meteorological conditions would be more uniform. The similarity in the curve shape indicates that the fluctuations in the 10 min averages are related to atmospheric effects and are not due to random instrument fluctuations.

As before, the standard deviations in refractivity obtained from the meteorological instruments are about a factor of four larger than those derived from the receiver fluctuations measured. This is attributed to the path-averaging experienced by the propagating microwave signals.

V. RESULTS AND DISCUSSION OF COMPARISONS OF PREDICTED AND MEASURED SIGNAL STRENGTH FLUCTUATIONS

Meteorological measurements at eight heights plus the surface made at one location near the RF signal propagation path provided the information required to calculate the refractive index profile and its time variations. The 10 min averaged data sampled on the hour provided the refractivity profile needed by the RF signal prediction model. Calculations were made to estimate the expected signal intensity changes from hour-to-hour at the ten receivers. The predicted changes were compared with the hour-to-hour changes obtained from the averaged 10 min data at each receiver.

There was also significant interest in evaluating the ground plane effects on the signal distribution at the receivers. The long term averaged receiver pattern did not correspond to the theoretical model using a ground reflection coefficient of $-1$. The peak in the measured signal distribution occurred at about 0.8 of the predicted height. The lowered peak could be the result of variations in the grazing angle over the Fresnel zone and the associated variations in reflection phase. The absence of sharp nulls could result from additional multipath signals and the fact that short term refractive changes would tend to cause rapid and extreme changes at the expected null points. The time averaging process used would result in null filling.

To establish a reference profile for the prediction model, the reflection coefficient was varied with grazing angle to obtain the best fit to the measured profile “unperturbed” by atmospheric inhomogeneities. Variations about this reference profile were then used for comparison of the predicted and measured effects of atmospheric inhomogeneities. This is shown in Fig. 6 for eight different levels. The shape of the curves compare best during the evening hours when there is usually less mixing. In addition, the predicted changes are generally a factor of two or more larger than the measured changes. This latter effect can be attributed to the fact that the model predictions are based on meteorological measurements made at one point, but the propagating signal experiences the average meteorological conditions existing over the propagation path.

It should be noted that the measured intensity changes at levels 2 and 10 are almost identical in shape and that the measured intensity changes at levels 2 and 6 are mirror images. This indicates that the dominant effect of the refractive change is to shift and stretch or compress the theoretical intensity distribution in the vertical, without distorting its basic shape.

REFERENCES


Electromagnetic Fields Near Rail Guns

WILLIAM G. SOPER

Abstract—The field in the neighborhood of an electromagnetic rail gun is computed for the case of constant current. Expressions are derived for electric field strength and magnetic flux density as functions of gun parameters. These formulas can be used to compute the electromagnetic pulse produced by a rail gun of given characteristics.

INTRODUCTION

The muzzle-velocity limitation imposed by gas dynamics on conventional powder-driven guns has during the past 60 years produced periodic interest in methods for the electromagnetic (EM) launch of projectiles [1]. Several techniques for EM launch have been studied in the past, but recent successes with a rail-type launcher [2] have spurred interest in EM guns in general and especially in rail guns. The Westinghouse EMACK launcher has accelerated a 317-gm armature to 4.2 km/s in a distance of 5 m.

The power and current characteristics of rail guns raise questions concerning the EM environment produced. In the EMACK device, for example, two rails of 5 m length carry a peak current of 2.1 MA, while a peak power of over 10 GW is delivered to the rails. The objective of this note is to estimate analytically the EM field about a rail gun of this size.

ANALYSIS

A rail gun consists of two parallel conducting rails that carry a sliding armature. Direct current is passed through the rail-armature circuit, producing a Lorentz force that accelerates the armature. As shown in the following figures, the gun is modeled here as a threessided circuit that grows in length with time. Constant current is assumed; this condition, in addition to simplifying the analysis, is a design goal of practical electric guns.

