Techniques to Screen for Moving and Large Stationary Discretes in Space-Time Adaptive Estimation Data

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Abstract—When the data samples used to estimate the interference covariance matrix in Space-Time Adaptive Processing (STAP) are assumed to be independent and identically distributed, an estimation loss indirectly proportional to the number of samples will occur. Measured data has shown that data samples are often nonhomogeneous and that these nonhomogeneities can cause losses beyond that from the estimation alone. Nonhomogeneities are of various types; here we have focused on moving and large stationary discretes. Numerical examples are presented that give a rationale for a screening technique. Then we present related screening techniques and performance simulations for the detection of moving and large stationary discretes in the estimation data. The screening techniques consist of pre-processing the samples by transforming them from the space-time domain to the angle-Doppler domain and exploiting the resulting interference characteristics.

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1. INTRODUCTION

Space-Time Adaptive Processing (STAP) is a candidate technology for airborne radar to improve the detection and tracking of slow moving targets in difficult clutter and jamming environments. STAP performance is determined in part by how closely its estimate of the interference covariance matrix, typically calculated from range samples surrounding a test range bin, matches the interference statistics of the test range bin. In this paper interference will mean ground clutter and receiver noise although in general it may also include other interference such as jamming and mutipath returns.

When the estimation samples are independent and identically distributed (i.i.d.) and the statistics match those of the test range bin, an estimation loss directly related to the number of samples, also called secondary data or sample support, will occur. Measured data, however, has shown that interference is often nonhomogeneous [1,2], and is characterized by varying amplitude and spectral statistics that depart from the i.i.d. assumption. These nonhomogeneities in the sample support will introduce losses in addition to the estimation loss, and will limit the range extent and number of samples that may be considered i.i.d. Many approaches have been proposed to meet sample support requirements in nonhomogeneous environments. These include screening the data to excise nonhomogeneities [3], selecting samples based on data dependent rules [4], using multiple transmit frequencies [5] and exploiting the structure of the interference covariance matrix [6,7]. Since the required sample support, as well as computational complexity, is directly related to the dimension of the covariance matrix, many partially adaptive and reduced rank methods have been introduced. With partially adaptive methods, the idea is to reduce a prohibitively large problem into a number of smaller problems via data transformation, while maintaining performance close to that of the optimum fully adaptive processor. A comprehensive discussion and taxonomy of partially adaptive techniques is given in [8].

For the reduced rank techniques a subspace of the covariance matrix is used to calculate the adaptive weights; the required sample support is related to the dimension of the interference subspace (often much less than the dimension of the estimated covariance matrix). An analysis of reduced rank techniques can be found in [9]. A systematic description, modeling and analysis of nonhomogeneities has only recently been addressed [10,11]. In [10] nonhomogeneities are classified by type, causes, and impact on adaptive radar performance. An overview of existing algorithmic solutions is also given. In [11] a variety of nonhomogeneities are analytically modeled and the resulting STAP performance analyzed.

Nonhomogeneities have been categorized into those having a gradual change over range and those with an abrupt change [1]. The former type are caused by range variations in the interference characteristics introduced for example by mismatched elevation patterns and the range-Doppler dependencies characteristic in non-sidelooking arrays. The abrupt nonhomogeneities are caused by clutter edges, moving discretes, coherent repeater jammers, and large stationary discretes. In this paper, we focus on moving and...
large stationary discretizes; that is, except for discrete nonhomogeneities the estimation data is assumed to be homogeneous and the secondary data statistics match those of the test bin. Previous results, both simulated and with measured data, have shown that when present in the estimation data these discretizes can lead to significant performance losses [3,10,11]. For example, in [11] it is shown that for a given airborne radar scenario, moving discrete scatterers corrupting the estimation data set result in performance losses from 2 to 20 dB, depending on the size of the moving discretizes and how close their directions of arrival are to the peak of the mainbeam. We will substantiate these results with numerical examples and present related techniques to screen range samples for the presence of undesired discretizes so that those samples can be removed before interference covariance matrix estimation.

The paper is organized as follows. Section 2 gives the rationale for a screening technique by first defining STAP signals and metrics and then illustrating with numerical examples the potentially significant losses caused by discretizes in the secondary data. Fully adaptive STAP and a reduced rank method are considered. Section 3 describes the screening techniques and is followed with numerical examples in Section 4. Section 5 is a conclusion and summary.

