Large-Aperture Sparse Array Antenna Systems of Moderate Bandwidth for Multiple Emitter Location

WILLIAM F. GABRIEL, FELLOW, IEEE

Abstract—Spatial linear prediction filter antenna systems of large aperture width (in wavelengths) are described for the superresolution estimation of multiple emitter locations over moderate bandwidths. To accomplish this difficult task with reasonably few degrees of freedom (DOF) from the large aperture, the system employs interferometer/sparse array techniques in conjunction with a shaped reference beam which may be steered to the spatial sector of interest. Computer simulations on several multiple source examples have demonstrated that the concept is feasible, provided that true time-delay steering is employed on the shaped reference beam. The optimal estimation system configuration results when time-delay steering is also applied to the interferometer elements because this "focuses" the sample covariance matrix and minimizes the DOF required. Even transversal filters may be dispensed with under "focused" conditions if the bandwidth is moderate. System performance goals included high resolution with few DOF, fast response/processing time, moderate cost, and ease of frequent calibration.

I. INTRODUCTION

The application of spectral estimation techniques to resolving multiple closely spaced sources of significant bandwidth is a challenging problem area which has been addressed by a number of investigators [1]–[6]. For example, Buckley [1] utilizes linearly constrained minimum variance beamformers in which the constraints are derived from source representation spaces and are termed eigenvector constraints. His paper contains a good description of the general broad-band source problem plus analyses of several approaches. Wang and Kaveh [3] proposed a coherent subspace approach to eigenvector-based broad-band spatial spectral estimation in which a "focused" matrix technique is employed, based upon approximate a priori source location information. The "focusing" corresponds to time-delay steering in broad-band beamforming. Wax et al. [4] utilize decomposition of the snapshot vectors into a discrete set of narrow-band frequency components which are then processed independently, their null spectra superimposed, and finally, inversion to form the broad-band spatial spectrum.

This paper is concerned with applying a few of these referenced broad-band concepts to a spatial linear prediction filter (SLPF) source-estimator antenna system which features a large aperture width (in wavelengths) but employs relatively few degrees of freedom (DOF) for accomplishing the estimation. Such a system would be a natural companion to the partially adaptive low-sidelobe phased-array by Gabriel [7], deriving considerable benefit from shared hardware and data processing equipment. The work reported by Mayhan [8] is also pertinent. It is helpful to review the primary objectives that led to the investigation of this particular type of SLPF source estimator antenna system.

1) Utilize the largest antenna aperture width available to maximize resolution capability [9].

2) Employ interferometer/sparse array techniques to minimize the antenna hardware DOF needed, consistent with the postulated source distribution [10], [11]. This objective is driven by speed of response and cost considerations.

3) Accommodate the moderate bandwidths associated with phased-array radar systems [12]. This implies processing received signal samples which are correlated in both the spatial domain and the time domain. Adaptive transversal filters are often employed in such processing [1], [2], [13], [14], but they have the inherent disadvantage of greatly increasing the number of adaptive DOF.

4) Restrict the source estimation to a narrow spatial sector that can be steered, with the sector position determined either from approximate a priori information or a deterministic search procedure. This particular sector shall be denoted as the assigned search (AS) sector.

5) Filter out or discriminate against all other sources that are located outside of the AS sector.

6) Be amenable to frequent calibration of the system by using strong sources of opportunity.

Section II develops the snapshot signal model which was utilized in conducting the investigation, and it illustrates the problem of processing received signal samples which are correlated in both the spatial domain and the time domain over a modest RF bandwidth. Section III discusses a constrained beamspace partially adaptive array system with an adaptive transversal filter on each beam. Here the reader is introduced to a broad-band adaptive system and becomes acquainted with dual-domain performance behavior against broad-band and distributed interference sources.

Section IV then leads into our first large-aperture interferometer-style SLPF source estimator antenna system wherein we demonstrate the estimation of closely spaced multiple sources of moderate bandwidth. The SLPF system features an interferometer beamformer and a shaped AS sector "reference" beam, both of which may be steered to the AS sector.
sector of interest. A transversal filter is connected to each assigned beam port. The initial source simulation example is chosen to produce a wide equivalent spatial bandwidth spread yet without any significant spatial “overlap” within the common bandwidth. This example is utilized to illustrate a number of principles including: pattern shift with frequency, notch cancellation versus source estimation, and the effective performance of three-tap transversal filters connected into a few beams.

Section V considers the more interesting source simulation example wherein the multiple sources have significant “overlap” of their equivalent spatial bandwidth spread and, in addition, are located so close together as to require super-resolution. This example is utilized to illustrate that simple phase-shifter steering of the beams may seriously degrade estimation performance and that, at the very least, the reference beam should be steered via true time delay. Optimal estimation performance is then demonstrated when time-delay steering is utilized on all element signals, and it is pointed out how this represents a practical implementation of the “focused” covariance matrix technique proposed by Wang and Kaveh [3].

Note that throughout this report source estimation is indicated by conventional spatial antenna pattern nulls. This physical graphical representation emphasizes that we are dealing with a linear prediction filter in which the null positions of the spatial filter function estimate the locations of the spatial point sources. Reference [9] is recommended for a review of this fundamental principle and a reminder that filter nulls can often be rather “fragile” estimates of the true spectrum.