The field vectors about the gun will be derived from the retarded vector potential $\tilde{A}$ defined as

$$\tilde{A}(x, y, z, t) = \frac{\mu_0}{4\pi} \int_{\text{volume}} \frac{\tilde{J}(x', y', z', t')}{r} \, d \text{volume}$$

where $r$ is the distance from a volume element at point $(x', y', z')$ to the point $(x, y, z)$, and $\tilde{J}$ is the vector current density at $(x', y', z')$. In the integration, $\tilde{J}$ is to be evaluated at time $t' = t - r/c$, where $c$ denotes wave velocity. Thus the magnitude of current used to form $\tilde{A}$ is that which produces the signal received at time $t$, not the current that exists at time $t$. This retardation effect will, however, be neglected in the following analysis because 1) each rail element, once energized, carries a current that is constant in time, and 2) for armature velocities of practical significance, the array of armature elements whose signals are received at time $t$ is negligibly different from the array which constitutes the armature at time $t$.

There is no accumulation of charge in the rail gun circuit. Thus the electrostatic potential is zero, and the electric and magnetic field

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The author is with the Naval Surface Weapons Center, Dahlgren, VA 22448.

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vectors \( \vec{E} \) and \( \vec{H} \) are given in terms of \( \vec{A} \) by

\[
\vec{E} = -\frac{\partial}{\partial t} \vec{A}
\]

(1)

\[
\vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{A}
\]

(2)

A coordinate system is assigned to the rail gun as shown in Fig. 1. From the definition of \( \vec{A} \) it is clear that the component \( A_x \) is zero everywhere, the component \( A_y \) is produced only by the armature current, and \( A_z \) only by the rail current. A general derivation of the field surrounding the gun will not be given here. Instead, \( \vec{E} \) and \( \vec{H} \) will be computed in two planes, the plane of symmetry and the plane of the rails.

The field in the plane of symmetry will be derived with the symbols defined in Fig. 1. The current in the armature provides \( A_y \) as follows:

\[
A_y = \frac{\mu i}{4\pi} \left. \frac{d\tau}{\tau} \right|_{\tau=0} - \frac{a+\sqrt{\gamma^2+z^2}}{r_0}
\]

(3)

where \( K = i/4\pi \). Contributions to \( A_y \) from the rails cancel here, giving \( A_y = 0 \). The single component of \( \vec{E} \) is found from (1) and (3) to be

\[
E_x = -\frac{2\mu Ka}{br_0^2} \sin \theta
\]

(4)

where

\[
S = dL/dt \quad \text{and} \quad b = (1+a^2/r_0^2)^{1/2}.
\]

The required derivatives of \( A_x \) for calculating \( \vec{H} \) are found directly from (3):

\[
\frac{\delta A_y}{\delta y} = -\frac{2\mu Ka}{br_0^2} \cos \theta
\]

\[
\frac{\delta A_z}{\delta z} = -\frac{2\mu Ka}{br_0^2} \sin \theta
\]

The term \( \delta A_x/\delta z \) is zero inasmuch as \( A_x \) is zero throughout the plane of symmetry. Computation of \( \delta A_x/\delta x \) requires consideration of a point \((\delta x, y, z)\) near the plane of symmetry; the value of \( A_x \) is

\[
A_x = \frac{\mu i}{4\pi} \int_{\tau=0}^{\tau=\infty} i \, d\tau \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

(5)

where

\[
R_1 = \sqrt{(\gamma+\epsilon)^2+(a-\delta x)^2+z^2}
\]

\[
R_2 = \sqrt{(\gamma+\epsilon)^2+(a+\delta x)^2+z^2}
\]

It can be shown that as \( \delta x \to 0 \),

\[
\frac{1}{R_1} - \frac{1}{R_2} = \frac{2a}{R^3} \delta x
\]

where

\[
R = \sqrt{(\gamma+\epsilon)^2+a^2+z^2}.
\]

The field in the plane of the rails will be derived with the additional symbols defined in Fig. 2. The contribution from the armature is

\[
A_y = \mu K \left[ \int_{\gamma-\epsilon}^{\gamma+\epsilon} \frac{d\eta}{\sqrt{x_1^2+(\gamma+\epsilon)^2+\eta^2}} - \int_{\gamma-\epsilon}^{\gamma+\epsilon} \frac{d\eta}{\sqrt{x_1^2+(\gamma+\epsilon)^2+\eta^2}} \right]
\]

or

\[
A_y = \mu K \ln \left( \frac{x_1+y_1}{x_1+y_1} \right)
\]