2. RATIONALE FOR A SCREENING TECHNIQUE

In this section we discuss and illustrate the rationale for a proposed technique that screens for the presence of moving and large stationary discretizes in the estimation data. First, we introduce signals that are relevant to STAP and discuss the need to estimate a covariance matrix from data samples prior to weight computation. Then we briefly discuss the STAP metric of Signal to Interference plus Noise Ratio and the expected losses introduced by the covariance matrix estimation. Discrete nonhomogeneities are then discussed and an expression for including them in the covariance matrix estimation is shown. Finally, we present numerical examples for both fully adaptive STAP and a reduced rank technique; it is seen that moving discretizes introduce significant losses beyond that introduced by estimation losses alone.

STAP Signals

Consider a linear array of N isotropic elements with each element followed by M time taps each delayed by the Pulse Repetition Interval (PRI) of the radar (Fig. 1).

The resulting signal at a given range, p, can be represented in matrix form as

$$X_p = [\bar{x}_{d,p}, \bar{x}_{l,p}, \ldots \bar{x}_{M-1,p}] \in \mathbb{C}^{N \times M} \quad (1)$$

where $\bar{x}_{m,p} \in \mathbb{C}^{N \times 1}$ is the spatial response vector across the array for the mth pulse. When vectorized as $\vec{x}_p = \text{vec}(X_p) \in \mathbb{C}^{MN \times 1}$, the resulting vector is called the space-time snapshot and is the superposition of all received signals from that range including clutter, $\bar{x}_{c,p}$, noise, $\bar{x}_{n,p}$, and target signal if present:

$$\vec{x}_p = \alpha_t \vec{v}(\phi_t, f_{d_t}) + \bar{x}_{c,p} + \bar{x}_{n,p} = \alpha_t \vec{v}(\phi_t, f_{d_t}) + \bar{x}_{n,p} \quad (2)$$

Fig. 1. Linear Array with PRI Delays

where $\alpha_t$ is the target amplitude and $\vec{v}(\phi_t, f_{d_t})$ is the array response to the target arriving from an angle $\phi_t$ and having Doppler frequency $f_{d_t}$. More specifically, $\vec{v}(\phi_t, f_{d_t})$ can be represented as

$$\vec{v}(\phi_t, f_{d_t}) = \vec{b}(f_{d_t}) \otimes \vec{a}(\phi_t) \quad (3)$$

with $\vec{a}(\phi_t)$ and $\vec{b}(f_{d_t})$ the spatial and temporal responses of the array given by

$$\vec{a}(\phi_t) = [1, e^{j2\pi f_r}, \ldots, e^{j2\pi f_r(N-1)}] \quad (4)$$

and

$$\vec{b}(f_{d_t}) = [1, e^{j2\pi f_{d_t}f_r}, \ldots, e^{j2\pi f_{d_t}f_r(M-1)}] \quad (5)$$

where $f_r = (d / \lambda) \sin \phi_t$ is the normalized spatial frequency, $f_{d_t}$ is the Pulse Repetition Frequency (PRF), and the quantity $f_{d_t} / f_r$ is the normalized Doppler frequency.

As indicated in figure 1 a scalar output for a given range is formed by the inner product of a complex weight vector with the space-time snapshot. The processor decides whether a target is present or absent according to the hypotheses:

$$\bar{x}_p = \bar{x}_u \quad : H_0 \text{ target absent}$$

$$\bar{x}_p = \alpha_t \bar{v}_t + \bar{x}_u \quad : H_1 \text{ target present}$$

The optimum weight vector to a scale factor is given by [12]:

$$\bar{w}_o = E_{[\bar{x}_u \bar{x}_u^H]}^{-1} \bar{v}_t \quad = R_p^{-1} \bar{v}_t \quad (6)$$

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where \( R_u \) is the covariance matrix of the clutter and noise signals, \( E \) is the expectation operator, and \( H \) denotes Hermitian transpose.

In practice, the covariance matrix is unknown and must be estimated. Various techniques for covariance matrix estimation are possible [see for example, 4] but nominally it is estimated by averaging snapshots located close to but not including the range bin currently under test for a target:

\[
\hat{R}_u = \frac{1}{P} \sum_{p=1}^{K} X_p X_p^H
\]

where \( P \) snapshots are used to estimate the covariance matrix. These estimation snapshots make up the sample support.

