Multicarrier Modulation as a Navigation Signal of Opportunity

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Abstract—The Global Positioning System (GPS) generally provides worldwide high-accuracy positioning. However, GPS requires lines of sight to multiple satellites and may be blocked or jammed, hence backup navigation techniques are of interest. Navigation via signals of opportunity uses existing radio infrastructure as ad hoc navigational beacons. A mobile receiver determines its position by computing a time difference of arrival (TDOA) between signal reception time at the mobile and at a reference receiver. The drawback is that this requires communication between the reference and mobile receivers. In this paper, we show that multicarrier modulation is an ideal candidate for a navigation signal of opportunity. We show that communication between the reference and mobile can be very minimal for multicarrier modulation compared to other modulation types, since the block structure inherently aids synchronization of the two received signals. Simulations are used to quantify the performance of the TDOA estimate as a function of the SNR and the bandwidth between the reference and the mobile.

1. INTRODUCTION

The advent of the Global Positioning System (GPS) has provided worldwide high-accuracy position measurements. However, GPS requires lines of sight to multiple satellites, hence it is ill-suited to use indoors, underground, or in urban canyons. Moreover, in the presence of radio-frequency interference or jamming, GPS may be unavailable. Thus, alternative methods of navigation and positioning are of interest, either as a backup or for use in areas unreachable by satellites.

One alternative method of navigation that has been proposed recently is “Navigation via Signals of Opportunity.” The idea is to use existing radio frequency signals that were not intended for navigation, and leverage knowledge of the transmitter locations to determine the position of the receiver. The Defense Advanced Research Projects Agency (DARPA) has recently awarded $5 million to a team of companies to simply evaluate the feasibility of this approach [1], with potentially more money to follow to develop the idea.

Measurements that can be taken from signals of opportunity include the angle of arrival, the received power level, or the time difference of arrival (TDOA) at multiple receivers [2]. We focus on TDOA, since it is difficult to get sufficient position accuracy from angle or power measurements. One difficulty encountered with using TDOA is that for each TDOA measurement, there must be either two transmitters sending the same signal or two spatially separated receivers measuring the same transmission. Usually only one transmitter is available, hence a “reference receiver” must be placed at a known location, and the mobile (whose position is to be determined) must cooperate with the reference in some way [3,4,5]. TDOA measurements generally require some form of correlation between the two received signals [6,7]. This requires that the reference retransmit much of its received signal to the mobile, using significant bandwidth. This paper discusses how multicarrier modulation can be used to ease this difficulty.

Multicarrier systems have the benefit of a well-defined signal structure, in which the beginning and end of each block of data are identical. Thus, a receiver can identify block boundaries by looking for this repetition, which does not require knowledge of the transmitted signal (i.e. it is non-cooperative, or “blind”). The mobile and reference receiver can each locate the block boundaries, and can each calculate some statistical feature (e.g. mean or variance) of each block. Then the reference can transmit the sequence of block reception times and the associated feature values to the mobile, rather than retransmitting the entire signal. This leads to a significant reduction in bandwidth. In this paper, we propose a two-step TDOA computation, in which we first use the block structure to align the block boundaries, and then use a feature correlation to determine the time difference in blocks. The performance of various candidate features is then assessed via simulations.

REFERENCES

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2. **System Model**

The geometry of the transceivers involved in a single TDOA computation from a signal of opportunity is depicted in Figure 1. Note that in general, either multiple transmitters or multiple reference receivers are required in order to obtain multiple TDOA measurements and solve for a position, but for simplicity we only show one of each. We assume that there is a transmitter at a known location with a line of sight to a reference receiver at a known location, and with a line of sight to the mobile whose location is to be determined. There must be a reliable communication link between the reference and the mobile, but it need not be a line of sight. The reference gathers information about the received signal and passes a portion of the information on to the mobile, and the mobile compares its received signal to the data from the reference in order to compute the TDOA. The intent is that the amount of bandwidth that the reference uses to transmit to the mobile be as small as possible.

