Comparison of Dynamic Inversion and LPV Tailless Flight Control Law Designs

James M. Buffington and Andrew G. Sparks
Air Force Research Laboratory (AFRL/VAC)
Wright Patterson AFB, OH 45433-7521

Abstract

This paper compares two inner loop flight control law designs for a tailless fighter aircraft model developed from aerodynamic wind-tunnel test data. Dynamic inversion and linear parameter varying control using parameter dependent Lyapunov functions are each used to design control laws for the nonlinear model. The aircraft design model and design requirements are the same for each method.

1 Introduction

Dynamic inversion has evolved into an important tool for inner loop flight control law design [1, 2]. Using dynamic inversion, the natural dynamics of the aircraft, which can be stored using on-board models, are subtracted off and augmented with desired dynamics to meet handling qualities requirements. The technique is very appropriate for designing full envelope control laws since typically very accurate models of aerodynamic data exist for implementation.

More recently, linear parameter varying (LPV) control using parameter dependent Lyapunov functions has become a topic of interest for designing gain-scheduled control laws [3, 4]. Using LPV control, the nonlinear dynamics across the flight envelope are approximated by a linear model whose coefficients are functions of measurable parameters such as Mach number and altitude. A control law meeting desired handling qualities can then be found by using implicit model following with a synthesis model based on the LPV plant model. A norm-based optimization procedure is used to find the control law, which is a function of the same parameters as the LPV plant model and hence is automatically gain-scheduled.

This paper compares inner loop flight control law designs for a tailless fighter aircraft model using dynamic inversion [5] and LPV control [6, 7]. The aircraft design model and design requirements for both designs are the same for consistency. The paper is organized as follows. The next section describes the tailless fighter aircraft model. The nonlinear simulation model contains nonlinear aerodynamic wind-tunnel force and moment data. The nonlinear aerodynamic data is used to develop an LPV aircraft model which is used in the dynamic inversion controller as the on-board model of the system dynamics and in the LPV controller as the basis for the synthesis model. The following section gives some background on the dynamic inversion and LPV control techniques and presents the individual control designs. A simple linear control allocator that provides redundant actuator commands for both controllers is also described. The next section contains the analysis results for both of the designs, including flying qualities analysis, time histories, and robustness analysis. A summary and conclusions are contained in the final section.

2 Tailless Aircraft Model

This section describes a tailless fighter aircraft model developed under the Innovative Control Effectors (ICE) program [8, 9]. The aircraft is a 65 degree sweep delta wing, single engine, multi-role supersonic fighter with internal weapons carriage. To reduce radar cross section and structural weight, the aircraft has no vertical tail. It has relaxed longitudinal stability for reduced supersonic trim drag and enhanced maneuverability and relaxed directional stability due to the absence of a vertical tail. The ICE aircraft model includes a large suite of conventional and innovative control effectors that provide forces and moments in multiple axes. The conventional effectors include elevons, pitch flaps, thrust vectoring, and outboard leading edge flaps. The innovative or unconventional control effectors include spoiler-slot deflectors and all-moving tips. The unconventional control effectors provide additional yaw control power to overcome deficiencies resulting from the absence of rudders.

2.1 Nonlinear Simulation Model

A high-fidelity six degrees-of-freedom nonlinear simulation was developed in a generic simulation environment. The simulation includes high fidelity atmosphere, sensor, actuator, and gust models. Mass data and scale model wind-tunnel test data from the ICE program were used to develop weight and aerodynamic models. The propulsion model is based upon an engine from an F-16 class fighter aircraft.
2.2 Linear Parameter Varying Model

Nonlinear aerodynamic wind-tunnel data from the simulation model is used to develop linear analysis and design models that are parameter dependent. The models take the form

\[
\dot{x} = A(\theta)x + B(\theta)u, \quad (1)
\]

\[
y = C(\theta)x,
\]

where \( \theta \) is a vector of parameters.

Ten state linear rigid body models are obtained by linearizing the equations of motion at 89 flight conditions across the flight envelope between sea level and 40,000 ft and Mach 0.3 and 0.9. The models are trimmed using symmetric elevons and pitch flaps. The ten state linear models are truncated to five state design models corresponding to the short period, roll and Dutch roll modes and the linear parameter varying model is constructed by combining these linear models. The dimensional stability and control derivatives are approximated using a polynomial least squares fit in Mach number and altitude and the approximate derivatives are used for control law design. In addition, the control surface deflections are replaced by the generalized moment commands \( d_q, d_p, \) and \( d_r. \) The control synthesis will be done using these inputs, and then the resulting control laws will be combined with a simple control allocation scheme to translate the generalized control commands to the individual surface commands. The longitudinal dynamics are approximated by the following LPV system

\[
\begin{bmatrix}
\dot{\alpha} \\
q
\end{bmatrix} = \begin{bmatrix}
Z_\alpha(\theta) & 1 \\
M_\alpha(\theta) & M_q(\theta)
\end{bmatrix} \begin{bmatrix}
\alpha \\
q
\end{bmatrix} + \begin{bmatrix}
0 \\
d_q
\end{bmatrix}, \quad (2)
\]

while the lateral-directional dynamics are approximated by the following LPV system