**SINR Metrics**

An important metric often used in STAP analysis is Signal to Interference plus Noise Ratio, SINR, at the processor output:

\[
\text{SINR} = \frac{\alpha_t^2 \bar{v}_t^H \bar{v}_t \| \bar{w}_t \|^2}{\bar{w}_t^H \bar{R}_u \bar{w}}.
\]

Using equation (6) in equation (8) gives the optimum SINR,

\[
\text{SINR}_o = \alpha_t^2 \bar{v}_t^H \hat{R}_u^{-1} \bar{v}_t.
\]

When the weights are calculated with the estimated covariance matrix we have the following expression:

\[
\text{SINR}_e = \frac{\alpha_t^2 \bar{v}_t^H \hat{R}_u^{-1} \bar{v}_t \| \bar{w}_t \|^2}{\bar{v}_t^H \hat{R}_u^{-1} \bar{R}_u \hat{R}_u^{-1} \bar{v}_t}.
\]

The SINR loss resulting from the covariance matrix estimate relative to \( \text{SINR}_o \) is denoted by

\[
\rho = \frac{\| \bar{v}_t^H \hat{R}_u^{-1} \bar{v}_t \|^2}{\bar{v}_t^H \hat{R}_u^{-1} \bar{R}_u \hat{R}_u^{-1} \bar{v}_t} = \frac{\text{SINR}_e}{\text{SINR}_o}.
\]

Under the assumption that i.i.d. data samples are used for the covariance matrix estimation and that the weight vector has MN components it has been shown that the expected value of the estimation loss, \( E[\rho] \), is \(-3 \) dB when the number of samples, \( P \approx 2MN \) [12].

**Discrete Nonhomogeneities**

We have previously mentioned that because of nonhomogeneities in the estimation data the i.i.d. assumption is often not met in practice. Here we will be mainly concerned with moving discrete and large stationary discrete nonhomogeneities. By moving discretes we mean ground vehicles such as jeeps, tanks and trucks which have shown measured median RCS values (averaged over angle) of 20, 50 and 100 m² respectively [13]; large stationary scatterers refers to man-made structures such as buildings. For the numerical examples in this section and in Section 4 we have given the moving discretes an RCS of 10 m² in all cases.

We may modify the covariance matrix estimation equation to include discrete nonhomogeneities as follows [3]:

\[
\hat{R} = \frac{1}{P} \sum_{p=1}^{P} \bar{y}_p \bar{y}_p^H
\]

where the double hat denotes discrete nonhomogeneities in the data. The data snapshot sample at range bin \( p \) is \( \bar{y}_p = \bar{X}_{u,p} + \delta \bar{y}_p \) where \( \delta \bar{y}_p \) represents the discrete nonhomogeneity and may be expressed as

\[
\delta \bar{y}_p = \alpha_d \bar{a}(\phi_d) \bar{b}(f_d) \delta \bar{y}_p
\]

where \( \alpha_d, \bar{a}(\phi_d), \) and \( \bar{b}(f_d) \) are the amplitude of the discrete and the spatial and temporal responses of the array to the discrete. The covariance matrix estimation in the nonhomogeneous environment is then

\[
\hat{R} = \frac{1}{P} \sum_{p=1}^{P} \bar{y}_p \bar{y}_p^H
\]

\[
= \frac{1}{P} \sum_{p=1}^{P} \bar{X}_{u,p} \bar{X}_{u,p}^H + \frac{1}{P} \sum_{p=1}^{P} (\delta \bar{y}_p \delta \bar{y}_p^H + \delta \bar{y}_p \bar{X}_{u,p}^H + \bar{X}_{u,p} \delta \bar{y}_p^H)
\]

\[
\hat{R} = \hat{R} + \Delta \hat{R}.
\]

In the remainder of this section we will show by numerical example the potentially significant SINR loss caused by discrete moving scatterers in the estimation data, and that this significance diminishes as the moving discrete moves outside the 3 dB mainbeam pattern of the array and away from the Doppler frequency of a desired target. SINR loss curves for fully adaptive and a reduced rank technique are shown and it is seen that the reduced rank technique is more sensitive to moving discretes than the fully adaptive STAP. We show no performance examples from large stationary discretes in the estimation data but note that they can increase the number of false alarms by crossing thresholds set too low for them. Having thus established a rationale for screening the secondary data for moving and large discretes we present such a screening method in Section 3.

**Numerical Examples**

To illustrate the potentially significant SINR loss caused by discrete moving scatterers in the estimation data we simulate an airborne radar. The parameters stated here will hold throughout the paper unless otherwise noted. The airborne radar is a side-looking, planar array of 18x4 elements where the columns are spaced at half wavelength, \( d = \lambda /2 \), and beamformed to give \( N=18 \) spatial channels. The transmit array has a Chebyshev weighting with 30 dB sidelobes and a peak gain of 22 dB; the receive gain for the columns is 10 dB. The radar processes \( M=16 \) pulses in a coherent processing interval, transmits at a frequency of 450 MHz, has a platform velocity of \( v_p = 50 \) m/s, and has a PRF of \( f_c =\).
The clutter to noise ratio is taken to be CNR = 50 dB. The signal to noise ratio of an unwanted discrete is taken to be SNR = 14 dB, and the signal to noise ratio of the desired target signal is taken to be SNR = 10 dB.