![Figure 1 - The geometry of TDOA computation.](image)

The "full knowledge" approach to TDOA computation would be to have the reference rebroadcast its entire received signal to the mobile, and then have the mobile cross-correlate the signal from the transmitter and the signal from the reference. However, this is wasteful of bandwidth. A simpler approach is to look for notable events that can be separately located in the received signals at the reference and the mobile. For example, occasional sharp spikes might occur in the transmitted signal, and both receivers can locate and compare the times when the received signal exceeds some threshold. Then the reference only needs to transmit the times at which large spikes occurred, and the mobile can correlate this with its record of when the spikes occurred.

The main drawback of this approach is that the events in question may be sensitive to noise, because in a low SNR environment, significant spikes are more likely to be due to the noise than the signal. Additionally, some signals do not contain identifiable "features," even in high SNR conditions. In the next section, we will discuss how the block structure of multicarrier modulation naturally lends itself to identification of events of this sort. First, however, we present the mathematical model of multicarrier modulation.

The block diagram of a multicarrier transmitter-receiver pair is shown in Figure 2. The idea behind multicarrier modulation is to break up a potentially frequency-selective multipath channel into a bank of narrow-band flat fading channels. This can be accomplished by parsing the source data into $N$ parallel lower-rate data streams, and modulating them with a bank of synchronized oscillators at linearly spaced carrier frequencies. Equivalently, we can use an inverse fast Fourier transform to modulate each successive block of $N$ data samples, convert the output $N$ samples from parallel to serial, and transmit them through the multipath channel. Then equalization can be done after a demodulating FFT at the receiver, simply by inverting the gain of each subchannel in the frequency domain.

![Figure 2 - Block diagram of a multicarrier transmitter and receiver.](image)

This method of equalization is enabled by the fact that a circular convolution in time is equivalent to an element-wise (Hadamard) multiplication in frequency. The caveat is that the physical multipath channel induces a linear (not circular) convolution with the transmitted data. The solution is to make the convolution appear circular by making the transmitted data appear periodic, at least over the duration of each block. This is done by inserting a cyclic prefix (CP) of length $v$ at the start of each block, as shown in Figure 3.

![Figure 3 - Insertion of the cyclic prefix, for $N=8$ and $v=2$.](image)

The CP is a copy of the last $v$ samples of each block, and it extends each block from $N$ to $M = N + v$ samples. The value of $v$ must be at least as large as the delay spread of the multipath channel. If this is true, then the last $N$ samples of the extended block that arrives at the receiver appear to be a circular convolution of the original $N$ samples and the multipath channel. Although the CP was put in place for
purposes of multipath mitigation, we will make use of it to ease the TDOA computation in the next section.

The notation is as follows. The discrete-time transmitted data stream will be denoted \( x(i) \). The redundancy induced by the CP causes the source data to obey

\[
x(Mk + i) = x(Mk + i + N)
\]

\[1 \leq i \leq N, \quad -\infty \leq k \leq \infty
\]

where \( k \) denotes the block index and \( i \) is the sample index within the block. We also assume an oversampling factor of \( P \) at the receiver, so that the sample period \( T \) is a fraction \( 1/(PM) \) of the OFDM block duration, and thus we have

\[M = PM \]

samples per OFDM block. In this work, we assume no multipath. Hence, incorporating the propagation delays and additive white Gaussian noise, the received signals at the reference and the mobile are

\[y_{\text{ref}}(i) = x(i - \delta_{\text{ref}}) + n_{\text{ref}}(i),\]

\[y_{\text{mob}}(i) = x(i - \delta_{\text{mob}}) + n_{\text{mob}}(i).
\]

where \( \delta_{\text{ref}} \) and \( \delta_{\text{mob}} \) are the propagation delays in samples, and \( n_{\text{ref}}(i) \) and \( n_{\text{mob}}(i) \) are the noise values. The powers of the signal, noise at the reference, and noise at the mobile are \( \sigma^2_s \), \( \sigma^2_{\text{ref}} \), and \( \sigma^2_{\text{mob}} \), respectively.