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
Y_\beta(\theta) & Y_r(\theta) & Y_p(\theta) \\
L_\beta(\theta) & L_r(\theta) & L_p(\theta)
\end{bmatrix} \begin{bmatrix}
\beta \\
\dot{\alpha} \\
\dot{q}
\end{bmatrix} + \begin{bmatrix}
0 \\
d_p \\
d_r
\end{bmatrix}. \quad (3)
\]

3 Control Design

This section describes the flight control designs for the ICE tailless fighter aircraft. The design requirements are posed as loop transfer response constraints for stability augmentation and flying qualities low order equivalent systems (LOES) parameter specifications for command augmentation. For simplicity, the roll, pitch, and yaw channels all have the same loop requirements except where noted. The controller structure is indicated in Figure 1. The signals available for feedback are angle-of-attack, pitch rate, sideslip angle, roll rate, and yaw rate. The controller is a function of Mach number and altitude to account for flight condition variations.

The same commanded variables are used for both design methods to be consistent. The commanded variables are chosen to reflect desirable flying qualities. It has been shown [1] that the following commanded variable responses are desired by pilots

\[
y = \frac{1}{\tau_c s + 1} y^*, \quad (4)
\]

\[
y = \begin{bmatrix}
p \cos \alpha + r \sin \alpha \\
q + K_d(\theta) \alpha \\
-p \sin \alpha + r \cos \alpha - \beta
\end{bmatrix},
\]

where \( \tau_c \) is the desired command bandwidth.

3.1 Design Requirements

In this section design requirements are stated in terms of loop gain and LOES parameter bounds.

3.1.1 Command Response

Command response requirements are in terms of LOES specifications. Level I flying qualities are required; specifically, the LOES parameters must lie in the following ranges

\[
0.28 \leq CAP \leq 3.6, \quad T_s \leq 0.5 \text{sec},
\]

\[
\omega_p > 1.0 \text{rad/sec}, \quad \zeta_d \geq 0.4,
\]

\[
0.35 \leq \zeta_p \leq 1.3, \quad \zeta_d \omega_d \geq 0.4 \text{rad/sec}.
\]

Finally, all channels are required to have equivalent time delays less than 0.10 seconds.

3.1.2 Stability and Robustness

The controller must provide internal stability. It is required to have an open loop bandwidth less than 10 rad/sec derived from F-16 actuator capabilities. The following high frequency loop transfer constraint is imposed to insure attenuation of unmodelled dynamics

\[
|L(s)| < \frac{s + 10000}{1000(s + 0.01)}, \quad \forall \omega \geq 10 \text{rad/sec}. \quad (5)
\]

Parametric uncertainty is approximated by a low frequency unstructured model. A low frequency loop transfer constraint is imposed to insure robustness to parametric uncertainty. The following unstructured model
approximates typical aerodynamic and propulsion parametric uncertainty for F-16 type fighter aircraft

\[ |L(s)| > \frac{0.001(s + 1000) .6(s + 4) 1}{s + 0.001 s + 0.6} s, \forall \omega \leq 1 \text{ rad/sec}. \quad (6) \]

3.2 Control Allocation

The ICE aircraft has redundant multi-axis control effectors to overcome yaw control power deficiency. A controller consisting of a separate control allocator provides optimal effector management without degrading feedback loop properties [5]. A pseudo-inverse of the control effectiveness is used for simplicity since the focus of this effort is not control allocation

\[ u^c = B_y^T(\theta) (B_y(\theta)B_y^T(\theta))^{-1} d^*_y, \quad (7) \]

where \( d^*_y \) is the generalized control vector

\[ d^*_y = \begin{bmatrix} d_y^1 \\ d_y^2 \\ d_y^3 \end{bmatrix}. \quad (8) \]

The stability and command augmentation control law is designed in terms of the generalized control vector and mapped to actuator commands by the control allocation function in (7).

3.3 Dynamic Inversion Control

Dynamic inversion control deaugments the commanded variable dynamics by subtracting the natural dynamics and augments the commanded variable dynamics by adding desired dynamics [1]. The desired dynamics have been chosen to have a proportional plus integral structure. The dynamic inversion control law for the LPV design plant in (2) is given in terms of the generalized control vector by

\[ \dot{d}_y^* = -C(\theta)A(\theta)x + \omega_c(x_1 - y) + \omega_c f_z y^*, \quad (9) \]

\[ \dot{x}_1 = -\omega_c f_i(y - y^*), \]

where \( d_y^* \) is the input to the control allocation and \( \omega_c = 5 \text{ rad/sec}, f_i = 0.25, f_c = 0.5. \)

