ENHANCED NDI STRATEGIES FOR RECONFIGURABLE FLIGHT CONTROL

Aaron J. Ostroff and Barton J. Bacon
NASA Langley Research Center, Hampton VA

Abstract

A recently proposed method of on-line control design for aircraft reconfiguration is modified to mitigate the effects of effector rate/position saturation and sensor noise in critical measurements while preserving some, perhaps reduced, level of flying qualities. The on-line control design, based on an incremental version of nonlinear dynamic inversion, does not require a complete aerodynamic model of the aircraft, but does require the local control derivatives along with feedback of accelerations and effector positions. Recovery from a variety of failure (stuck or missing effectors) is possible under the original design as long as the working effectors do not enter saturation for extended periods and critical measurements are relatively noise free—an unlikely situation. Here, an improved control allocator minimizes both effector rate and position, utilizing a multi-pass strategy to restore lost control power due to saturation using the remaining unsaturated controls. Command model flying parameters are adaptively manipulated online to comply with reduced levels of control power further reducing saturation. A classically designed compensator placed around each actuator underpins strategy to reduce jitter due to sensor noise in the control variable responses while preserving decoupling of original control. Improvements due to these modifications are demonstrated on an advanced tailless fighter.

Introduction

A reconfigurable flight control is expected to maintain stability and some acceptable level of handling qualities in the presence of actuator failure, missing surfaces, and wing damage. On-line control design is one of three elements making up an indirect adaptive approach to reconfigurable flight control. The other two elements are failure detection and isolation for sensors and actuators, and parameter identification for updated models of the damaged vehicle. Robustness is tied to the exchange of information between elements and the ability of the on-line control design to tolerate information errors from the other two elements. Robustness is also tied to internal properties of the on-line control design element including managing reduced control power reserves in failure subject to rate/position limits of working effectors and dealing with noise in measurements critical to reconfiguration. This paper deals with these internal problems for on-line control design.

Over the past decade, the literature concerning the on-line design portion of reconfigurable controls has been dominated by two concepts: dynamic inversion [1] and receding horizon optimal control [2]. Both can be readily modified to handle changing dynamics with updates to the control’s required onboard model. Both can also produce desired closed-loop dynamics to satisfy handling qualities. Flight experiments with various versions of these concepts, however, underscored the methods’ sensitivity to onboard models as summarized in [3].

Two approaches have been proposed to desensitize dynamic inversion to onboard model error. One approach uses an on-line neural net [4] to adaptively regulate the error in the plant inversion to yield the desired response of selected control variables. The other less complicated approach considered here is based on an incremental version of nonlinear dynamic inversion (NDI) and does not require a complete aerodynamic model of the aircraft, but does require the local control derivatives along with feedback of accelerations and effector positions [3].

The original approach in [3] provides smooth recovery for aircraft that have suffered either actuator damage, missing effector surfaces, or any combination thereof only if the remaining working effectors do not enter saturation for extended periods of time and critical measurements are relatively noise free—an unlikely situation. To reduce the likelihood of effector rate/position saturation, the control allocator has been reformulated to minimize both effector rate and position. One additional benefit is that actuator windup [5] from an incremental control implementation is suppressed. This new solution is coupled with a multi-pass control allocation strategy that mitigates the effects when one or more effectors enter saturation.

Saturation proved to be a problem for the neural net correction [4] as well and lead to the development of a hedging strategy [6] that moved the command model “backwards” by an estimate of the unachieved desired control variable rate. The desired control variable rate is manipulated here too, but in a different manner by modifying the command model flying quality parameters at the onset of failure.

Sensor noise on certain critical measurements, specifically noise on the measured angular rates used to obtain the required angular accelerations, is also problematic for the incremental NDI approach. A classically designed compensator placed around each actuator enables the bandwidths of the washout filters used in generating the angular accelerations to be cut in half while preserving the decoupling aspects of the original
control. The result is a vast reduction in the jitter due to sensor noise in the control variable responses.

Improvements due to control allocator modifications with command model adjustment are demonstrated on an advanced tailless fighter. The preserved decoupling behavior under the new washout filters is apparent. Due to space limitations, the reduction in jitter is not demonstrated.

Modified Incremental Version of Nonlinear Dynamic Inversion

For this discussion, let \( x \) and \( u \) denote the state and control/effector position vectors of the aircraft whose motion is governed by

\[
\dot{x} = f(x, u) + g(x, u)
\]

(1)

where \( dim(u) = m > 3 \). The control objective here is to obtain \( u \) such that three state-dependent control variables corresponding to the longitudinal, lateral, and directional axes,

\[
y = [y_{lon} \ y_{lat} \ y_{dir}]^T = h(x)
\]

(2)

have some desired behavior under both nominal and failed conditions. The desired behavior of \( y \) to some commanded input \( y_c \) is typically defined by

\[
y = y_{des} \quad \dot{y} = y_{des}' (y, y_c)
\]

(3)

yielding low-order responses that are decoupled along orthogonal axes and compliant with military specifications [7].