Fig. 2. Varying Angle of Arrival for Single Discrete in the Estimation Data.

Figures 2-4 show SINR loss performance relative to the noise limited environment:

\[
\text{SINR}_{\text{Loss}} = \frac{\bar{v}_t^H \hat{R}_u^{-1} \bar{v}_t}{\bar{v}_t^H \tilde{R}_a^{-1} \hat{R}_u^{-1} \bar{v}_t} (\text{MN})
\]

where unit variance noise is assumed. The upper performance limit will be zero dB since noise sets an upper limit on performance. A desired target is assumed to arrive from broadside; its Doppler is varied and the adaptive weights are calculated for each Doppler. Figure 2 shows the results of several experiments in which the angle of arrival of a single moving discrete is varied from 1 to 4 degrees relative to broadside. The 3 dB half beamwidth of the transmit pattern is about 5 degrees. In figure 2 the solid line shows SINR_{Loss} when the covariance matrix is known. The remaining lines show SINR_{Loss} when the weights are calculated with a covariance matrix estimated with P = 2MN = 576 range snapshots; note the estimation loss relative to the known covariance matrix. Also note the additional loss introduced by a single moving discrete in the secondary data. In all cases the moving discrete has a normalized Doppler of \( f_d / f_r = 0.1108 \) (24.8 mph). Depending on the angle of arrival these additional losses can be significant and we certainly would like to remove the moving discrete from the estimation data.

Figure 3 illustrates what may happen when multiple moving discretes are present in the estimation data. Here 3 moving discretes are included in the estimation data; each has an angle of arrival of 1 degree and their respective normalized Doppler shifts are 0.1108 (24.8 mph), 0.0997 (22.3 mph), and -0.0997 (-22.3 mph). It does not take many moving discretes in the estimation data to significantly degrade the detection of a desired target.

Fig. 3. Multiple Moving Discretes in the Estimation Data.

Fig. 4. Comparison of MME to Fully Adaptive STAP.

Reduced Rank Technique—In the preceding discussions and in calculating the \( \text{SINR}_{\text{Loss}} \) curves in figures 2 and 3 we have been using fully adaptive STAP. That is, the dimension of the weight vector is MN. In practice fully adaptive STAP requires tremendous computational complexity and a large secondary data set subject to many nonhomogeneities; consequently, a number of partially adaptive and reduced rank techniques have been proposed to lessen the computational burden and the required sample support. The Minimum Norm Eigencanceller (MME) is a reduced rank technique \([14]\). In its single constraint form the MNE is formulated as the following constrained optimization problem:

\[
\min \tilde{w}^H \tilde{w} \text{ subject to } Q^H \tilde{w} = 0 \text{ and } \bar{v}_i^H \tilde{w} = 1. \quad (16)
\]

Here, \( Q \) is an MNxM matrix representation of the interference subspace and \( r \) is the dimension of the
interference subspace. The resulting optimum weight vector is given by

\[ \tilde{w}_o = (I - Q_r Q_r^H)^{-1} Q_r (I - Q_r Q_r^H)^{-1} \tilde{v}_r. \]  

(17)

The rank of the subspace is often much less than \( MN \) and under certain limiting assumptions \( r \approx [N + (M-1)] \) (this is our assumption; also see [8]). Moreover, it has been shown that for the single constraint MNE the secondary data support requirement is \( 2r \) [15].

Figure 4 compares the SINR\(_{\text{est}}\) performance of MNE to fully adaptive STAP when the number of secondary samples is \( 2r \) and \( 2MN \) respectively; a single moving discrete with angle of arrival of 1 degree and normalized Doppler of 0.1108 (24.8 mph) is put in the estimation data for both cases. Because of its smaller sample size the MNE performance is much more sensitive to the moving discrete; however, when the samples are homogeneous MNE and fully adaptive STAP have similar performance. Note that for reduced rank and partially adaptive methods a screening technique may be particularly beneficial by allowing the relatively small secondary data set to be chosen from a much larger set of available data. In the next section we present related techniques to screen for the presence of moving and stationary discretes in the secondary data.

3. SCREENING TECHNIQUES

This section presents related techniques to screen range samples for the presence of undesired discretes both moving and stationary; those samples containing discretes can then be removed prior to covariance matrix estimation. First, we describe a two stage method for detecting moving discretes. The first stage is a frequency domain Adaptive Matched Filter and the second stage is a detector based on the Cell-Averaging Constant False Alarm Rate (CA-CFAR) detector.