Assuming a sample period of \( T \) seconds, the TDOA can be written as

\[T = (\delta_{\text{mob}} - \delta_{\text{ref}}) T_z.
\]

In this paper, we assume that the TDOA is an integer multiple of samples. Fractional TDOA values could be handled by comparing the sampling phase in the two receivers.

3. TDOA Computation

In this section, we discuss how the TDOA is computed from the signals received at the reference and the mobile. The computation is a two-step process:

Step 1 (block boundaries): The reference uses the CP to locate the block boundaries within the signal that it receives. Simultaneously and independently, the mobile performs the same task on its received signal.

Step 2 (feature extraction): The reference and the mobile each compute a single, scalar statistical feature from each block. The reference transmits the feature values and boundary times of the associated blocks to the mobile, which then correlates the sets of feature values in order to line them up.

The first step is a fairly common method of blind (non-cooperative) block synchronization [8]. In conventional communications applications, block synchronization must be performed in order to successfully demodulate the data. However, we perform the same task here in order to transform the TDOA computation from the time scale of samples to the time scale of blocks. For clarity of presentation, we review the blind block synchronization method of [8] here, but the novel work lies primarily in step 2, which we will discuss later in this section.

Given a received block, the maximum likelihood estimate of the block boundaries is given by [8]

\[\hat{\delta}_{\text{ML,rx}} = \arg \max_{0 \leq m \leq M-1} \left| \gamma(m) - \left( \frac{\sigma^2_s}{\sigma^2_x + \sigma^2_n} \right) \Phi(m) \right| \]

where the subscript "rx" can be replaced by either "ref" or "mob", and where

\[\gamma(m) = \sum_{i=m}^{m+N} y_{\text{rx}}(i) y_{\text{rx}}^*(i + N)
\]

\[\Phi(m) = \frac{1}{2} \sum_{i=m}^{m+N} \left( |y_{\text{rx}}(i)|^2 + |y_{\text{rx}}(i + N)|^2 \right)
\]

denote the prospective CP-to-block correlation and the power in the CP and the end-of-symbol. As opposed to [8], in our work we average over many blocks. This causes the power term \( \Phi(m) \) to be nearly constant as a function of \( m \), hence we omit it. Also, [8] does not account for the fact that the desired signal component only contributes to the real part of \( \gamma(m) \). Including the averaging and approximation, and replacing the absolute value with a real operator, the estimate of the block boundaries becomes

\[\hat{\delta}_{\text{ML,rx}} = \arg \max_{0 \leq m \leq M-1} \Re \{ \gamma_{\text{avg}}(m) \}
\]

\[\gamma_{\text{avg}}(m) = \sum_{k=0}^{K-1} \sum_{i=m}^{m+N} y_{\text{rx}}(Mk + i) y_{\text{rx}}^*(Mk + i + N)
\]

where \( \Re \{ \} \) denotes taking the real part and \( K \) is the number of blocks included in the averaging. However, there is an ambiguity in this estimate, since at this point we cannot tell one block from another—we simply know where the boundaries are. In a standard communications application, this is not a problem, since synchronization is only performed to be able to demodulate the data, and the
value of the delay involved is not of interest in and of itself. However, for TDOA computation, we must move on to the second step of the process described at the start of this section.

Given the block boundaries, we parse the received signals into blocks, with the $k^{th}$ block given by

$$ y_{rx}(k) = \begin{bmatrix} y_{rx}(Mk + 1 + \delta_{ML,rx}) \\ \vdots \\ y_{rx}(Mk + M + \delta_{ML,rx}) \end{bmatrix}. \quad (7) $$

The task now is to compute some scalar feature for this block, which can be used to label it and discriminate it from other arbitrary received blocks. Thus, a desirable feature will vary significantly from block to block. It should depend on the underlying signal as much as possible and on the noise as little as possible.

