noise level. Although we can still utilize the feature given by the left side of (6), this is insufficient to discriminate similar fighter aircraft.

In general, both the wing tip and the wing edge of fighter aircraft are difficult to discern. When their positions are measurable, it is often with large uncertainty, so that the features described by (5), (6), and (7) provide limited discrimination capability. Thus, when the comparison database contains similar fighter aircraft, identification must be based primarily on the positions of dominant scatterers [1].

IV. CONCLUSIONS

We have shown that it is relatively easy to identify an aircraft whose image reveals the shape of the aircraft, using features related to length and wing span, without the need for an (usually unavailable) accurate aspect angle measurement. A high image quality also provides additional features for identification. In practice, the conditions for this simple type of identification are typically met by large commercial aircraft, which in addition to having a good backscattering behavior also fly quite smoothly. In by far most situations, such a simple identification procedure cannot be used for such small planes as jet fighters.

A. W. RHACZEK
S. J. HERSHKOWITZ
MARK Resources, Inc.
3878 Carson Street
Suite 210
Torrance, CA 90503
E-mail: (mri@markres.com)

REFERENCES

Theory and Practice of Radar Target Identification.

Cascaded Adaptive Canceler Using Loaded SMI

A fast-converging, highly parallel/pipeline cascaded canceler which uses the 2-input loaded sample matrix inversion (SMI) algorithm as the fundamental building block is developed which has convergence performance almost identical to one of the standards of a fast-converging adaptive canceler, the fast maximum likelihood (FML) canceler. Furthermore, the new algorithm, denoted as the cascaded loaded SMI (CLSMI), does not require the numerically intensive singular value decomposition (SVD) of the input data matrix as does the FML algorithm. For both the FML and CLSMI developments it is assumed that the unknown interference covariance matrix has the structure of an identity matrix plus an unknown positive semi-definite Hermitian (PSDH) matrix. The identity matrix component is associated with the known covariance matrix of the system noise and the unknown PSDH matrix is associated with the external noise environment. For narrowband (NB) jamming scenarios with /j jammers it was shown via simulation that the CLSMI and FML converge on the average ~3 dB below the optimum in about 2/ independent sample vectors per sensor input. Both the CLSMI and FML converge much faster than the standard canceler technique, the SMI algorithm.

I. INTRODUCTION

The use of adaptive linear techniques to solve signal processing problems is needed particularly when the interference environment external to the signal processor (such as for a radar or communication system) is not known a priori. Due to this lack of knowledge of an external environment, adaptive techniques require a certain amount of data to cancel the external interference. The amount of data (the number of independent samples per input sensor) required so that the performance of the adaptive processor is close (nominally within 3 dB) to the optimum is called the convergence measure of effectiveness (MOE) of the processor. The minimization of the convergence measure is important since in many environments the external interference changes rapidly with time.

Manuscript received January 24, 2000; revised August 24 and December 13, 2000; released for publication January 29, 2001.

IEEE Log No. T-AES/37/2/04469.

Refereeing of this contribution was handled by L. M. Kaplan.
This work was supported by the Office of Naval Research, ONR 31.
U.S. Government work not protected by U.S. copyright.
With assumption that the input interference is Gaussian, the classical adaptive linear processor with $N$ inputs is based on forming the sample covariance matrix (SCM) via maximum likelihood (ML) estimation. The linear weight vector is found by multiplying the inverse (assuming it exists) of the SCM by a desired steering vector. The convergence MOE of this technique which is called the sample matrix inversion (SMI) algorithm [1] is (with some assumptions) independent of the external noise environment and is approximately twice the number of independent sensor inputs. However, if fewer samples are available, performance will degrade. For example, if there were only one narrowband (NB) jammer present and $N = 50$ sensors, the SMI requires roughly $K = 2N = 100$ samples per input channel. Intuitively only a few samples should be required since only one NB jammer is present.