II. Snapshot Signal Model

This signal model is developed in a tutorial manner because it is the key to understanding the behavior of our rather complex dual-domain antenna system. Consider a linear array of $K$ elements as shown in Fig. 1, wherein the received signal samples are correlated in both space and time. The postulated signal environment on any given observation consists of $L$ plane waves of $L$ spectral lines arriving from distinct spatial directions $\theta_i$. The RF phase at the $k$th antenna element resulting from the $l$th spectral line of the $i$th source will be the product

$$2\pi X_{kl} \sin \theta_i$$  \hspace{1cm} (1)  

where $X_{kl}$ is the location of the element phase center with respect to the midpoint of the array in wavelengths. All element phasing is referenced to the midpoint or center of the array. Note that if our elements are equally spaced by a distance, $d$, $X_{kl}$ may be written

$$X_{kl} = \left( \frac{d}{\lambda_l} \right) \left[ k - \left( \frac{K + 1}{2} \right) \right]$$  \hspace{1cm} (2)  

where $\lambda_l$ is the RF wavelength of $l$th spectral line.

The complex amplitude of the $l$th spectral line of the $i$th source at the array midpoint reference is $p_i(l)$ such that we can express the $l$th frequency component of the $n$th time-sampled signal at the $k$th element as

$$E_k(n,l) = \eta_k(n,l)$$

$$+ \sum_{i=1}^{L} p_i(n,l)g_{ki}(\theta) \exp(j2\pi X_{kl} \sin \theta) \hspace{1cm} (3)$$

where

$$p_i(n,l) = P_i(l) \exp(j\phi_i(q,l)), \quad q = \text{INT}(n/T).$$

$P_i(l)$ is the amplitude of the $l$th spectral line of the $i$th source, $\phi_i(q,l)$ is a random phase variable associated with the $l$th spectral line of the $i$th source during the $q$th time interval of length $T$, $\text{INT}(x)$ means integer value of $x$. $g_{ki}(\theta)$ is the element pattern amplitude in the direction $\theta_i$, and $\eta_k(n,l)$ is the $l$th frequency component of the $n$th sample from the $k$th element independent Gaussian receiver noise. The receiver noise is assumed to be a random process with respect to the frequency spectral line index $l$, the time index $n$, and the element index $k$. It is also assumed that the source signals are uncorrelated with receiver noise. Equation (3) permits us to construct a convenient column vector of observed data in the form

$$E(n,l) = D(l)p(n,l) + \eta(n,l) \hspace{1cm} (4)$$

where $D(l)$ is a $K \times L$ matrix containing a column vector $d_i(l)$ for each of the $L$ source directions, i.e.,

$$d_{ki}(l) = g_{ki}(\theta) \exp(j2\pi X_{kl} \sin \theta) \hspace{1cm} (5)$$

Equation (4) separates out the basic variables of source direction in the direction matrix $D(l)$, source baseband signal in the column vector $p(n,l)$, and element receiver channel noise in the column vector $\eta(n,l)$. The vector $E(n,l)$ is defined as the $l$th spectral line component of the $n$th snapshot, i.e., a simultaneous signal sampling across all $K$ array elements at the $m$th time instant. These snapshots would nominally occur at the Nyquist sampling rate corresponding to our receiver bandwidth.

For our simple analysis purposes the random variable, $\phi_i(q,l)$, in (3) is utilized to ensure that the spatial point source signals are uncorrelated with one another. A total of $L = 21$ spectral lines were utilized in the simulations, equally spaced over the RF bandwidth, and the index $l$ was stepped sequentially on each snapshot to produce “sawtooth” swept-frequency signals. Thus the index $l$ is related to the index $n$

$$l = n \pmod{(21)}. \hspace{1cm} (6)$$
This simulated signal design is somewhat fictitious, but it permits separating out the frequency parameter for discussion. The resulting bandwidth behavior should be similar for normal signal waveforms which have the same instantaneous bandwidth and an equivalent data averaging time.

To illustrate the problem of processing data which is correlated in both the frequency/time domain and the spatial domain, consider forming a uniform-illumination beam pointed in the spatial direction \( \theta_0 \) at band-center wavelength \( \lambda_0 \). For elements equally spaced by the distance \( d \), the element weights \( b_k \) may be written

\[
b_k = \left( \frac{1}{\sqrt{K}} \right) \exp \left\{ -j2\pi \left( \frac{d}{\lambda_0} \right) \sin \theta_0 \left[ k - \frac{K}{2} \right] \right\}.
\]

(7)

The output of this beam for a single point-source data vector constructed per (4) would be

\[
V(n, l) = [b^I \cdot E(n, l)] = p(n, l) [b^I \cdot D(l)] + [b^I \cdot \eta(n, l)]
\]

where \( b \) is a column vector of beam weights and \( V(n, l) \) is the \( l \)th frequency component of the \( n \)th snapshot output voltage. The term of interest is the dot product of \( b \) and \( D(l) \)

\[
[b^I \cdot D(l)] = \sum_{k=1}^{K} b_k d_k(l).
\]

(9)

Substituting from (5) and (7) we have products of the form

\[
b_k d_k(l) = \left( \frac{g_k(\theta)}{\sqrt{K}} \right) \exp \left\{ j2\pi \left[ k - \frac{K}{2} \right] \right\}
\]

\[
\left[ \frac{d}{\lambda_0} \sin \theta - \frac{d}{\lambda_0} \sin \theta_0 \right] \left[ \frac{f_i}{f_0} \sin \theta - \sin \theta_0 \right].
\]

(10)

The two domains interact in the sine terms which may be factored to yield the expression

\[
\left[ \frac{d}{\lambda_0} \sin \theta - \frac{d}{\lambda_0} \sin \theta_0 \right] \left[ \frac{f_i}{f_0} \sin \theta - \sin \theta_0 \right] = \frac{d}{\lambda_0} \sin \theta - \frac{d}{\lambda_0} \sin \theta_0.
\]

(11)

where \( f_0 \) is the band-center RF frequency and \( f_i \) is the \( l \)th RF frequency within our passband.