The candidate features considered in this work are

- the first four normalized central moments (mean, variance, skewness, and kurtosis)

$$ \begin{align*}
\mu_{rx}(k) &= \frac{1}{M} \sum_{i=1}^{M} [y_{rx}(k)] \\
\sigma^2_{rx}(k) &= \frac{1}{M} \sum_{i=1}^{M} [y_{rx}(k)]^2 - \mu_{rx}(k)^2 \\
\gamma_{1,rx}(k) &= \frac{1}{\sigma^2_{rx}(k) M} \sum_{i=1}^{M} [y_{rx}(k)] - \mu_{rx}(k)^3 \\
\gamma_{2,rx}(k) &= \frac{1}{\sigma^4_{rx}(k) M} \sum_{i=1}^{M} [y_{rx}(k)] - \mu_{rx}(k)^4 
\end{align*} \quad (8) $$

- the average symbol’s phase

$$ \Phi_{rx}(k) = \text{atan} \left( \frac{\sum_{i=1}^{M} \text{Im} \left( [y_{rx}(k)] \right)}{\sum_{i=1}^{M} \text{Re} \left( [y_{rx}(k)] \right)} \right) \quad (9) $$

- the root-mean-square of the signal (preserving phase)

$$ \text{RMS}_{rx}(k) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} [y_{rx}(k)]^2} \quad (10) $$

- the standard deviation (no phase)

$$ \sigma_{rx}(k) = \sqrt{\sigma^2_{rx}(k)} \quad (11) $$

- and the peak–to–average power ratio

$$ PAPR_{rx}(k) = \max_{i=1}^{M} \frac{\left[ y_{rx}(k) \right]^2}{\frac{1}{M} \sum_{i=1}^{M} \left[ y_{rx}(k) \right]^2} \quad (12) $$

where $[\cdot]$ refers to the $i^{th}$ element of a vector. The features can also be computed in the frequency domain by discarding the CP, taking the FFT, and then averaging with respect to the resulting $N$ samples. However, we found that the results were almost identical to their time-domain counterparts. We omit them here, but they are available in [9]. Other statistical features could be chosen as well; these are just some of the most common, so we will use them to form a baseline for future comparisons. Note that many of the features are related, for example the variance and the standard deviation. The reason for considering both of these is that the square of the correlation of the standard deviation is not the same as the correlation of the square of the standard deviation, so we may potentially get different results.

The reference calculates the value of a particular feature for each of $K$ blocks, then transmits these $K$ values and its estimate $\hat{\delta}_{ref}$ to the mobile. Depending on the feature the values may be real or complex, so the amount of data that needs to be sent is either $K$ or $2K$ real numbers, plus one integer.

Now the mobile can compute the covariance of the features. Generically denoting the feature values as $f_{ref}(k)$ and $f_{mob}(k)$, the mobile computes

$$ R_F(d) = \frac{K}{K-1} \left( \frac{f_{ref}(k + d) - f_{ref} (1 + d)}{f_{ref}(k) - f_{ref}(1)} \right) \quad (13) $$

where

$$ f_{ref}^{arg}(j) = \frac{1}{K} \sum_{k=j}^{j+K-1} f_{ref}(k) \quad (14) $$

and similarly for the mobile. The mobile must compute (13) for all anticipated valid ranges of the block arrival time difference, say $-D \leq d \leq D$. This, while the reference computes the feature values for $K$ blocks, the mobile must compute them for $K+2D$ blocks. Once (13) has been computed over this range, the TDOA can be computed as
\[
\Delta = \arg \max_{-\frac{D}{2} \leq d \leq \frac{D}{2}} \Re \{ R_d (d) \}
\]
\[
\delta = \delta_{\text{mob}} - \delta_{\text{ref}}
\]
\[
TDOA = (\delta + M \Delta) T_s
\]

where \(\delta\) is the offset in samples, modulo \(M\); \(\Delta\) is the offset in blocks, which accounts for the modulo \(M\) ambiguity; and \(T_s\) is the sample period.