The detector is not optimized here but used only to demonstrate the concept. Then we consider a related screening method for large stationary discretes and note that it is computationally much simpler than that for moving discretes since it requires no adaptation.

**Moving Discretes**

**Stage 1**—Let us refer back to an individual space-time data sample in its matrix form:

\[ X_p = [\tilde{x}_{0,p}, \tilde{x}_{1,p}, \cdots \tilde{x}_{M-1,p}] \in \mathbb{C}^{N \times M}. \]

Now transform \( X_p \) from the space-time domain to the angle-Doppler domain via a 2-dimensional Fast Fourier Transform:

\[ \tilde{X}_p = F_{2D} X_p \]  

(18)

where \( F_{2D} \) is the 2-dimensional FFT transform, and the tilde denotes a frequency domain quantity. The rows of \( \tilde{X}_p \) correspond to spatial frequencies and the columns to Doppler frequencies. A notional power spectral density plot of \( \tilde{X}_p \) for a sidelooking array is shown in figure 5. The diagonal line represents the clutter return or clutter ridge; when the parameter \( \beta = 2v_s/f, d \) is equal to one, in our numerical examples, the Doppler is unambiguous and the clutter ridge will just completely fill the Doppler space. (A detailed description of \( \beta \) and its effect on the angle-Doppler return is given in [8]). The shaded area of the figure, labelled \( \alpha_o \), shows the spatial frequency extent of the mainlobe clutter return where return from the boresight has zero spatial frequency. When a moving discrete is present in the data sample its motion gives it a Doppler offset relative to the stationary clutter ridge and potentially places it in an area relatively clear of interference and accessible for screening. The dots labelled 1 and 2 represent moving discretes of different Dopplers but at spatial frequencies close to the boresight. The moving discrete labelled 3 is well off boresight and will have little effect on SINR as figure 2 illustrated. Dots 4 and 5 represent stationary discretes and will be described in the next section. Thus, since we wish for the screening technique only to detect moving discretes in the secondary data it will suffice to use only that portion of \( \tilde{X}_p \) containing spatial frequencies near the boresight. It is reasonable for our simulation to assume that these frequencies are confined to the single spatial frequency bin represented by \( \alpha_o \). If we extract and transpose bin \( \alpha_o \) from \( \tilde{X}_p \) then we have the data vector, \( \tilde{X}_{\alpha_o,p} \in \mathbb{C}^{M \times 1} \), of Doppler frequencies. The frequency domain optimum weight vector at frequency \( f_i \) to within a scale factor is given by

\[ \tilde{w}_{\alpha_o,f_i} = \tilde{R}_{\alpha_o}^{-1} \tilde{v}_{f_i} \]  

(19)

where \( \tilde{R}_{\alpha_o} \in \mathbb{C}^{M \times M} \) is the covariance matrix from the transformed data at spatial frequency \( \alpha_o \) and \( \tilde{v}_{f_i} \in \mathbb{C}^{M \times 1} \).
is the transformed steering vector at frequency \( f \) defined as

\[ \tilde{\mathbf{v}}_f = F_{1D} \tilde{\mathbf{b}}(f) \]  

(20)

where \( F_{1D} \) is 1-dimensional FFT transform and \( \tilde{\mathbf{b}}(f) \) is the temporal response vector defined in equation (5). The covariance matrix can be estimated from the transformed secondary data by

\[ \hat{\mathbf{R}}_{ao} = \frac{1}{P} \sum_{p=1}^{P} \tilde{x}_{ao,p} \tilde{x}_{ao,p}^H \]  

(21)

where the notation \( p \neq \text{test} \) indicates that the estimate does not include data from the range bin under test. The resulting estimated weights at frequency \( f \) are

\[ \hat{\mathbf{w}}_f = \hat{\mathbf{R}}_{ao}^{-1} \tilde{\mathbf{v}}_f \]  

(22)

If a moving discrete is present in the test bin then its Doppler frequency will be unknown so it is necessary to vary the steering vector over a number of frequencies, \( K \), such that all Doppler frequencies of interest are covered. Thus, an \( M \times K \) matrix of weight vectors,

\[ \hat{\mathbf{W}}_{ao} = \begin{bmatrix} \hat{\mathbf{w}}_{ao,f_1} & \ldots & \hat{\mathbf{w}}_{ao,f_i} & \ldots & \hat{\mathbf{w}}_{ao,f_K} \end{bmatrix} \in \mathbb{C}^{MK} \]

is calculated by

\[ \hat{\mathbf{W}}_{ao} = \hat{\mathbf{R}}_{ao}^{-1} \tilde{\mathbf{v}}_f \]  

(23)

where \( \tilde{\mathbf{v}}_f = [\tilde{v}_{f_1}, \ldots, \tilde{v}_{f_i}, \ldots, \tilde{v}_{f_K}] \in \mathbb{C}^{MK} \) is a matrix of steering vectors.