The peak gain of the uniform-illumination beam occurs at the spatial angle \( \theta \) where (11) goes to zero, i.e.,

\[
\left( \frac{f_i}{f_0} \right) \sin \theta = \sin \theta_0.
\]

(12)

There are two conditions of interest here. 

1) \( \theta_0 = 0^\circ \): The beam is steered to boresight and the direction angle \( \theta \) will remain at boresight regardless of the frequency. Boresight is defined as the spatial direction perpendicular to the linear array aperture line.

2) \( \theta_0 \neq 0^\circ \): The beam will point in its intended steered direction \( \theta_0 \) only when \( f_i = f_0 \). Otherwise, for \( f_i > f_0 \) the beam shifts toward boresight, and for \( f_i < f_0 \) the beam shifts away from boresight.

A little thought on (10) shows that this shifting versus frequency occurs throughout the sidelobe regions of the beam, i.e., it affects all lobes and nulls except for the boresight direction. Examples of this pattern shifting are illustrated in the following sections. The significance of off-boresight spatial direction angle shifting versus frequency is that spatial point sources “spread out” and appear to the array as an equivalent spatially distributed source over their bandwidths. This effect has serious ramifications in attempting to estimate the true spatial positions of point sources and is the dominant problem area addressed in this paper.

Finally, let us consider the outputs obtained from attaching a tapped delay line to a beam output such as \( V(n, l) \) given in (8). Two options have been utilized in simulations for delay lines of \( M \) taps: 1) sample for \( M \) snapshots at each spectral line, thus “filling up” the delay line with bona fide signal samples, or 2) compute the tap outputs from knowledge of the spectral line and intertap delay. In the latter option, the \( m \)th tap will carry the output voltage

\[
V_m(n, l) = V(n, l) \exp \left\{ -j2\pi(m - 1) \left( \frac{\Delta f_i}{f_i} \right) \right\}
\]

(13)

where \( f_i \) is the sampling rate and \( \Delta f_i \) is the translated signal band spectral line corresponding to the \( l \)th RF frequency. The unit delay \( T \) between taps is assumed to be exactly equal to the sampling period, and it is further assumed that the signal energy is bandlimited by a rectangular filter to prevent “aliasing” [15].

The number of taps needed in our transversal filter may be estimated from the aperture-bandwidth product for a given source situation. Define this product as

\[
B_r \left( \frac{D}{\lambda_0} \right) \sin \theta_i
\]

(14)

where \( D \) is the total antenna aperture width, and \( B_r \) is the RF bandwidth normalized to the midband frequency. This product must be approximated by the delay-bandwidth product of the transversal filter

\[
B_r \left( \frac{\tau}{T} \right) \equiv B_r \left( \frac{D}{\lambda_0} \right) \sin \theta_i
\]

(15)

where \( \tau \) is the total length of the delay line and \( B_r \) is the transversal filter bandwidth normalized to the sampling rate. The value of \( \tau/T \) then estimates the number of taps required. Applying the values for the multiple source examples in this paper—\( T = 1, B_r = 1, B_f = 0.1, (D/\lambda_0) = 25, \theta_i = 48^\circ \)—we compute \( \tau/T = 1.85 \). Thus at least two delay sections (three taps) will be needed to cover the delay-bandwidth product involved. References [14] and [15] are recommended for those readers who wish to review the principles of tapped delay line filters or their digital equivalents.

III. CONSTRAINED BEAMSPACE SLC WITH A TRANVERSAL FILTER ON EACH BEAM

The spatial/time/frequency domain adaptive performance is reviewed for a constrained beamspace sidelobe-canceler (SLC) antenna system wherein each auxiliary SLC beam feeds into a multiple-tap delay line (transversal filter). The purpose
of the review is threefold: 1) to illustrate the interaction between the spatial domain and the time/frequency domain in cancelling broad-band spatial point sources; 2) to illustrate the special problem area encountered when spatially distributed sources are involved; and 3) to become acquainted with an adaptive SLC antenna system which utilizes similar principles of operation and would constitute a natural application area for the postulated source estimation processor.

Fig. 2 illustrates a schematic diagram of the particular type of partially adaptive array system under discussion. It consists of two beamformer subsystems which connect into common array aperture elements. The use of common elements helps to avoid polarization problems in the adaptive nulling performance, i.e., if the auxiliary elements are different, then variable source polarization can spoil the nulling even though the geometry remains fixed. The main beam subsystem (on the left side in Fig. 2) is intended to function as a conventional low-sidelobe phased array capable of electronic scan over the region of interest. Sidelobe level is determined by the usual quality of the elements, phase shifter modules, and corporate feed. The second beamformer subsystem is intended to furnish a set of SLC beams for selective subtraction from the main beam. Thus it is referred to as the SLC subsystem. This SLC subsystem is auxiliary to the main beam subsystem and consists of a beamformer which is coupled into the aperture at the elements, prior to the TR module phase shifters. This beamformer does not require low-sidelobe design and may consist of either a Butler matrix type or a lens type such as a Rotman lens. A very favorable feature from the standpoint of potential bandwidth is that all beams have the same phase center, i.e., the geometric center of the array aperture. Orthogonality in the family of beams is desirable but need not be precise for our purposes.

The output beam ports connect into a "beam assignment selector" wherein they are electronically switched into the SLC algorithm processor based upon source location estimates. The idea is to connect in only enough SLC beams to cancel a given source distribution situation, thus minimizing sidelobe degradation. The general principles of beamspace SLC have been described in the literature [7] and shown to offer advantages of a stable main beam, retention of low sidelobes, very fast adaptive response, and no adaptive grating lobes.