Due to the failure of the existence of the ML solution for $K < N$ samples (the SCM is singular), there have been several techniques proposed to improve convergence. The loaded sample matrix inversion (LSMI) technique [2] is implemented by approximating the optimal weight as the matrix inverse of the sum of the SCM and a scaled identity matrix times the desired steering vector. This form results from assuming that the input covariance matrix is known and constraining the adaptive weight vector magnitude to be less than a given constant while maximizing the output signal-to-noise ratio. The given constant and the scalar on the identity matrix are related. In actual implementation, the input covariance matrix is estimated. Techniques such as the adaptive-adaptive technique [3] and subspace techniques [4—6] have also been proposed. However, these techniques are based on heuristic constructions. Most subspace techniques such as in [4—6] are based on utilizing only the first $J$ (or $J + 1$ if signal is present) dominant eigenvectors, where $J$ is the number of independent NB jammers. Hence, they often assume knowledge of $J$ or require that $J$ be estimated by semi-heuristic techniques such as the AIC or MDL [7]. However, these techniques are not derived via fundamental criteria such as a ML estimate, as is the SMI. Also they are often limited in their applicability or require intuitive rules to determine the dimension and/or basis of the subspace. In [8, 9] we presented a new technique that provides typically similar performance as the heuristic techniques, and does not require a priori knowledge of $J$. The new fast maximum likelihood (FML) technique assumes only knowledge of the receiver thermal noise levels of the various input channels. The convergence MOE is similar to many of the fast-converging heuristic techniques, e.g. in an NB jamming scenario, the convergence MOE is on the order of twice the number of NB jammers. The technique also works for any external interference environment (for example for wideband jammers and clutter) without requiring modification. In addition in [8, 9], it was shown that the LSMI technique’s convergence performance was almost identical to the FML’s.

An adaptive canceler is a particular form of adaptive linear processor (ALP). In general, the ALP assumes that the desired input signal has the form of an $N$-length steering vector $\mathbf{s}$, where $N$ is the number of sensor inputs to the ALP and it is assumed $\mathbf{s}'\mathbf{s} = 1$ (‘ denotes conjugate transpose). An adaptive canceler assumes that $\mathbf{s} = (1 \; 0 \; 0 \; \cdots \; 0)^T = \mathbf{I}_0$ (where $T$ denotes transpose) and the adaptive weight on the main channel equals one. For this form, it is seen that all of the desired signal energy is assumed to be in the first input of the ALP. The first input is called the main channel and the other inputs are called auxiliary channels. For example, a common implementation of an adaptive canceler is the adaptive sidelobe canceler (ASLC) of a radar system. For this configuration the main channel is the output of the radar’s main antenna and the auxiliary channels are formed from much physically smaller auxiliary antennas which are in close proximity of the main antenna.

If digital processing is employed, any ALP can be transformed into an adaptive canceler configuration. This is because the multiplication of the $N$-inputs to the ALP by any $N \times N$ nonsingular matrix does not change the signal-to-noise (S/N) performance measure of the ALP. It can be shown that there always exists a nonsingular matrix $A$ such that $\mathbf{As} = \mathbf{I}_0$.

One of the advantages of an adaptive canceler configuration is that it can be laid out functionally in a highly numerically efficient parallel/pipeline cascaded signal processing architecture as illustrated by the generic cascaded canceler (GCC) in Fig. 1. For example, it is known that the SMI algorithm for $\mathbf{s} = \mathbf{I}_0$ can be replicated exactly by using the Gram–Schmidt (GS) canceler configuration [10, 11]. For this configuration, canceler inputs (from the right in the figure) are sequentially orthogonalized (decorrelated) with respect to other canceler inputs.
Data is inputted as an $N \times K$ block of data where $K$ is the number of samples per input channel used to calculate the weights. After the $N - 1$ weights are calculated at the 1st level of GS canceler, the $N \times K$ data block is weighted properly and passed as an $(N - 1) \times K$ data block to the second level for processing. Thereafter a new $N \times K$ input data block can be processed by the 1st level. This sequential processing, whereby input data blocks are processed and passed from level-to-level, results in an enhanced processing throughput rate. The advantage of the cascaded canceler configuration is that it requires significantly less numerical operations (which could be implemented as software) but that it can be laid out in a highly parallel/pipeline structure in hardware.