(6)

The contribution from the rails is

\[
A_y = \mu K \left[ \int_{\gamma-\epsilon}^{\gamma+\epsilon} \frac{d\eta}{\sqrt{x_2^2+(\gamma+\epsilon)^2+\eta^2}} - \int_{\gamma-\epsilon}^{\gamma+\epsilon} \frac{d\eta}{\sqrt{x_2^2+(\gamma+\epsilon)^2+\eta^2}} \right]
\]

or

\[
A_y = \mu K \ln \left( \frac{x+y+\epsilon+y_1}{x+y+\epsilon+y_1} \right)
\]

(7)
The electric field vector has two components given by (7), (8), and (9) as

\[ E_x = \frac{\mu KS}{r} \left( \frac{x_1 - x_2}{r_1 - r_2} \right) \]

and

\[ E_y = -\frac{\mu KS}{r} \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \].

Symmetry requires that the derivatives $\frac{\partial A_x}{\partial z}$ and $\frac{\partial A_y}{\partial z}$ be zero in the plane of the rails. Thus, the magnetic vector has only a $z$-component given by

\[ H_z = \frac{1}{\mu} \frac{\partial A_x}{\partial x} - \frac{1}{\mu} \frac{\partial A_x}{\partial y} \]

where, from (7) and (8),

\[ \frac{\partial A_x}{\partial y} = \frac{\mu K}{\gamma} \left( \frac{x_1 - x_2}{r_1 - r_2} \right) \]

and

\[ \frac{\partial A_x}{\partial x} = \mu K \left[ \frac{1}{\gamma (x_1 r_2 - x_2 r_1)} + y \left( \frac{1}{x_1 r_2} - \frac{1}{x_2 r_1} \right) \right] \].

**Far-Field Approximation**

The preceding expressions for $\vec{E}$ and $\vec{H}$ are exact within the conditions stated at the outset. Simpler forms are obtained by restricting all $r$'s to be much greater than $a$. For the plane of symmetry, this approximation leads to $b = 1$ in (4), (5), and to the following modification of (6):

\[ H_z = \frac{2Ka}{r_0^2} \cos \theta + \frac{2Ka}{a^2 + z^2} (\cos \alpha - \cos \theta). \]

For the plane of the rails, the following are obtained from (9) and (10), where $r$ denotes $\sqrt{r_1 r_2}$:

\[ E_x = -\left( \frac{\mu KS}{r^2} \right) \sin \phi \]

\[ E_y = \left( \frac{\mu KS}{r^2} \right) \cos \phi. \]

Equations (12) and (13) simplify to

\[ \frac{\partial A_x}{\partial y} = -\frac{\mu Ka}{r^2} \sin \phi \]

and

\[ \frac{\partial A_y}{\partial x} = -\frac{\mu Ka}{x_1 x_2} \left[ (1 + \cos^2 \phi) \sin \phi - (1 + \cos^2 \beta) \sin \beta \right]. \]

**Discussion**

From (4) it is seen that, in general, $E_x$ in the plane of symmetry exhibits a "figure of eight" polar diagram, with maximum intensity directly ahead of and behind the armature and zero intensity directly above it. In the plane of the rails, (14) and (15) show that the $E$-vector in the far field has a constant magnitude around any circle centered on the armature, and a direction tangential to that circle. These features of $E$ are depicted in Fig. 3.