Each of the selected beams feeds into a tapped delay line, usually referred to as a transversal filter. It is this latter arrangement that permits adaptivity in the time/frequency domain [14], [16], [17] in addition to adaptivity in the spatial domain. The transversal filter (TF) output taps then feed into the black box labeled "SLC algorithm signal processor," which applies an adaptive algorithm to obtain the TF tap weights for achieving cancellation. This type of system is amenable to any of the current adaptive processing algorithms, including even analog versions.

Two different interference scenarios were selected for illustrating cancellation performance of the Fig. 2 SLC antenna system: spatial point-sources of broad bandwidth and spatially distributed interference sources. These were evaluated via computer simulations which utilized a 16-element linear array with half-wavelength element spacing as the common aperture. The main beam subsystem was given a quiescent Taylor illumination taper designed for 30-dB sidelobes, and the SLC subsystem beamformer was chosen to be a Butler matrix type. Fig. 3 shows a plot of a typical quiescent pattern for this array, with the mainbeam steered to \( -15^\circ \). The adaptive weights for the two scenarios were computed from the sample covariance matrix inverse (SMI) algorithm [7].

### A. Spacial Point Source with Bandwidth

This simulation involved a single 33-dB point-source located at \( +44^\circ \) azimuth, with the bandwidth parameter varied. A single beam, 14, from the Butler matrix SLC beamformer, was assigned to cover this point source. Cancellation performance for this single beam was evaluated with and without a transversal filter. Fig. 4 is a plot of typical adapted patterns with a three-tap filter at the low, midband, and high frequencies of the ten-percent RF bandwidth example. Note the considerable shift in the sidelobes versus frequency, but yet the null remains steady at the location of the point source. Fig. 5 then shows a summary of performance for this case, where we plot the adapted output in dB above receiver noise level versus the percent RF bandwidth multiplied by \( \sin(\theta) \), where \( \theta \) is the angular location of the point source. Note that the performance of the SLC beam plus a transversal filter is much superior to the SLC beam alone, which indicates that the extra DOF's from the transversal filter taps are providing effective adaptivity in the frequency domain to cancel the broad-band interference.

The curves in Fig. 5 are dependent upon the location of the point-source with respect to both the main beam sidelobes and the auxiliary SLC beam. Thus they may be used only for qualitative comparison purposes.
plot the adapted output power in dB above receiver noise level two, and three SLC beams. Note the considerable increase in performance obtained by utilizing three beams instead of just one or two. Fig. 7 illustrates the particular reference beam that was utilized for the simulations contained in this report. It is based upon a rectangular shape which passes the AS spatial sector but interferometer beams. The purpose of the phase shifters is to permit steering the beamformer beams to an assigned search (AS) spatial sector. For example, Fig. 10 illustrates a typical midband cluster of nine beams formed by a Butler matrix beamformer and steered to an azimuth center position of 42°. The details of the interferometer beamformer weights are contained in the Appendix, where it is noted that the cluster is not unique and will replicate itself several times throughout visible space because of the wide element spacing of 3λ.

The output beam ports connect into a "beam assignment selector" wherein they are electronically switched into a few transversal filters (TF) in a manner similar to the Fig. 2 system. The TF output taps then feed into the SLPF algorithm signal processor. In this manner we funnel down to the few DOF that will be utilized in performing source estimation over a moderate bandwidth. The number of TF taps is selectable.

A final similarity between Fig. 9 and Fig. 2 is that a "main beam" is included, albeit of much different characteristics. For source estimation we desire a shaped beam, hereafter denoted as the AS sector reference beam, which is of constant gain across the AS sector, but of low sidelobe level elsewhere. In other words, the ideal reference beam would be a rectangular shape which passes the AS spatial sector but filters out all sources which are located outside it. Fig. 11 illustrates the particular reference beam that was utilized for the simulations contained in this report. It is based upon a 19-element filled linear array whose phase center is located 7.5λ away from the phase center of the interferometer beams and which can be electronically steered to the AS sector of interest. The phase center of the reference beam must be separated significantly from the phase center of the interferometer beamformer to achieve robust spatial estimation performance. Details of the reference beam element weights are contained in the Appendix.

B. Spatially Distributed Interference

This simulation involved eleven 40-dB sources distributed uniformly in angle about the center position of +44° azimuth, with the angular width parameter varied. Beams 13, 14, and 15 from the Butler matrix SLC beamformer were assigned to cover this distributed interference. These beams are illustrated in Fig. 6. Cancellation performance was evaluated with and without a transversal filter on each beam. Fig. 7 illustrates a typical adapted pattern for the example wherein the 11 sources were distributed over a sector width equal to 1.25 beamwidths. Note that the three beams readily handle this rather wide angular distribution of interference. Fig. 8 then shows a summary of performance for this scenario, where we plot the adapted output power in dB above receiver noise level versus the distributed sector width in beamwidths, for one, two, and three SLC beams. Note the considerable increase in performance obtained by utilizing three beams instead of just one or two.

A significant observation from these simulation runs was that the addition of a transversal filter on each beam had virtually no effect on the cancellation performance, i.e., the extra DOF's from the transversal filter taps did not provide any additional adaptivity in the spatial domain to counter distributed interference. The curves in Fig. 8 are dependent upon the location of the distributed interference with respect to both the main beam sidelobes and the auxiliary SLC beams. Thus they may be used only for qualitative comparison purposes.

The SLC system of Fig. 2 has also been found to be very effective in the cancellation of delayed multipath signals which get into the mainbeam (time domain adaptivity) [16] and the cancellation of channel mismatch errors (frequency domain adaptivity within the receiver system passband [17].