In [18], a fast-converging, highly parallel/pipeline cascaded canceler (FCC) was developed which has convergence performance almost identical for many pertinent jamming scenarios to the FML canceler. The FCC used a modified 2-input FML as the fundamental building block. Again for both the FML and FCC developments it was assumed that the unknown interference covariance matrix has the structure of an identity matrix plus an unknown positive semi-definite Hermitian (PSDH) matrix. For NB jamming scenarios with $J$ jammers where $J$ is less than some upper bound, it was shown via simulation that the FCC and FML converge on the average $-3$ dB below the optimum in about $J/2$ independent sample vectors per sensor input. Both the FCC and FML converged much faster than the standard canceler technique, the SMI algorithm.

In this work, we develop what we call a cascaded loaded SMI (CLSMI). Just as the GS canceler is a highly parallel/pipeline implementation of the SMI canceler and has the same convergence performance as the SMI canceler, we show that the highly parallel/pipeline CLSMI canceler’s convergence performance closely approximates the FML’s. The fundamental building block of the CLSMI is a modified 2-input LSMI where the loading matrix will no longer necessarily be a weighted $2 \times 2$ identity matrix. The form of the loading matrix is clarified in a subsequent section. The motivation for using the CLSMI was the observation made in [8, 9] that the LSMI’s convergence performance is almost identical to the FML’s. It has been shown empirically [8, 9] that the FML canceler has one of the fastest convergence performance of all contemporary adaptive canceler algorithms. Thus a cascaded form of LSMI should have good convergence qualities. Indeed, it was found that the CLSMI has almost identical convergence performance as the FML canceler. Finally, the CLSMI is superior to the FCC in the sense that the convergence performance relative to the FML canceler is no longer limited by number of jammers in the interference scenario. For the CLSMI development we assume (as was done for the FML) that the unknown interference covariance matrix has the structure of an identity matrix (associated with system noise) plus an unknown PSDH matrix (associated with the external noise).

II. REVIEW OF FML CANCELER

The ML estimate for the $N$ by $N$ covariance matrix that results under the Gaussian assumption and $K \geq N$ is the SCM. The convergence MOE is on the order of $K \approx 2N$. We derived in [8, 9] the ML estimate for $K < N$, assuming a structured covariance matrix of the form

$$R = \Omega + R_0$$

where $R_0$ is PSDH, the rank of $R_0 = M_0 < N$, and $\Omega$ is the $N \times N$ diagonal matrix covariance matrix of thermal noise levels of the input channels. We assume that the thermal noise levels are known and that $M_0$ is unknown. Because a given input data channel can be normalized with respect to the known thermal noise level of that channel, without loss of generality, we take $\Omega = I_N$ in the above equation where $I_N$ is the $N \times N$ identity matrix. In much of the adaptive literature, the thermal noise level has been assumed unknown. Justification for this assumption can be traced to several papers [12, 13]. Certainly, the thermal noise level is unknown in many time-series problems such as in the analysis of sun-spot data. However, in systems operating at the microwave frequencies, the thermal noise level is dominated not by the unknown external thermal noise but by the receiver thermal noise [14, p. 1.4]. Hence, the thermal noise power level is known a priori for many practical interference cancelation problems. Under these conditions we have found a fast converging ML solution for the case $K < N$.

The ML solution for the structured matrix $R = I_N + R_0$ has been derived in previous published work for the case $K \geq N$ and the SCM is nonsingular. In [8, 9] we showed that a solution also surprisingly exists for $K < N$. In fact, a solution can be found even for a single sample. There are only a few cases [15] of constrained covariance matrices where a ML solution exists for such small sample cases, and most of these are not relevant to existing problems. Our solution is, however, directly relevant to many systems that operate in the microwave region, since the thermal noise is dominated by the receiver noise, of which the statistics can be assumed known.