3.4 LPV Control

An LPV synthesis model is developed from the LPV plant model, an ideal model containing the desired dynamics from handling qualities specifications, and the proportional plus integral structure of the dynamic inversion controller. The error between the closed loop aircraft and the ideal model is minimized to achieve implicit model following. Unstructured uncertainty at the plant input is included for robustness to high frequency modelling error and control inputs are weighted to penalize control surface deflections. Details are given in [7]. The LPV control law is given in the following form

\[ \dot{x}_c = A_c(\theta)x_c + B_c(\theta) \begin{bmatrix} \omega_c(x_1 - y) + \omega_c f_z y^* \\ x \end{bmatrix}, \]

\[ \dot{x}_i = -\omega_c f_i(y - y^*), \]

\[ d^*_y = C_c(\theta)x_c, \]

where the control law matrices \( A_c(\theta), B_c(\theta), \) and \( C_c(\theta) \) are functions of the LPV synthesis model and solutions to a set of linear matrix inequalities [3, 4].

3.5 Prefilter

To meet handling qualities requirements, a prefilter is added to the command path of each axis. Identical prefilters were used for each of the designs.

4 Analysis of Controllers

This section provides stability and performance analysis of both control designs. Due to space limitations, analysis results are only given for the flight condition at Mach 0.35 and 15000 feet altitude. Note that the model used for analysis was not the approximate LPV design model but rather the actual linearized data at the chosen point. In addition, high fidelity actuator models are used.

4.1 Flying Qualities Analysis

The flying qualities LOES parameters at Mach 0.35 and 15000 feet altitude are shown in Tables 1 and 2. Comparison with the requirements in Section 3.1.1 demonstrates that all parameters lie within Level 1 regions except for the LPV equivalent time delays. This is due to the large dynamic order of the LPV control laws; the longitudinal control law has seven states while the lateral directional has fifteen.

<table>
<thead>
<tr>
<th>Method</th>
<th>( \omega_{\text{LP}} ) (rad/sec)</th>
<th>( \zeta_{\text{LP}} )</th>
<th>( \tau_{\text{LP}} )</th>
<th>CAP</th>
<th>( \tau_{\text{LP}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DI</td>
<td>3.14</td>
<td>0.77</td>
<td>1.82</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>LPV</td>
<td>4.16</td>
<td>0.78</td>
<td>2.30</td>
<td>0.155</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>( \omega_{\text{LP}} ) (rad/sec)</th>
<th>( \zeta_{\text{LP}} )</th>
<th>( \tau_{\text{LP}} )</th>
<th>( \tau_{\text{LP}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DI</td>
<td>2.48</td>
<td>0.78</td>
<td>0.088</td>
<td>0.386</td>
</tr>
<tr>
<td>LPV</td>
<td>2.46</td>
<td>0.78</td>
<td>0.158</td>
<td>0.655</td>
</tr>
</tbody>
</table>

4.2 Simulation Analysis

The controllers are implemented into the nonlinear aircraft simulation model. Simple lateral and longitudinal
stick commands drive the following coupled roll and pitch maneuver. The aircraft is initialized at Mach 0.35 and 15,000 feet altitude, and a 5 deg/sec pitch step command lasting four seconds is given at 1 second followed by a 20 deg/sec roll doublet command lasting four seconds given at 2 sec. The roll command variable responses for the both controllers are shown in Figure 2. It is seen that the LPV controller response is slightly more sluggish in the roll channel than the dynamic inversion controller. The pitch command variables for both controllers are shown in Figure 3 and the sideslip angle is shown in Figure 4. It is seen that the LPV controller responses exhibits slightly better sideslip response. The RMS response of the control vector provides a measure of control activity. The RMS control responses for both controllers are shown in Figure 5. It is seen that the control activities are similar.

4.3 Robustness Analysis

The dynamic inversion and LPV control loop shapes are plotted in Figure 6 with the desired open loop transfer function constraints (5) and (6). Note that while the high frequency bounds are satisfied for both designs, the low frequency bounds are violated for both designs. It is seen that the two loop shapes are similar for the dynamic inversion and the LPV designs and that the LPV design has a higher crossover frequency.

Next, the structured singular values for the longitudinal and lateral-directional control laws are computed for each design. Performance is measured as a weighted error between the closed loop response and an ideal model and unstructured uncertainty at the input is included. In addition, based on the curve fitting errors in the dimensional derivatives, the parameters $Z_{a}, M_{a}, L_{\beta}$, and $N_{\beta}$ are allowed to vary by 0.1, 1.0, 0.5, and 0.2, respectively, in either direction. The structured singular value plots are shown in Figure 7. The peak values for the longi-
Tudinal control laws are 1.1651 and 3.5605 for the LPV and dynamic inversion controllers, while for the lateral directional control laws the peak values are 1.3356 and 1.7944. This suggests that the LPV control law is more robust to model uncertainty.

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**References**