An output vector, \( \tilde{\mathbf{z}}_{\text{test}} \), is calculated by operating on the test bin vector with the weight matrix:

\[ \tilde{\mathbf{z}}_{\text{test}} = \hat{\mathbf{W}}_{ao}^H \tilde{x}_{ao,\text{test}} \in \mathbb{C}^{K1}. \]  

(24)

Thus, processing a block of \( L \) range bins will yield \( L \) output vectors, \( \tilde{\mathbf{z}}_i, i = 1:L \) each of length \( K \).

Before describing the second stage we emphasize that the frequency domain Adaptive Matched Filter just described is used only to screen secondary data for moving discretes and need not resemble the STAP architecture that will be used to adaptively determine the presence or absence of targets. Moreover, a far greater number of false alarms can be tolerated at this screening stage because false alarms will only result in the loss of secondary data as opposed to false target declarations.

**Stage 2**—After processing a block of potential secondary data samples with the frequency domain Adaptive Matched Filter those samples are processed by a detector to test for the presence of a moving discrete. One possible detector, shown in figure 6, is based on the Cell Averaging CFAR detector. Referring to figure 6 the output vectors, \( \tilde{\mathbf{z}}_f \), form the columns of the data block; the column labelled \( T \) represents the test bin currently under test. The first step of the detector is to average each frequency, \( f_i \), of the output vectors across all \( L \) range bins in the block except the test bin at each frequency, \( f_i \). Mathematically, we have:

\[ D_{f_i} = \frac{1}{L-1} \sum_{l=1; l \neq \text{test}}^L |\tilde{z}_{f_i,l}|^2, \quad i = 1: K. \]  

(25)

The second step is to average the outputs, \( D_{f_i} \), across all frequencies:

\[ \bar{D} = \frac{1}{K} \sum_{f=1}^{K} D_{f_i}. \]  

(26)

and to average the test bin across all frequencies:

\[ \bar{T} = \frac{1}{K} \sum_{i=1}^{K} |T_{f_i,\text{test}}|^2. \]  

(27)

where \( \bar{D} \) and \( \bar{T} \) are scalars. A difference, \( \Delta = \bar{T} - c\bar{D} \), is calculated where \( c \) is a threshold adjustment used to control false alarms. If \( \Delta \) exceeds zero a moving discrete is declared. In this way each of the \( L \) range bins is tested for the presence of moving discretes. In section 4 we will show numerical examples for the two stage screening method just described.

**Large Stationary Discretes**

In contrast to moving discretes, large stationary discretes lie along the clutter ridge. In the notional PSD plot of figure 5, the dots labelled 4 and 5 represent large stationary discretes in the mainlobe and sidelobe clutter respectively. Figure 7 shows a range/angle representation of the large stationary discretes. For our example sideloooking array, the velocity vector lies along the x-axis and the mainlobe lies along the y-axis. Angle from boresight is measured relative to the y-axis and range is represented by the concentric rings.
4. NUMERICAL EXAMPLES

In this section we demonstrate the data screening concept with numerical examples. Cases of interest for moving discretes and large stationary discretes in the secondary data are described and representative results discussed.

**Moving Discretes**

The radar parameters used for these numerical examples are those given in Section 2 with the following change. Recall the covariance matrix estimated from the transformed secondary data:

$$\hat{R}_{\alpha_0} = \frac{1}{P} \sum_{p=1}^{P} \tilde{X}_{\alpha_0,p} \tilde{X}_{\alpha_0,p}^H \in \mathbb{C}^{M \times M}.$$  

The dimension of $$\hat{R}_{\alpha_0}$$ and thus the number of samples required to estimate it is determined only by the number of pulses, M; this is in contrast to fully adaptive STAP where the dimension of the covariance matrix is the product of spatial channels and pulses, MN. We may increase the number of pulses processed for the screening technique without affecting the actual STAP processing; it is advantageous to do so because the increased Doppler resolution will reduce the amount of interference that the moving discretes compete with. Thus, for the screening technique numerical example we increase M from 16 to 32 and estimate $$\hat{R}_{\alpha_0}$$ with 2M=64 range samples located symmetrically about the range bin under test.