IV. A Shaped-Beam SLPF Estimator with TF

Building upon the introductory material of the previous sections, we now proceed to large-aperture interferometer/sparse array techniques and evaluate their estimation performance against closely spaced multiple sources of moderate bandwidth. The particular type of SLPF source estimator antenna system under investigation is shown in Fig. 9. Aperture width was chosen to be 25λ (wavelengths) midband, which would accommodate 49 elements of half-wavelength spacing, if filled. From this aperture we select nine elements, spaced 3λ apart, which feed their signals through a phase shifter into an interferometer beamformer. The purpose of the phase shifters is to permit steering the beamformer beams to an assigned search (AS) spatial sector. For example, Fig. 10 illustrates a typical midband cluster of nine beams formed by a Butler matrix beamformer and steered to an azimuth center position of 42°. The details of the interferometer beamformer weights are contained in the Appendix, where it is noted that the cluster is not unique and will replicate itself several times throughout visible space because of the wide element spacing of 3λ.

The output beam ports connect into a "beam assignment selector" wherein they are electronically switched into a few transversal filters (TF) in a manner similar to the Fig. 2 system. The TF output taps then feed into the SLPF algorithm signal processor. In this manner we funnel down to the few DOF that will be utilized in performing source estimation over a moderate bandwidth. The number of TF taps is selectable.

A final similarity between Fig. 9 and Fig. 2 is that a "main beam" is included, albeit of much different characteristics. For source estimation we desire a shaped beam, hereafter denoted as the AS sector reference beam, which is of constant gain across the AS sector, but of low sidelobe level elsewhere. In other words, the ideal reference beam would be a rectangular shape which passes the AS spatial sector but filters out all sources which are located outside it. Fig. 11 illustrates the particular reference beam that was utilized for the simulations contained in this report. It is based upon a 19-element filled linear array whose phase center is located 7.5λ away from the phase center of the interferometer beams and which can be electronically steered to the AS sector of interest. The phase center of the reference beam must be separated significantly from the phase center of the interferometer beamformer to achieve robust spatial estimation performance. Details of the reference beam element weights are contained in the Appendix.
A. Equation Formulation

An equation formulation for the SLPF antenna/processor system of Fig. 9 may be developed in terms of the same pattern subtraction principles as utilized for the similar beamspace partially adaptive array system of [7], where the optimum weight column vector \( W_0 \) was expressed for \( K \) beamformer beams as

\[
W_0 = S^* - \sum_{k=1}^{K} W_k b_k
\]  

(16)

where \( S^* \) is a column vector of the element weights associated with the AS sector reference beam, \( b_k \) is the \( k \)th Butler matrix beamformer beam column vector of element weights, and \( W_k \) is the adaptive weight applied to the \( k \)th beam. The solution for \( W_k \) may be arrived at via any of the current adaptive processing algorithms, recognizing that there are performance differences.

Pointing toward the sample matrix inverse (SMI) algorithm, we take advantage of the fact that our sample covariance matrix of signal inputs \( \mathbf{R} \) involves only the \( J \) assigned beams needed, and its dimensions reduce from \( K \times K \) down to \( J \times J \), thereby easing the computation burden involved in obtaining its inverse [14]. The equivalent "steering vector" \( \mathbf{\Lambda} \) per Applebaum [18] is also reduced to dimension \( J \) and consists of the cross correlation between the reference beam signal \( V \) and \( J \) assigned beam outputs \( Y \)

\[
\mathbf{\Lambda} = \frac{1}{N} \sum_{n=1}^{N} V(n, l) Y^* (n, l)
\]  

(17)
Fig. 7. Typical adapted pattern response for 11 40-dB sources distributed uniformly over spatial sector width of 1.25 beamwidths (37.7° to 50.2°); three SLC beams 13, 14, and 15 without transversal filters.

Fig. 8. Summary of adaptive cancellation performance for 11 40-dB sources distributed uniformly over spatial sector; utilizing one, two, or three auxiliary SLC beams without transversal filters.

Fig. 9. Schematic diagram of SLPF estimator incorporating sparse-array beamformer, shaped reference beam, and transversal filter structure.
Fig. 10. Midband cluster of nine beams formed by Butler matrix interferometer beamformer, steered to center position of 42° azimuth.

Fig. 11. Typical AS sector reference beam at midband steered to center position of 42° azimuth.

where $N$ is the number of snapshots and index $l$ is related to $n$ (see (6)). The $j$th assigned beam output for the $n$th snapshot signal sample is simply

$$Y_j(n, l) = (E'(n, l) \cdot b_k), \quad \text{k set by j}$$

where $E(n, l)$ is the signal snapshot vector described in Section II, and the particular beam index $k$ must be selected for the $j$th assigned beam. $V(n, l)$ is the $l$th frequency component of the $n$th snapshot output voltage of the reference beam computed from

$$V(n, l) = (E'(n, l) \cdot S^*)$$

Our $J$ dimension adaptive weight solution thus becomes

$$W = [\hat{R}^{-1} A]$$

where

$$\hat{R} = \frac{1}{N} \sum_{n=1}^{N} [Y(n, l)Y^*(n, l)].$$

Finally, if transversal filters with $M$ taps are being used, then the beam output snapshot vector becomes a partitioned or stacked vector of $M$ segments, i.e.,

$$Y'(n, l) = [Y'_1(n, l), Y'_2(n, l), \ldots, Y'_M(n, l)]$$

wherein the vector components are defined as in (13), and the foregoing expressions must be modified to accommodate the partitioned matrices. Needless to say, the matrix dimensions are greatly increased when the transversal filters are employed, thus substantially increasing computation burden.