Let \( x_o, u_o, \) and \( \dot{x}_o \) denote the state, the control effector position, and the corresponding acceleration vectors from the previous control update. The modified dynamic inversion approach [3] incrementally updates the previous control position to realize some desired control variable rate \( \dot{y} = \dot{y}_{des} \). The control is defined as \( u = u_o + \delta u \) where \( \delta u \) must satisfy

\[
B_o \delta u = y_{des} - h_o \dot{x}_o = \dot{y}_e
\]

(4)

where \( h_o = \partial h(x)/\partial x \big|_{x=x_o} \) and the elements of \( B_o \) are the control derivatives, or

\[
B_o = \frac{\partial}{\partial \delta} (g(x,u)) \big|_{x=x_o,u=u_o}.
\]

(5)

In the past, a minimum norm solution of (4) has been used

\[
\delta u = (h_x B_o)^+ \dot{y}_e
\]

(6a)

\[
(h_x B_o)^+ = W^{-1}(h_x B_o)^T [h_x B_o W^{-1}(h_x B_o)^T]^T_1
\]

(6b)

minimizing \( \delta u^T W \delta u \) with respect to the control correction \( \delta u \). This effectively minimizes control rate but neglecting control position. A control solution that minimizes both rate and position is considered.

Position and Rate Weighting with Nonlinear Function

The following quadratic cost function combines control rate and position

\[
J = \frac{1}{2} (\delta u)^T W_{rr} \delta u + \frac{1}{2} (u_o + \delta u)^T W_{pp}(u_o + \delta u)
\]

(7)

where \( W_{rr} \) and \( W_{pp} \) are rate and position diagonal weighting matrices respectively. To minimize (7) subject to (4), define the Lagrangian equation

\[
L = \frac{1}{2} \delta u^T \dot{W}_{rr} \delta u + \frac{1}{2} (u_o + \delta u)^T W_{pp}(u_o + \delta u) + \lambda^T (B \delta u - \dot{y}_e)
\]

(8)

where \( B = h_x B_o \) and \( \dot{W}_{rr} = W_{rr}/(\Delta T)^2 \). Following the standard procedure for optimization with equality constraints [8] yields

\[
\delta u = W_{rr}^{-1} B^T (B W_{rr}^{-1} B^T)^{-1} \dot{y}_e -
\]

\[
[I_m - W_{rr}^{-1} B^T (B W_{rr}^{-1} B^T)^{-1}] W_{pp}^{-1} W_{pp} u_o
\]

(9)

where \( \overline{W} = W_{rr} + W_{pp} \). Note, the correction due to the previous control position is in the null space of \( \text{col}\{B\} \) modifying \( \delta u \) in such a way to maintain (4) and reduce \( \|u_0 + \delta u\|_W^2 \). Setting \( W_{pp} = 0 \) yields the previous solution (6) minimizing control rate.

In the control's implementation, the effector position \( u \) is the output of a set of actuators, limited in both rate and position, driven by the previously defined control command \( u_{cmd} = u_o + \delta u \). Although (9) minimizes effector rate and position activity, tending to zero counterbalanced effectors (providing equal and opposite moments), it does not guarantee the commanded controls will be within their saturation limits. Violations lead to clipped control commands that do not satisfy the required equality constraint (4), i.e. \( y \neq y_{des} \). The control effectors can be adaptively penalized as they approach their saturation limits. Redefining \( \dot{W}_{rr} \) and \( W_{pp} \) as

\[
\dot{W}_{rr} = \dot{W}_{rr} (1 - f_o(z_i)) \quad i = 1...m
\]

(10)

\[
W_{pp} = W_{pp} (1 + f_o(z_i)) \quad i = 1...m
\]

(11)

then the nonlinear function \( f_o \), chosen here to be quadratic, can be implemented as

\[
f_o(u_o) = (u_{o,i}/u_{o,i,max})^2 \quad 0 \leq f_o \leq 1.
\]

(12)

When the actuator position reaches maximum, the rate weighting goes to zero and the position weighting is twice the value at zero position.

It should be mentioned that (9-12) still do not guarantee effector commands will not violate saturation limits. Moreover, these limits can be violated even when sufficient control power exists in the remaining
unconstrained controls to satisfy (4). A multi-pass strategy added to the allocator is proposed to address this problem.