For the first example, we arbitrarily choose to screen a block of 20 range bins where each range bin in the block undergoes the two stage screening process described in Section 3. Recall that all moving discretes are given a SNR=14 dB. We place a single moving discrete in the 16th range bin of the block and experiment by varying its angle of arrival and normalized Doppler frequency. These experiments, or cases, are summarized in Table 1.

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</tbody>
</table>
The angle of arrival is denoted by $\phi$ and the normalized Doppler by $f_d/f_r$. Each case was run 100 times and the results are presented in the last column of Table 1 and in figures 8 and 9. Table 1 shows the number of times the moving discrete is detected in range bin 16 out of 100 trials for each of the nine cases. The numbers show that as the relative velocity of the moving discrete increases and it breaks free of the mainlobe clutter it becomes easier to detect; it is also apparent that for a given normalized Doppler frequency, the moving discrete is easier to detect as its angle of arrival moves closer to broadside. More to the point we may say that it is easier to detect as its Radar Cross Section increases, although we have not explicitly varied the RCS for these cases. The array transmit taper will also significantly influence detection performance by reducing the sidelobe clutter interference that the moving discrete must compete with.

Figures 8 and 9 are meant to illustrate the screening technique; they show the respective outputs of the detector for test range bins 16 and 17 plotted versus frequency. That is, the quantities, $D_{f_r}$, and $\left| T_{f_r, \text{test}} \right|^2$ are plotted after the adaptive matched filtering but before being averaged across frequency with equations (25) and (26). The single moving discrete is in range bin 16 and has a normalized Doppler frequency of 0.0997 and an angle of arrival of $2^\circ$ (Case 5, Table 1). In figures 8 and 9 the solid line is a plot of the test range bin and the dashed line the average of the remaining range bins at each frequency in the block of 20. Figure 8 clearly indicates the peak caused by the matched filter when it is tuned to the moving discrete frequency of 0.0997. After averaging across frequency and setting a reasonable threshold adjustment the moving discrete is detected 95% of the time as shown in Table 1. Figure 9 shows the result of a test bin which has no moving discrete; false alarms occurred 3% of the time using the chosen threshold adjustment. Note that the threshold settings are not optimized and that false alarms occurred up to 10% of the time for all cases. False alarms that happen during the screening technique will result in the loss of possibly good secondary data.

A good question to ask is what happens if moving discretes are in more than one range bin. If a moving discrete is present in the bin under test then moving discretes in other range bins serving as estimation data for $\tilde{\mathbf{R}}_{\alpha_m}$ are expected to cause cancellation of the test bin target and thus degrade detection performance as implied by the numerical examples in Section 2. The degree of degradation will depend on many factors including the number of moving discretes as well as their sizes and relative velocities. The interaction between these factors may be complex. Here we only briefly consider the question of multiple moving discretes with the single example summarized in Table 2.

\begin{table}[h]
\centering
\caption{Multiple Moving Discretes in the Range Block}
\begin{tabular}{|c|c|c|c|}
\hline
Range Bin & $\phi$, degrees & $f_d/f_r$ & Detections per 100 trials \\
\hline
6 & 2 & 0.0886 & 56 / 20 \\
11 & 3 & 0.0997 & 83 / 20 \\
16 & 1 & 0.1108 & 100 / 92 \\
20 & 2 & 0.1108 & 100 / 72 \\
\hline
\end{tabular}
\end{table}

A moving discrete is placed in each of the range bins indicated by Table 2 and given the corresponding angle of arrival and normalized Doppler. The experiment was run 100 times and the results are presented in Table 2. The last column in Table 2 compares the results of the multiple moving discrete experiment with the corresponding single moving discrete case. For example, 20 out of the possible...
100 detections occurs in range bin 6; the moving discrete in range bin 6 has the parameters corresponding to case 4 which yielded 56 detections when it was the only moving discrete in the range block. In other words, the interference caused by the multiple moving discretes has caused a loss of 36 detections. The performance in range bins 16 and 20 fares somewhat better, but it is clear that for these numerical examples the screening performance is degraded and some strategy for dealing with multiple moving discretes must be considered.

Large Stationary Discretes

We now consider a numerical example for the large stationary discrete screening technique described in Section 3. This time let us choose an arbitrary block of 50 range bins and run the experiment summarized in Table 3.

Table 3. Large Stationary Discretes

<table>
<thead>
<tr>
<th>Range Bin</th>
<th>$\phi$, degrees</th>
<th>RCS, $m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>0</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>36</td>
<td>16</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>41</td>
<td>0</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>46</td>
<td>16</td>
<td>$3 \times 10^5$</td>
</tr>
</tbody>
</table>

Fig. 10. Baseline: Output Across Range Bins with no Discretes in the Data for $\phi=0^\circ$ (Mainlobe Look Direction). Thresholds: $c=1.5$ ('-'); $c=1.0$ ('- -').