B. Estimating Multiple Sources with No “Overlap”

The first estimation example to be presented involves a simulation in which we have three 16.7-dB spatial point sources of ten-percent swept RF bandwidth, located at 37°, 42°, and 48° azimuth. These parameters were deliberately chosen so that there would be a continuous “equivalent” spatial interference distribution spread over the rather wide angular region extending from about 35° to 51.1° azimuth (roughly five beamwidths in extent), yet without any significant equivalent spatial overlap within their bandwidths. To graphically demonstrate the importance of the transversal filters, our first estimation pattern was run with no TF present. All nine beams from the interferometer beamformer were utilized, thus giving us an $\hat{R}$ matrix dimension of 9. Also, both the AS sector reference beam and the interferometer beamformer were phase-steered to a center position of 42° azimuth as shown in Figs. 10 and 11.

The sample covariance matrix $\hat{R}$ and the steering vector $A$ ((17) and (21)) were accumulated and averaged over a total $N = 1260$ snapshots. The resultant adaptive weights $W$ computed from (20) then give us the spatial pattern source estimate shown in Fig. 12(a). Several comments are in order regarding this adaptive pattern.

1) There is no spatial spectrum estimate per se because with no TF present the array processor cannot separate the two domains. It “sees” a distributed source and utilizes its nine DOF to form a wide spatial filter cancellation notch.

2) The adaptive pattern is shown plotted at three different frequencies within the ten-percent bandwidth: lowest, midband, and highest. Note that for $f_1 > f_0$ the pattern shifts toward boresight, and for $f_1 < f_0$ the pattern shifts away from boresight. Recall the discussion in Section II.

3) This performance is identical to what we would get if the processor operated from the nine interferometer element signals directly, i.e., the beamformer operation is a linear transformation of the element signals.

4) It demonstrates the same principle of distributed source notch cancellation as shown in Fig. 7 for the SLC system.

5) The eigenvalues computed for the particular $\hat{R}$ of this case, normalized to unity receiver noise power level, were 186.1, 160.0, 151.3, 128.4, 112.9, 86.1, 24.0, 2.8, and 1.1. Note that there are eight unique eigenvalues represented here and one “noise” eigenvalue (see [7]).

Next, the same case was run with the TF included, but with fewer beams. Beams 3, 4, 5, 6, and 7 from the interferometer beamformer were assigned to cover this source situation, with a three-tap TF selected for each of the five beams. Thus our matrix dimension for data processing is now 15, and we must incorporate partitioned vectors per (22). The resultant adap-

1See Section II for an explanation of equivalent spatial bandwidth spread.
tive weights spatial patterns are shown in Fig. 12(b) where we note the following points.

1) There is now a valid spectrum estimate indicated by the rather consistent positioning of three nulls in the adapted patterns, shown plotted at three frequencies within the ten-percent bandwidth.

2) Despite the considerable equivalent bandwidth spread, the three-tap TF permit separation of the frequency/time domain from the spatial domain in this case. Note the absence of pattern shifting as compared to Fig. 12(a).

3) This is an excellent illustration of a fundamental difference between source estimation and source cancellation systems. Both systems filter source signals out of their outputs, but in the former case we desire that the filtering be performed via reasonably distinguishable nulls located at the source positions—a much more difficult task.

4) The eigenvalues computed for the particular $\mathbf{R}$ of this case, normalized to unity receiver noise power level, were 469.6, 407.5, 288.7, 275.7, 229.1, 189.1, 157.7, 82.8, 38.3, 33.0, 9.3, 1.7, 1.6, 1.3 and 1.1. Note that there are about 11 unique eigenvalues represented here.

V. ESTIMATING MULTIPLE SOURCES WITH SIGNIFICANT "OVERLAP"

We now consider the more interesting case of estimating closely spaced multiple sources of moderate bandwidth wherein significant equivalent source overlap exists within their bandwidths. Our test simulation example was formulated by using the same three 16.7-dB spatial point-sources of ten-percent swept bandwidth discussed in the previous section but moving them closer together to locations of 40°, 42°, and 44° azimuth. Since our aperture beamwidth is 3.1° at these off-boresight locations, there is the added challenge of super-resolution, i.e., attempting to resolve multiple sources which are separated by less than a beamwidth.

The SPLF processor system conditions are the same as for Fig. 12(b) wherein both the AS sector reference beam and the Interferometer Beamformer are phase-steered to a center po-
Fig. 13. Typical adapted patterns for case of three 16.7-dB sources of ten-percent bandwidth located at 40°, 42°, and 44° with significant spatial "overlap." Five interferometer beams with three-tap transversal filter on each beam. (a) AS sector reference beam steered via phase shifters. (b) AS sector reference beam steered via true time delay.

The failure to resolve the sources is caused by two factors which were simultaneously present: 1) equivalent spatial domain overlap within the common bandwidth, and 2) the AS sector reference beam undergoes both pattern shifting and phase-center shifting versus frequency. The estimator system with TF can tolerate one of these factors at a time, as in Fig. 12(b), but not both. To demonstrate this point with the second option, let us next remove the equivalent source shifting in the AS sector reference beam by employing true time-delay steering instead of phase steering.

Time-delay steering produces element weights which vary with frequency. Recalling the discussion in Section II, it is readily seen that (7) would become

\[ b_k(l) = \left( \frac{1}{\sqrt{K}} \right) \exp \left\{ -j2\pi \left( \frac{d}{\lambda} \right) \cdot \sin \theta_0 \left[ k - \left( \frac{K + 1}{2} \right) \right] \right\} \] (23)

where we note the frequency dependence thru \( \lambda \). Substituting this change into (10), we see that there would be no beam shifting versus frequency in the steering direction \( \theta_0 \). Furthermore, the shifting will remain minimal for directions within a beamwidth of \( \theta_0 \), provided the RF bandwidth is modest. The Appendix contains pattern plots of the AS sector reference beam for both phase steering and true time-delay steering to a center position of 42° azimuth.