Note that this procedure includes the oversampled case because \(M\) refers to the total number of samples per block. However, the simulation results given later assume Nyquist sampling.

4. \textbf{BANDWIDTH AND COMPLEXITY}

In this section, we discuss what resources are available and how much of each is used. This includes a discussion of how many blocks \(K\) can be used in the feature correlator, the bandwidth between the reference and the mobile, and the computational complexity at the mobile.

The parameter \(K\) is of great importance because increasing \(K\) will improve the estimate of the TDOA. However, if the mobile is moving quickly, the true value of the TDOA will change over time, and if too many blocks are used, the TDOA will not be approximately constant within the estimation interval. Assume that the reference receiver is stationary and that the mobile is moving at a velocity \(V\).

The total length of the estimation interval in (13) is \(KMT_s\) seconds, and during this time the mobile will move \(KMT_sV\) meters. Depending on the geometry, the maximum change in propagation time from the transmitter to the mobile is \(\pm KMT_sV/c\) seconds, where \(c\) is the speed of light. The worst case scenario is such that the mobile is moving directly towards or away from the transmitter. We would like the total change in TDOA to be much less than the resolution of our sampling, i.e.

\[
KMT_sV/c \ll T_s
\]

which bounds \(K\) by

\[
KM \ll \frac{c}{V}.
\]

If the mobile is on an aircraft traveling at the speed of sound, \(V = 343\) m/s (or about 767 mph), then \(KM \ll 875,000\). For a block size of \(M = 80\) as used in our simulations, this means that \(K \ll 10000\) blocks can be used. If \(M = 2560\) (among the largest used in existing OFDM systems), then \(K \ll 340\) blocks can be used. Of course, this is all for a worst case scenario and a very fast mobile, so in practice more blocks may be used. On the other hand, if we oversample, then the oversampling factor will be incorporated into \(M\), and the upper bound on \(K\) will scale down accordingly.

The bandwidth between the reference and the mobile should be as small as possible. For scalability of comparison, we will evaluate the reference-to-mobile bandwidth in terms of the bandwidth used by the transmitted signal of opportunity, rather than in an absolute sense. The transmitter transmits \(M\) complex samples per block. The reference receiver transmits one feature value per block, and it may be real or complex. If it is a complex-valued feature, the bandwidth ratio is

\[
\frac{BW_{\text{ref}}}{BW_{\text{tx}}} = \frac{1}{M},
\]

and for a real-valued feature, it is

\[
\frac{BW_{\text{ref}}}{BW_{\text{tx}}} = \frac{1}{2M}.
\]

Typical values of \(M\) range from 80 to 2560, hence the proposed technique uses two to three orders of magnitude less bandwidth than the brute force approach of having the reference receiver simply retransmit the data it receives.

At the mobile, the computational complexity of the proposed approach is dominated by three contributions:

- computation of \(\gamma_{\text{avg}} (m), 0 \leq m \leq M - 1\),
- computation of \(\{ f_{\text{mob}}(k), 1 \leq k \leq K \}\),
- and computation of \(\{ R_f(d), -D \leq d \leq D \}\).

We will ignore additions and subtractions, since they are negligible compared to multiplications. First, consider computation of \(\gamma_{\text{avg}} (m)\). It is a sum of \(K\) dot products of a pair of windows that slides with \(m\). Thus, for the first value of \(m\), \(K\) complex multiplies are needed. However, as \(m\) is incremented, each dot product can be updated by appending and removing one term from each dot product. The elements to be removed have already been computed, hence only a total of \(K\) complex multiplies are needed for each additional value of \(m\). Thus, in total, computing \(\gamma_{\text{avg}} (m)\) requires approximately \(K (M + V)\) complex multiplies.

The complexity of the feature computation depends on the feature. The mean is the simplest feature to compute, and it requires a total of about \(MK\) complex additions. (Note that in practice we ignore constant scale factors such as \(1/M\)
since they scale the signal and noise components equally.) The most complex feature to compute is the kurtosis, which requires about 3MK real multiplies in an efficient implementation (a complex multiply is equivalent to 4 real multiplies). Thus, the complexity of this step can range from negligibility to MK complex multiplies.

