k) - C_{i,n}(k)A_{i,n}(k-1)\hat{z}_{i,n}(k-1)]
\]

(35)

together with

\[
L_{i,n}(k) = [A_{i,n}(k-1)P_{i,n}(k-1)(A_{i,n}(k-1))^\top + G_{i,n}(k-1)(G_{i,n}(k-1))^\top G_{i,n}(k-1)C_{i,n}(k-1)]^{-1}
\]

(36)

and

\[
P_{i,n}(k) = A_{i,n}(k-1)P_{i,n}(k-1)(A_{i,n}(k-1))^\top + G_{i,n}(k-1)(G_{i,n}(k-1))^\top G_{i,n}(k-1)C_{i,n}(k-1)
\]

(37)

And so, it is important to realize that the optimal emission power adjustment (24) now involves the state estimates governed by (35); i.e.,

\[
u^*_i(n) = K^*_i(n)\hat{z}_i(n)
\]

(38)

where \(\hat{z}_i(n) = [I_{4x4} 0_{4x4}]^\top \hat{z}^i_{i,n}(k)\) to confront with the reality of communication losses, quantization errors and waveform shaping filtering from terminal \(i\) on channel \(n\) and at activation epoch \(k\).

5. CONCLUSIONS

Closed-loop dynamical feedback continued to underpin dynamic radio resource allocation, providing terminal communication return access assignments over the differentiated services architecture for recourse to support delay sensitive and best effort traffics. A compelling review and adaptation of the development of minimal-cost-variance control theory was essential to the satellite system controllers, whose designs for reliability were concerned about quality of service data transmission, total power conservations, and the power emission updates. Another research contribution to the issue of intermittent terminal reports from remote terminals to the satellite system controller for closed-loop resource allocation was to emphasize a simple yet non-standard state-estimation problem, characterized by Brownian motion noise term. From such a traditional state augmentation, the solution to the problem of state estimation was expected to yield the optimal Kalman estimates of the terminal report messages.

In addition, the paper also leaves several open issues worthy of investigation. Whether the total cost for the use of transmission policies related to various traffic classes will turn into an iterative multi-stage optimization dedicated to the finding of feasible power adjustments and optimal logical transmission policies, will perhaps be the most profound question of the next research direction.

REFERENCES


**Biography**

*Khanh D. Pham* is a senior aerospace engineer at the Space Vehicles Directorate within the Air Force Research Lab. He is a senior member of Institute of Electrical and Electronics Engineers (IEEE), a Fellow of Society of Photo-Optical and Instrumentation Engineers (SPIE), and an Associate Fellow of American Institute of Aeronautics and Astronautics (AIAA). His research interests include statistical optimal control; decision analysis of adversarial systems; fault-tolerant control; dynamic game decision optimization; security of cyber-physical systems; satellite cognitive radios; and control and coordination of large-scale dynamical systems.