This "fixing" or stabilization of the multiple swept sources in the reference beam can be better appreciated via close examination of (17) in the previous Section IV, where we see that the equivalent steering vector \( \Lambda \) consists of the cross cor-
relations between the reference beam signals \( V(n, l) \) and the interferometer beam outputs \( Y(n, l) \). Obviously, if the sources remain constant in the \( V(n, l) \) signal with respect to the interferometer array phase center, then all of the amplitude/phase variations versus frequency will be confined to the interferometer beam outputs and can be compensated for by the adaptive transversal filters. Fig. 13(b) demonstrates the effective compensation by the TF under this condition wherein the reference beam is steered via true time delay rather than via phase shifters. Note that we now have a valid superresolution spatial spectrum estimate of the multiple swept-bandwidth "overlapped" sources and, furthermore, that there is very little pattern shifting throughout the entire AS sector (compare against Fig. 12(b)).

The eigenvalues computed for the particular \( \hat{R} \) of this case, normalized to unity receiver noise power level, were 648.7, 551.7, 403.2, 365.4, 199.4, 125.2, 80.5, 45.8, 34.1, 16.4, 6.2, 2.9, 2.4, 1.2, and 1.0. Note that there are 13 unique eigenvalues represented here.

A. Utilizing Time-Delay Steering on All Element Signals

If it is beneficial to minimize beam shifting of the AS sector reference beam by employing time-delay steering, then it should be even more beneficial to extend this characteristic to the interferometer beamformer as well by utilizing time-delay steering on all element signals. This does indeed prove to be the case for our SLPP estimator/processor system, and we end up with a practical manifestation of the "focused" covariance matrix technique proposed by Wang and Kaveh [3]. Essentially, the time-delay steering minimizes equivalent spatial bandwidth spread to such a small value within the AS sector that transversal filters are not really needed (assuming modest RF bandwidths). This benefit is demonstrated in Fig. 14(a) where we plot typical adapted patterns using time-delay steering on all element signals with no transversal filters present. All nine interferometer beams were used, thus giving an \( \hat{R} \) matrix dimension of 9, and the total number of snapshots averaged was \( N = 315 \).
The most remarkable feature about this technique is that we are getting an estimation performance fully equal to the performance demonstrated in Fig. 13, but with only three DOF needed as compared to 13 for the TF configuration. The DOF information is contained in the eigenvalues, which computed for the particular \( \mathbf{\tilde{R}} \) of this case as 400.3, 345.9, 101.8, 1.4, 1.2, 1.1, 1.0, 0.95, and 0.91. Note that there are only three unique eigenvalues represented here, which also happens to be the minimum number associated with three spatial point sources. This clearly indicates a “focused” covariance matrix and would permit us to reduce the matrix dimension down to three DOF, i.e., only three assigned beams for this particular case. Additional benefits include fewer data snapshots required and faster speed of response.

Time-delay steering to a narrow AS sector is a fundamental technique for separating the spatial domain from the frequency/time domain. When combined with a shaped AS sector reference beam, it functions in much the same manner as a narrow-band filter in the frequency domain, i.e., either technique can separate the two domains within the limitations of their “passbands” and “sidelobe levels.” To illustrate this point, the final simulation example deliberately includes two additional 20-dB sources of ten-percent swept bandwidth located at \(-13.5^\circ\) and \(-12.2^\circ\) azimuth. Referring to Fig. 11, note that these sources will suffer at least 24 dB of attenuation in the reference beam sidelobes and, therefore, will contribute very little to the cross-correlated steering vector \( \mathbf{A} \) of (17). In addition, they undergo an equivalent spatial spreading because they are about 55° away from the time-delay steering direction of 42°. The resultant adapted patterns are shown in Fig. 14(b) where we note only secondary disturbances due to the two extra swept sources. If significant correlation were present, there would be a cancellation notch extending over the indicated aliased region.

An interesting aspect of this particular case is that the two extra swept sources are presented at full gain in some of the interferometer beamformer weights because of the wide element spacing of \( 3\lambda_0 \), i.e., we have replication of the set of beams in Fig. 10 throughout visible space. This effect is threefold.

1) They contribute terms of high power level in \( \mathbf{\tilde{R}} \).

2) They increase and dominate the eigenvalues, which were computed as 703.6, 575.8, 419.5, 388.1, 287.6, 192.2, 35.3, 2.3, and 1.1. Note that there are now eight unique eigenvalues instead of three.

3) They have an equivalent spatial bandwidth spread in the AS sector because of the aliasing from their actual location. This equivalent spread is indicated in Fig. 14(b).

The aliasing intrusion into our AS sector can be harmful if it overlaps true sources in the sector, because then the adapted weights will be affected by the extra signal power coming from “common” directions, and it may be sufficient to distort or even destroy the null in those directions.

VI. CONCLUSION

We have investigated the concept of performing SLPF source estimation in a narrow steerable spatial sector by utilizing a large antenna aperture width (in wavelengths) but requiring relatively few DOF from it via operation in an interferometer beamformer space. Our objectives included high resolution, the accommodation of moderate bandwidths, fast response time, moderate cost, and ease of frequent calibration. Computer simulations on several multiple-source examples have demonstrated that the concept is indeed feasible provided true time-delay steering can be employed on the AS sector reference beam. This provision retains a reasonably stable spatial reference for each source within the sector over the common bandwidth.