Let $z_n$ ($n = 1, 2, \ldots, N$) represent data from the $n$th sensor (or equivalently the $n$th input to the adaptive linear processor), $\hat{R} = Z^*Z/K$ where $Z$ is the $N$ by $K$ sample data matrix, $\hat{R}$ is the SCM, and $*$ denotes conjugation. The $K$ columns of $Z$ are assumed to be zero-mean, independent, and identically distributed (IDD) $N$-length complex Gaussian random vectors.
The individual elements of \( Z \) are complex circular random variables (i.e., the real and imaginary parts are IDD). The \( n \)th \((n = 1, \ldots, N)\) row of \( Z \), denoted by \( z_{i}^{T} \) (a \( K \)-length vector), represents the sampled data on the \( n \)th sensor. The data in a given column of \( Z \) is assumed to be time (or range) coincident. Hence, each \( N \)-length random vector (a column of \( Z \)) is often called a snapshot of input data.

The joint probability density function (pdf) of the data under the Gaussian assumption and \( R \) nonsingular is

\[
p(Z; R_{0}) = \pi^{-KN} |R|^{-K} \exp(-Tr(KR^{-1} \hat{R}))
\]

(1)

where \( R = I_{K} + R_{0}, \) \( R_{0} \) is PSDH, and \( | \cdot | \) and \( Tr(\cdot) \) denote determinant and trace, respectively. The ML estimate (if it exists) for the covariance matrix is given by

\[
R_{ML} = \arg \min R \ln |R| + Tr(R^{-1} \hat{R}).
\]

(2)

It is known that the ML solution exists for \( K \geq N \) and the SCM is nonsingular \([16]\). Consider an eigenvalue decomposition (EVD) of the SCM, \( \hat{R} = \Phi \Lambda \Phi^{T} \), where \( \Lambda \) is an \( N \times N \) diagonal matrix with diagonal entries \( \lambda_{1} \geq \lambda_{2} \cdots \geq \lambda_{N} \) that are eigenvalues of \( \hat{R} \) and \( \Phi \) is an \( N \times N \) unitary eigenmatrix. Denote \( M \) as the number of eigenvalues that are greater than one. Set \( \Lambda_{0} = \text{Diag}(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{M}, 1, 1, \ldots, 1) \) where \( \text{Diag}(\cdot) \) denotes a diagonal matrix with elements and ordering specified by its arguments. The ML estimate for \( R \) exists for \( K \geq N \) and is given by \([16]\)

\[
R_{ML} = \Phi \Lambda_{0} \Phi^{T}.
\]

(3)

In \([8, 9]\), we showed that this same estimate can be used for \( 1 \leq K < N \). This result is stated as the following theorem.

**THEOREM 1** *Under the condition \( R = I_{K} + R_{0}, \) where \( R_{0} \) is a PSDH matrix, the ML solution, \( R = \arg \min_{R} \ln |R| + Tr(R^{-1} \hat{R}), \) is given by (3) for any \( K \geq 1, \) and \( \Phi \) and \( \Lambda_{0} \) are as defined above.*

The FLM \( K \)-length-vector weight, denoted by \( \hat{w}_{FML} \), is proportional to \( R_{ML}^{-1} s \) where \( s \) is the \( K \)-length steering vector of the desired signal. For the canceler configuration, \( s = I_{K} \). In addition for the canceler configuration, the first element of \( w_{FML} \) is constrained to be equal to one. Let \( r_{ML}^{11} \) be the \((1,1)\) element of \( R_{ML}^{-1} \). It follows that

\[
\hat{w}_{FML} = \frac{1}{r_{ML}^{11}} R_{ML}^{-1} I_{0}.
\]

(4)

We apply the weight \( \hat{w}_{FML} \) to data (a snapshot) that is statistically independent of the data that was used to calculate \( \hat{w}_{FML} \) and denote this snapshot as \( z \). This is referred to as nonconcurrent processing. The snapshot of the data \( z \) is also identically distributed as the snapshots of the data that are used to calculate \( \hat{w}_{FML} \).

We apply \( \hat{w}_{FML} \) to \( z \) to form the scalar output residue

\[
r_{z} = \hat{w}_{FML}^{T} z.
\]

(5)

We examine and compare the statistical characteristics of \( r_{z} \) with the CLSMI output residue in the subsequent analysis.