**Multi-pass Allocator Solution for Constraints**

To satisfy both position and rate constraints of the actuators, the incremental control is constrained prior to actuators as

\[ u_{\text{lim},i} - u_0 \leq \delta u \leq u_{\text{lim},i} - u_0 \]
\[ u_{\text{lim},i\Delta T} - u_0 \leq \delta u \leq u_{\text{lim},i\Delta T} \]

(13)

where \((u_{\text{lim},i}, u_{\text{lim},i\Delta T})\) are respectively the position and rate limits of the actuators applied to the control command \(u_{\text{cmd}} = u_0 + \delta u\). Let \(k = 1\) correspond to the first pass solution given by (9), i.e., \(\delta u (1) = \delta u\), and assume at least one control effecter command \(\delta u_i (1)\) violates a saturation limit. For any \(k \geq 1\), define \(\delta u_{i \text{lim},k} \in R^m\) as follows. If \(\delta u_{i,k} (k)\), \(i = 1, m\), violates or equals one of the constraints in (13), denoted as \(\delta u_{i,k} (k) = \delta u_{i,k}\). Otherwise, set \(\delta u_{i \text{lim},k} = 0\).

The portion of \(\dot{y}_e\) achieved by the constrained effecters is

\[ \dot{y}_e (k) = B \delta u_{i,k} \] (14)

For the \((k + 1)\)-pass solution, solve first for \(\delta u_{o,k+1}\)

\[ B \delta u_{o,k+1} = \dot{y}_e (k) - \dot{y}_e (k) \] (15)

using (9) with \(\dot{y}_e (k)\) replacing \(\dot{y}_e\) and the constrained allocator weighting elements set to zero, i.e.,

\[ W^{-1}(i,i) = 0 \text{ for all } i \text{ such that } \delta u_{i \text{lim},k} \neq 0 \]. From (9), only the unconstrained controls are used in the solution: elements of \(\delta u_{o,k}\) corresponding to constrained effecters are zero. The incremental control solution for the \((k + 1)\)-pass is

\[ \delta u_{i,k+1} = \delta u_{i,k} (k) + \delta u_{i \text{lim},k} \] (16)

If \(\delta u_{i,k+1}\) satisfies (13), set \(\delta u_{i,k+1} = \delta u_{i,k+1}\). Otherwise, set \(k = k + 1\) and repeat. To avoid driving all controls into saturation when excessive control power is required, limit \(k \leq 2\). Note the first pass corresponds to \(\delta u_{i \text{lim},1} = 0\) for all \(i = 1, m\).

As stated, the control command \(u_{\text{cmd}}\) drives the actuators that set the effector positions \(u\). The previous control \(u_0\) is actually the position of the effectors at the last control update. It is not the previous control command.

One problem with constraining the control command prior to the actuator is that dynamics, hinge moments, nonlinear elements, and other non-modeled elements are not included in the controller. Actuator dynamics will reduce the input amplitude, depending upon the frequency content, and may not result in a constrained situation even though the input amplitude may be greater than a saturation limit. To avoid this situation, a check is made on both the previous actuator rate and position and the corresponding actuator command. If the actuator rate is within 95% of the no-load rate limit and the command is greater than the no-load rate limit, then the signal is constrained. Similarly, if the actuator position is within 95% of the saturation limit and the command is greater than that limit, then the signal is also constrained.

**Command Model Flying Qualities Parameter Adjustment**

After failure, sufficient control power may not be available to satisfy (4) and maintain the level 1 flying qualities of (3). To preserve stability in this case, \(\dot{y}_{des}\) of \(\dot{y}_e\) in (4,9) must be relaxed through an adjustment of flying qualities (FQ) parameters. The \(h_{x_{lo}}\) portion of \(\dot{y}_e\) in (9) produces a de-coupled integrator block relating \(\dot{y}_{des}\) to \(\dot{y}_e\). This relationship is exploited in the definition of \(\dot{y}_{des}\) to produce the following de-coupled closed loop responses in the longitudinal, lateral and directional axes.

\[ \dot{y}_{lon}/\dot{y}_{lon,c} = \frac{\omega_{lon}^2}{(s^2 + 2\zeta_{lon}\omega_{lon}s + \omega_{lon}^2)} \] (17)
\[ \dot{y}_{lat}/\dot{y}_{lat,c} = \frac{\omega_{lat}^2}{(s^2 + 2\zeta_{lat}\omega_{lat}s + \omega_{lat}^2)} \] (18)
\[ \dot{y}_{dir}/\dot{y}_{dir,c} = \frac{\omega_{dir}^2}{(s^2 + 2\zeta_{dir}\omega_{dir}s + \omega_{dir}^2)} \] (19)

directly utilizing the flying quality parameters \(\zeta_{lon}, \omega_{lon}\)
\[ \zeta_{lat}, \omega_{lat}, \zeta_{dir}, \omega_{dir} \] in the outer loop of the control. One approach to maintaining stability after failure reconfiguration is to reduce the desired bandwidth of (17-19). This has been implemented by arbitrarily defining some bandwidth reduction values \(\omega_{fac}\) for each of the control effectors from .25 to .5 in the coming example.