The optimum estimation system results when time-delay steering can be applied to all element signals, i.e., including the interferometer elements, because this “focuses” the sample covariance matrix for sources which are within the AS sector and minimizes the DOF required. Even the transversal filters may be dispensed with under “focused” conditions if the bandwidth is modest. When combined with a shaped AS sector reference beam, this technique functions in much the same manner as a narrow-band filter in the frequency domain, i.e., it separates the spatial domain from the frequency/time domain within the limitations of its “passband” and sidelobe levels.

APPENDIX I

A. Interferometer Beamformer Weights

The beamformer indicated in Fig. 9 may consist of a Butler matrix design [19], a Rotman lens design [20], or any other feasible implementation [21]. Orthogonality in the family of beams is desirable but not essential, and low sidelobe design is not required. A Rotman lens would offer the inherent advantage of incorporating true time-delay beam formation such that in the light of the “focused” sector discussion of Section V it would probably be a preferred candidate in an actual system design. However, for the purposes of this report, where it was necessary to evaluate both phase steering and time-delay steering, the Butler matrix was a better choice.

A Butler matrix essentially sets up a spatial discrete Fourier transform in which the transformation vector for a linear array with equal element spacing will have individual weights of the form

\[
b_{km} = \left( \frac{1}{\sqrt{K}} \right) \exp \left\{ -j2\pi \left( \frac{k - K + 1}{2} \right) \left( m - K + X \right) \right\}
\]

where

\[
X = \left( \frac{2Kd}{\lambda_0} \right) \sin \theta_b
\]

and

- \( m \) beam index,
- \( k \) element index,
- \( K \) total number of elements,
- \( d \) element spacing,
- \( \theta_b \) vernier beam adjustment angle.

For our case where \( K \) is an odd number (\( K = 9 \)) we chose \( \theta_b \) such that \( X = 1 \), and this automatically adjusts the center beam to boresight position. Note that the element weights
expressed in (24) are independent of the spacing of the elements except for the vernier adjustment factor $X$. The complete transformation matrix $B$ then consists of $K$ column vectors of $K$ element weights, where the $k$th column vector $b_k$ is associated with the $k$th beam

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1K} \\ b_{21} & b_{22} & \cdots & b_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ b_{K1} & b_{K2} & \cdots & b_{KK} \end{bmatrix}.$$  

The beamformer output vector $\hat{E}$ is expressed

$$\hat{E} = B^T \mathbf{E}$$  

where $\mathbf{E}$ is the column vector of input signals from the array elements.

This beamformer is referred to as an “interferometer beamformer” because the input elements from the aperture are spaced $d = 3\lambda_0$ apart. The implication of this wide spacing is that the cluster of beamformer beams is not unique and will replicate itself several times throughout visible space. For example, if we consider just the center beam and its boresight position, we will get its full uniform illumination peak gain at multiples of $2\pi$ interelement phasing

$$2\pi \frac{d}{\lambda_0} \sin \theta_i = 2\pi i$$  

or $\sin \theta_i = i/3$ for $i = -3, -2, -1, 0, 1, 2, 3$. Therefore, the center beam replicates at the azimuth spatial angles of $-90^\circ, -41.8^\circ, -19.5^\circ, 0^\circ, +19.5^\circ, +41.8^\circ$, and $+90^\circ$.

**B. AS Sector Reference Beam**

The reference beam is intended to be a spatial filter which passes the AS sector but filters out all sources that are located outside it. In attempting to approximate this ideal rectangular-shape filter without getting into sophisticated design procedures, the method of separating two sin $x/x$ beams was utilized. It is readily shown that the element phasing required
for separating the two beams by $\Delta$ beamwidths is
\[
\phi_k = \frac{\pi \Delta}{K} \left[ k - \frac{K + 1}{2} \right],
\]
and adding the two beams results in cosine element weights $W_k$
\[
W_k = \frac{1}{2} \left[ e^{j \phi_k} + e^{-j \phi_k} \right] = \cos \phi_k.
\]
The array parameters chosen for this simple cosine-type weighting were $K = 19$ elements spaced a half-wavelength apart, and a beam separation of $\Delta = 1.5$ beamwidths where a beamwidth is defined as the arc sin $(2/K)$. By itself, this weighting produces a double-hump pattern with a 2.2-dB dip in the middle, so the end element weights were increased by $+0.546$ to flatten and extend the central constant-gain region. Final weights obtained for these two methods is readily determined for steering the beam, phase-shifters, and half-wavelength spaced elements

\[
\text{phase shifters} = \exp \left\{ -j\Psi_k \right\}
\]
\[
\text{time delays} = \exp \left\{ -j(f_i/f_0)\Psi_k \right\}
\]
where
\[
\Psi_k = \pi \sin \theta_0 \left[ k - \frac{K + 1}{2} \right].
\]
$f_0$ is the midband RF frequency and $f_i$ is the $i$th frequency within the bandwidth. The essential difference is that the phase shifter is a fixed phase setting $\Psi_k$, whereas time delay offers a countering frequency compensation to the midband setting.

A final point to be emphasized in regard to the AS sector reference beam is that its phase center is displaced $-7.5 \lambda_0$ away from the phase center of the interferometer beams. This displacement is essential to achieving robust spatial estimation performance in our SLPF systems because it produces a phase sensitivity to source location in a beamspace system in addition to the amplitude sensitivity derived from beamshapes. One consequence of this displacement is that it must be taken into account in referring the AS reference beam output signal to the phase center of the interferometer beamformer. Thus it adds an additional term to (31) and (32):
\[
\text{phase shifters} = \exp \left\{ -j(\Psi_k + \alpha) \right\}
\]
where
\[
\alpha = 2\pi(7.5) \sin \theta_0.
\]

**References**