The final correlation step requires computing \( f_{\text{avg}} \) and \( f_{\text{mob}} \), which are of negligible complexity. For each value of \( d \), computing \( R_f(d) \) requires 2K subtractions followed by K multiplies, for a total of about 2DK multiplies (real or complex depending on the feature). In total, the computational complexity is upper bounded by \( K(2M+2D+\nu) \) complex multiplies. The “mean” feature, which is shown to have the best performance in Section 5, only requires \( K(M+2D+\nu) \) complex multiplies. Since the total number of samples is \( KM \) and since \( D \) is typically small compared to \( M \), the total computational complexity is roughly one to two complex multiplies per sample, which is a reasonable level for a realizable system.

5. SIMULATION RESULTS

This section provides a performance analysis via Matlab® simulations. The transmitter uses multicarrier modulation with an FFT size of \( N=64 \), a CP length of \( \nu=16 \), and a block size of \( M=80 \), which are consistent with the IEEE 802.11a, HIPERLAN/2, and MMAC standards for wireless LANs. Throughout, we assume that step 1 (the CP correlation) is 100% accurate, since as many symbols as needed can be used without increasing the bandwidth from the reference to the mobile, thus, the bottleneck is the accuracy of step 2 (the feature correlation).

When computing (13) and looking for the peak value at the desired delay, we noticed that for several of the features there was a spike at the correct location but it was not always the global maximum of the function \( R_f(d) \). In order to magnify the desired spike, we convolved the function \( R_f(d) \) with a simple high pass second-order difference filter, with impulse response \([-1, 2, -1]\). This made the desired peak much more prominent.

Plots of the probability of correctly determining the overall TDOA are given in Figure 4, 5, and 6, for the best feature (the mean), a feature with moderate performance (the phase), and one of the worst features (the skewness). Performance of the variance, standard deviation, RMS, and peak-to-average power ratio was comparable to that of the phase; and performance of the kurtosis was comparable to that of the skewness.

![Figure 4 - Probability of synchronization error vs. SNR, for the “mean” feature.](image)

![Figure 5 - Probability of synchronization error vs. SNR, for the “phase” feature. Comparable performance occurred for the “variance,” “standard deviation,” “RMS,” and “peak-to-average power ratio” features.](image)

In these simulations, \( \sigma_{\text{ref}}^2 = \sigma_{\text{mob}}^2 \) and \( \text{SNR} = \frac{\sigma_r^2}{\sigma_{\text{ref}}^2} \). For the “mean” feature, perfect synchronization is achieved 95% of the time at 10 dB SNR with only 10 blocks. If 100 blocks are available, perfect synchronization is almost always achieved at -3 dB SNR; and if 1000 blocks are available, perfect synchronization is almost always achieved at -10 dB SNR.

If only a limited number of blocks are available, these results could be improved by combining several features into the correlator of (13). This would allow a tradeoff between the bandwidth used between the reference and the mobile and the synchronization performance.
6. CONCLUSIONS AND FUTURE WORK

Multicarrier modulation can be used to perform accurate TDOA computation with an SNR as low as -3 dB when 100 blocks are available. No training signal was required, although we did assume knowledge of the block structure of the transmitted signal. The only communication required between the reference and the mobile for this level of performance was the transmission of 100 complex numbers and one integer over the course of the 100 block (8000 sample) time window, hence the bandwidth from the reference to the mobile was almost two orders of magnitude smaller than the bandwidth of the signal of opportunity.

In many broadcast multicarrier systems (e.g., the terrestrial repeaters for satellite radio or European digital television), multiple transmitters each transmit the same signal. This cannot be done with single carrier systems because it creates multipath in the received signal. However, multicarrier systems are very robust to multipath. The result is that the reference receiver may no longer be needed. Possible future work could include assessing the feasibility of using multiple terrestrial repeaters to perform positioning.

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