When multiple failures occur, the individual reduction values are added to create a total reduction \(\omega_{fac,tot}\) as

\[ \omega_{fac,tot} = 1 + \sum_{f=1}^{n_f} \omega_{fac,j} \] (20)

where \(n_f\) is the number of failures and \(\omega_{fac,j}\)
corresponds to the \(j\)th effector failed. There probably should be some maximum level of reduction, but that has not been implemented. The flying-qualities bandwidth in each axis is divided by the value in (20). Targeting this bandwidth reduction specifically to the closed-loop response associated with the failed effector’s dominant axis is currently being considered.

The complete control is shown in figure 1 with a couple of additions, \(H_c(z)\) and \(H_ip(z)\). Thus far, the discussion corresponds to the case \(H_c(z) = H_ip(z) = I_m\). In
At or exceeds limit  
\( k = 0 \)
\( \delta t_{lim} = 0 \)
First Pass

\[ k = k + 1 \]
All Constraints Satisfied
Yes

\[ \delta t = \delta t^{(k+1)} \]

Multi Pass Allocator

\[ B \]

\[ y_{lim} \]

Desired Dynamics

\[ h(x) \]

FQ parameter Adjustment

Failure Flag

\[ x_{des} \]

\[ \dot{x}_{des} \]

\[ [B_{WW}] \]

Position/Rate Allocator

Failure Flag, Control Derivatives

\[ B_{WW} \]

\[ W_{pp} \]

\[ H_{C}(s) \]

\[ H_{c}(z) \]

\[ X_o \]

\[ a_{i,Sen} \]

\[ x_{Sen} \]

\[ x \]

\[ \dot{x} \]

\[ f(x) + g(x,u) \]

Acceleration Calculation

\[ H_{act} \]

\[ H_{c}(s) = I_m \]

where \( H_{c}(s) = I_m \) . In the absence of position weighting, \( W_{pp} = 0 \), first-order actuator models \( \omega_a/(s + \omega_a) \) become integrators \( \omega_a/s : \) one for each effector. This shows that allocation exclusively based on actuator rate could lead to uncorrected drift problems in the incremental NDI control. The integrator poles disappeared with position weighting, replaced instead by low frequency complex poles exhibiting decreasing damping ratios with increasing actuator bandwidth and position weightings. Various lag-lead and lag compensations were investigated to replace the unity diagonal entries of \( H_{c}(s) \) in attempts to reduce the bandwidth of the washout filter used for angular rate differentiation. A simple lag filter with a bandwidth of 30 rad/sec was shown to closely preserve the nominal flying quality poles while enabling a dramatic reduction in washout filters frequency from 90 rad/sec to 40 rad/sec.

**Example**

A nonlinear simulation containing a highly maneuverable tailless fighter aircraft model with Innovative Control Effectors (ICE)\(^1\) expressed in the ATLAS (Aircraft Trim, Linearization, And Simulation) environment is used to demonstrate the proposed enhancements to the incremental NDI reconfigurable control. The vehicle considered has eleven controls, but four of these controls are unilateral (operate only in one

---

\( ^1 \) Property of the Flight Controls Branch of Lockheed Martin Tactical Aircraft Systems.

---

### Figure 1. Enhanced Incremental NDI Reconfigurable Control

The implementation, \( H_{IP}(z) \) is a low-pass filter block with a bandwidth of 5 rad/sec designed to smooth \( u_o \) prior to assigning allocator inputs and weightings. In the next section, \( H_{C}(z) \) is added to improve the acceleration calculation of \( \dot{x}_o \) subject to sensor noise.

#### Actuator Compensation for Noise Suppression in NDI Feedback

In [9], a second-order washout filter with a natural frequency of 90 rad/sec approximately differentiates the angular rates to yield the angular acceleration components of \( \dot{x}_o \). With this approximate angular acceleration, the sensed angular rates, and the sensed accelerations \( a_{i,Sen} \), a kinematic expression yields the required linear acceleration components \( \dot{x}_o \) at the cg. One potential problem is that this high frequency differentiation allows a wide frequency range for rate gyro noise to influence control system performance. Additional compensation, \( H_{C}(s) \) is classically developed to enable a substantial reduction in the bandwidth of the washout filters.