### III. CASCADED LSMI

The general functional structure of a cascaded canceler was shown in Fig. 1. The fundamental building block of the structure is the 2-input canceler whereby data in the right input of the 2-input canceler is weighted by a complex scalar and thereafter subtracted from the left input. For example if \( u \) and \( v \) denote the \( K \)-length vectors of the left and right inputs, \( w \) denotes the complex scalar weighting, and \( r \) the \( K \)-length output residue vector, then \( r = u - wv \). (Note from now on for notational purposes we write without loss of generality \( r = u + \bar{w}v \) where \( \bar{w} = -w \)). We associate the \( i, j \) 2-input canceler with \( w_{ij} \), i.e. the \( i, j \) 2-input canceler can be found at the \( k \)th level and \( j \)th column of the GCC seen in Fig. 1. Assume that the input to GCC are the \( N K \)-length input vectors \( z_{n} \) \((n = 1, 2, \ldots, N)\). Define \( x_{j}^{(i)} \) \( j = 1, 2, \ldots, N - i + 1 \) as the \( K \)-length output vector of the \( i, j \) 2-input canceler with

\[
x_{j}^{(i)} = z_{j}, \quad j = 1, \ldots, N.
\]

(6)

Thus

\[
x_{j}^{(i+1)} = x_{j}^{(i)} + \hat{w}_{ij} x_{N-i+1}^{(i)}, \quad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, N - i + 1.
\]

(7)

The weight \( \hat{w}_{ij} \) is a function of the inputs of the \( i, j \) 2-input canceler. For example, for the GS 2-input canceler

\[
w_{ij} = -(x_{N-i+1}^{(i)} x_{j}^{(i)})/(\|x_{N-i+1}^{(i)}\|^{2})
\]

(8)

where \( \| \cdot \| \) denotes the vector magnitude.

For the CLSMI, we use the 2-input modified LSMI (MLSMI2) as a building block as illustrated in Fig. 2. The MLSMI2 is described in detail in the following section. The 2-input MLSMI2 is used to find the scalar complex weight for each \( i, j \) canceler. However the 2-input LSMI algorithm must be modified because the covariance matrix of the 2 inputs into the \( i, j \) 2-input canceler associated with the internal thermal noise is no longer a \( 2 \times 2 \) identity matrix \( I_{2} \), except on the first level. Furthermore, it can be derived exactly because we know how the input
noises on the various channels have been weighted as they traverse through the cascaded canceler structure up to the \(i,j\) 2-input canceler. In order to calculate the \(2 \times 2\) internal noise covariance matrix at the 2-input cancelers, it is necessary to know how each input channel \(z_n\) \((n = 1, \ldots, N)\) is weighted at the output of the \(i,j\) 2-input canceler. Let \(y_{ij}\) represent the output residue of the \(i,j\) MFML2. We define \(h(i,j)\) to be weighting on the \(z_n\) at the \(i,j\) output. Simply stated

\[
y_{ij} = \sum_{n=1}^{N} h(i,j,n)z_n. \tag{9}\]

Since \(y_{i+1,j} = y_{ij} + \hat{w}_{ij}y_{i-N-i+1}\), it can be shown using (9) that the \(h(i,j,n)\) can be found reiteratively as follows:

\[
h(i+1,j,n) = h(i,j,n) + \hat{w}_{ij}h(i)(N - i + 1,n),
\]

\[
j = 1,2,\ldots,N - i
\]

\[
n = 1,2,\ldots,N. \tag{10}\]

Initial condition: \(h(i,j,n) = \delta_{jn}, j,n = 1,\ldots,N\) where \(\delta_{jn} = 1\) if \(j = n\), 0 otherwise.

How the \(h(i,j,n)\)s are used is described in the next section.

If we are performing noncurrent processing then we are interested in the equivalent weighting of the cascaded canceler structure illustrated in Fig. 1 with the MLSMI2 as the 2-input canceler building block. We desire to know the equivalent \(N\)-length weighting vector, denoted by \(\hat{w}_{\text{MLSMI}}\) of passing the \(N\)-length nonconcurrent data vector \(z\) through the MLSMI. It is straightforward to show that

\[
\hat{w}_{\text{MLSMI}} = (h(N)(1,1),h(N)(1,2),\ldots,h(N)(1,N))^T \tag{11}\]

with \(h(N)(1,1) = 1\).