Fig. 11. Shows Output $\tilde{\Delta}$ for $\phi=0^\circ$: Large Discretes in Range Bins 31 and 41.

Fig. 12. Shows Output $\tilde{\Delta}$ for $\phi=16^\circ$ (First Sidelobe); Large Discretes in Range Bins 36 and 46.

A large stationary discrete is placed in each of the range bins indicated in Table 3 and given the corresponding angle of arrival and RCS. The peak of the mainlobe is of course at $0^\circ$; $16^\circ$ degrees is also chosen because that is the location of the first sidelobe peak. The results for a single run are presented in figures 10-12. Figure 10 is a baseline plot used to determine a reasonable threshold adjustment, $c$. No discretes are present in the data. The solid line is a plot of the difference vector, $\tilde{\Delta} = \tilde{X}(\phi) - c\tilde{X}_{avg}(\phi)$, defined in Section 3. The angle of arrival being checked is $\phi=0^\circ$.

The lower and higher horizontal lines represent threshold settings with $c=1$ and $c=1.5$ respectively. Next, the large stationary discretes are added to the data and the screening technique described in Section 3 is carried out. It should be noted that in order to eliminate the interference caused by the 2-D FFT spatial frequency sidelobes a sidelobe taper
was applied to the space-time snapshot before the frequency processing. Figure 11 is a plot of $\Delta$ for $\phi=0^\circ$. The large discretes in range bins 31 and 41 cross the threshold and are easily detected. Figure 12 is a plot of $\Delta$ for $\phi=16^\circ$; that is, the angle cut corresponding to the peak gain of the first sidelobe. Not surprisingly, since the first sidelobe is 30 dB down from the mainbeam the output magnitudes are much smaller. The relatively smaller discrete in range bin 46 is just barely detectable with the given threshold setting.

5. CONCLUSIONS

STAP processing requires an estimation of the interference covariance matrix. The estimation is typically calculated with range samples surrounding a range bin under test for the presence of a target. When these range samples are i.i.d. and the statistics match those of the test range bin, an estimation loss indirectly proportional to the number of samples will occur. When they are not i.i.d. additional losses may occur. The departure from i.i.d. is caused by a variety of possible nonhomogeneities in the estimation data. In this paper we have focused on discrete nonhomogeneities and have shown with numerical examples that the losses caused by moving discretes can be quite significant and have stated that large stationary discretes can result in an increased false alarm rate.

Having provided the rationale to identify and remove discretes from the estimation data the paper presents related screening techniques to do so. The techniques are based on pre-processing the range samples; specifically, this means transforming space-time range samples to the angle-Doppler domain via a 2D FFT. For moving discretes, an adaptive matched filter is used that exploits the characteristic Doppler offset from mainlobe clutter; large stationary discretes are detected by simply comparing the return from one range at a given angle to the average from a number of other ranges at that same angle assuming that the antenna gain is approximately the same for all ranges.

The numerical results shown in this paper indicate that the proposed screening techniques are effective under certain limiting assumptions and warrant further study. The screening techniques are conceptually simple since they are based on Fourier Transforms and known adaptive processing and detection schemes; they also allow a far greater number of false alarms than the final STAP processing because this only results in the loss of estimation data as opposed to a false target detection. There are possible limitations. The ability to detect a moving discrete will depend on its velocity and size as well as the parameters of the radar. The additional processing loads and possible delays caused by the screening techniques need careful consideration. Multiple moving discretes in the estimation data can significantly interfere with the range bin under test and pose a limitation that merits further investigation. The limitations are not necessarily prohibitive and these screening techniques could find useful application for example in Slow Ground Moving Target Indication modes where the ground environment has moving discretes and large man-made structures present at many ranges.

Future work should include a performance analysis for a variety of radar parameters and for both forward and sidelooking array configurations. A careful analysis of multiple moving discretes in the estimation data and methods to overcome their adverse effects should be considered. Processing loads and delays to the STAP processing need further consideration. Most importantly, these techniques should be implemented on measured data to determine if the feasibility holds.

REFERENCES


Thomas Morton was born in Rumford, Maine. He received a B.A. degree from the Ohio State University, Columbus, OH, in 1985. He received a B.S.E.E. from Wright State University, Dayton, OH in 1989 and an M.S.E.E. from the University of Dayton, Dayton, OH, in 1996. He has been working at Wright-Patterson Air-Force Base since 1989 in various areas of radar acquisition and research and development. He currently works at the Air Force Research Laboratory where his research interests include Space-Time Adaptive Processing.