From figure 1, if the multi-pass strategy is neglected, i.e. \( \delta u = \delta u_0 \), the actuator dynamics are modified by the following positive feedback signal

\[ u_{act,fb}(s) = [H_c(s) - (I_m - P_{WW}B)(W_{pp}H_{IP}(s))]u_o(s) \]  \( (21) \)
angular direction) leaving nine effective controls. These effective controls are: 1) left elevon (LE), 2) right elevon (RE), 3) symmetric pitch flap (SPF), 4) all-moving tip (AMT), 5) spoiler-slot-deflector (SSD), 6) left outboard leading edge flap (LOLEF), 7) right outboard leading edge flap (ROLEF), 8) pitch vectoring (PV), and 9) yaw vectoring (YV). The AMT and the SSD controls are unilateral with control effectors on both the left side (LAMT and LSSD) and right side (RAMT and RSSD) of the airplane. A second-order, overdamped actuator drives each effector. The control variables selected include pitch rate \( q_b \), stability axis roll rate \( p_s \), and a directional mix \( \beta - 2\psi \) consisting of side-slip and stability axis yaw rate.

For this example, the vehicle is initially trimmed straight and level at 25000 feet and Mach .7. A directional channel doublet of amplitude 10 degrees is commanded over the first five seconds. In the other channels, \( q_b \) and \( p_s \) should both remain zero. A second outer loop, not shown in figure 1, commands \( p_s \) to control wind-axis bank angle \( \mu \), also zero. Another outer loop, also not shown, commands \( q_b \) to control \( n_z \): unity in this example. To introduce failure, at .25 seconds the yaw nozzle is stuck at 5° left, and at one second the left AMT goes to a stuck maximum position of 60°. For this example, sensor noise has been omitted.

Results are illustrated in figure 2. The solid lines correspond to the one pass-allocator with no adjustment of FQ parameters. The dashed lines correspond to the one pass–allocator with adjustment of FQ parameters. The dotted lines correspond to a two-pass allocator with adjustment of FQ parameters. The FQ adjustment parameter \( \omega_{\text{fac}} \) is .5 for each of the failed effectors so the FQ natural frequencies are reduced by a third after .25 seconds and a half after one second. It is clear that each added feature improves performance. The critical point in the simulation occurs after 2.5 seconds in response to –20 degree change in commanded \( \beta - 2\psi \). The one pass allocator is unacceptable with large excursions from the desired values for each control variable. Adding the FQ parameter adjustment improves the control variable performance, but the 30° excursion in wind-axis bank angle is not desirable. It is however better than the divergent response using the one-pass allocator solution alone. Note that amount of saturated control activity in AMTR and TAILR is greatly reduced with the addition of the FQ parameter adjustment. More improvement is obtained with a two-pass allocator/FQ parameter adjustment combination. Control variable tracking through failure is improved with a good \( \mu \) response. The TAILR spends less time in position saturation after the critical point at 2.5 seconds. The effector rate responses AMTRR and DPNOZR show unconstrained controls being utilized, to their respective rate limits, in the second pass to better satisfy (4).

Conclusions

Modifications to a reconfigurable flight control based on an incremental NDI controller implementation are outlined to mitigate the deleterious effects of effector rate/position saturation and sensor noise during reconfiguration. Proposed modification to the control allocator includes a new pseudo-inverse solution minimizing both effector rate and position. The solution corrects drift problems in the NDI incremental control, suppressing actuator windup that leaves counterbalanced effectors extended increasing drag. Admittedly, the solution does not guarantee that actuator saturation limits will not be violated. A multi-pass strategy is added to restore lost control power due to saturation with remaining unsaturated effectors. Reduction in demanded control variable rates using a judicious adaptation of command model flying parameters lead to less actuator saturation in failure. Other modifications include compensation added about the actuators to enable bandwidth reduction in washout filters used for approximate differentiation.

The combined effect of the control allocator modifications with FQ parameter adaptation to two failed actuators is shown in simulation. The improvement is significant. The added compensation to reduce jitter due to noise does not change the desired decoupled nature of the control variables.

Acknowledgement

The nonlinear simulation ATLAS used in this research is the property of the Flight Controls Branch of Lockheed Martin Aeronautics Company in Forth Worth, Texas.

References


Figure 2. Directional Doublet Example (Control Failures at 0.25 and 1.0 sec.)