From (5) and (6), it is seen that the contribution of the armature to $\vec{H}$ in the plane of symmetry is distributed in a manner similar to the far field $\vec{E}$ in the plane of the rails. The contribution from the rails tends, for $y = L/2$ and $L \to \infty$ to $\alpha / \pi z^2$, the "transmission line" field. The distribution of $H_z$ in the plane of the rails, as given by (11), (16), and (17), does not have a simple form. There are singularities in $\frac{\partial A_y}{\partial x}$ at $x = \pm a$ for $\phi$ negative and $\beta$ positive, i.e., at points on the rails. However, when both $\phi$ and $\beta$ have the same sign and $x \to$...
A Procedure for Calculating the Atmospheric Mutual Coherence Function Via the Statistical Fourier-Optical Method

JEFFREY J. SITTERLE, MEMBER, IEEE, PAUL C. CLASPY, SENIOR MEMBER, AND FRANCIS L. MERAT, MEMBER, IEEE

Abstract—An algorithm for computing the atmospheric mutual coherence function from flux measurements taken at the focal plane of a reflector antenna is presented. The procedure consists of first computing the inverse Abel transform of the flux, taking the Fourier transform of the result, and then dividing by the aperture pupil function. It is shown that when flux measurements contain additive noise, the Abel inversion is an ill-posed problem. Therefore, calculation of the inverse Abel transform is accomplished via a Kalman filtering algorithm. Results of the mutual coherence function estimator are given for simulated flux measurements.

I. INTRODUCTION

The mutual coherence function (MCF) of a propagating electromagnetic wave is a measure of the long-term spatial coherence of the complex fields in a plane transverse to the direction of propagation. It is an extremely useful quantity that arises in propagation studies [1] (e.g., it provides the degree of coherence of the field at two points, the angular distribution of the field, and the field's mean energy) and it determines the signal-to-noise ratio (SNR) in an optical heterodyne receiver [2], [3].

The experimental determination of the MCF can be accomplished through two methods; the statistical Fourier-optical method (SFOM) [4] and the long baseline interferometric method (LBIM) [5]. The LBIM has been used extensively at microwave and millimeter wave frequencies and consists of a multiple-phased receiving array. By employing a variable phase shifter in each antenna output and mixing various combinations of antenna outputs together, measurements of both the amplitude and phase fluctuation, along with the amplitude correlation of the wave front, are made. However, because the antennas are at fixed positions during the measurements, the MCF can only be determined for those spatial separations.

The SFOM, first introduced by Land [4] for visible wavelengths, is the two-dimensional continuous analogue of the LBIM with each point of a large spherical reflector acting as a separate point antenna. Using the basic principles of optics for reflector antennas, we know that the instantaneous intensity distribution appearing at the focal plane of the reflector is equal to the magnitude-squared spatial Fourier transform of the electromagnetic field at the antenna's aperture plane. Therefore, the position and shape of the focal plane distribution is directly related to the angle-of-arrival and intensity fluctuations, and the MCF is computed directly from the temporal average of these fluctuations [6]. Here, the MCF is given over a continuous interval which is limited by the aperture size of the reflector.

In this communication we present a method for computing the MCF at optical and millimeter wavelengths from a receiver based on the SFOM. The receiver consists of a parabolic reflector (or lens) with a spatial sampling device located at the focal plane. The spatial sampling device scans the focal plane diffraction pattern and allows measurement of the spatial and temporal fluctuations of the pattern.

II. THE STATISTICAL FOURIER-OPTICAL METHOD

For the case of a parabolic reflector antenna (or lens) of radius R and focal length F, the average intensity \( I(q) \) at a point \( q \) in the focal plane is given by

\[
\langle I(q) \rangle = \frac{k}{2\pi F} \int_{-\infty}^{\infty} \Gamma(r) M_{\text{L}}(\rho) \exp \left[ -i \frac{k}{F} \cdot (q - \rho) \right] d\rho
\]

where the MCF, \( \Gamma(r_1, r_2) \), is taken to be homogeneous (i.e., the MCF depends only on the difference coordinate \( \rho = r_1 - r_2 \)), \( r_1 \) and \( r_2 \) are coordinates in the aperture plane, \( E(r) \) is the stochastic aperture plane field, \( \langle \cdot \rangle \) denotes an ensemble average, \( k = 2\pi/\lambda \) where \( \lambda \) is the wavelength, and \( M_{\text{L}}(\rho) \) defined as the convolution of the receiver aperture function, \( W(r) \), with itself, is given by

\[
M_{\text{L}}(\rho) = \int_{-\infty}^{\infty} W(r) W(r - \rho) d^2r.
\]