IV. 2-INPUT MODIFIED LSMI

In this section, we discuss in more detail the implementation of the 2-input MLSMI2. As previously mentioned, the \(2 \times 2\) internal noise covariance matrix of the inputs of the \(i,j\) 2-input canceler is no longer equal to \(I_2\). Denote this \(2 \times 2\) covariance matrix as \(\hat{R}_{ij}\).

We can derive in simple fashion this covariance matrix with knowledge of \(h(i,j,n)\) and \(h(N - i + 1,n), (n = 1,2,\ldots,N)\). Set \(h(i) = [h(i)(1,1),h(i)(1,2),\ldots,h(i)(1,N)]^T\).

From (9) and the fact that the internal noise power on \(z_n(n = 1,\ldots,N)\) is 1, it follows that

\[
\hat{R}_{ij}(1,1) = h(i)h(i)^T \tag{12a}
\]

\[
\hat{R}_{ij}(2,1) = h(N-i+i)h(i)^T \tag{12b}
\]

\[
\hat{R}_{ij}(1,2) = \hat{R}_j(2,1) \tag{12c}
\]

\[
\hat{R}_{ij}(2,2) = h(N-i+i)h(N-i+i)^T. \tag{12d}\]

Let \(\hat{R}_{ij}\) be the \(2 \times 2\) SCM of the \(i,j\) 2-input canceler. The elements of \(\hat{R}_{ij}\) are found as

\[
\hat{R}_{ij}(1,1) = x_j^{(j)}x_j^{(j)}/K \tag{13a}
\]

\[
\hat{R}_{ij}(2,1) = x_{N-i+i}^{(j)}x_j^{(j)}/K \tag{13b}
\]

\[
\hat{R}_{ij}(1,2) = \hat{R}_j(2,1) \tag{13c}
\]

\[
\hat{R}_{ij}(2,2) = x_{N-i+i}^{(j)}x_{N-i+i}^{(j)}/K. \tag{13d}\]

Normally for the 2-input LSMI algorithm, a weighted \(2 \times 2\) identity matrix would be added to \(\hat{R}_{ij}\). However we form the following \(2 \times 2\) matrix:

\[
R_{ij} = \hat{R}_{ij} + \alpha \hat{R}_{ij} \tag{14}\]

where \(\alpha\) is defined as the loading factor which we set equal to one (\(\alpha = 1\) is an effective loading factor which we verified via simulation). The weighting on the \(i,j\)th MLSMI2 \(\hat{w}_{ij}\) is given by

\[
\hat{w}_{ij} = \frac{R_{ij}(2,1)}{R_{ij}(2,2)}. \tag{15}\]

For no diagonal loading (\(\alpha = 0\)), the GS weight would be given by (15). A block diagram of the \(i,j\)th MLSMI2 is given in Fig. 2 where the inputs and outputs are shown. The weighting on the \(i,j\)th MLSMI2, \(\hat{w}_{ij}\), is found via (12b), (12d), (13b), (13d), (14), and (15). The outputs, \(x_j^{(j+1)}\) and \(h_j^{(j+1)}\), are found using \(\hat{w}_{ij}\) and the inputs to the \(i,j\)th MLSMI2 via (7) and (10), respectively.

The number of extra complex multiplies per 2-input building block necessary to implement the CLSMI versus the GS is approximately \(3N\); at most \(N\) multiplies to find \(h_j^{(j)}\) (see (10)) and \(2N\) multiplies to find \(R_{ij}(2,1)\) and \(R_{ij}(2,2)\). In normal operation of a GS canceler the number of complex multiplies per building block is \(3K\) (\(2K\) multiplies to compute the weight and \(K\) multiplies to apply the weight). For reasonably fast convergence of a GS canceler, \(K = 2N\). Hence a GS canceler would require approximately \(6N\) multiplies per building block where a CLSMI canceler with the same number of input samples per channel would require \(9N\). However we point out that a CLSMI would normally require \(K < N\) in order to achieve the same normalized residue performance as the GS canceler.

V. RESULTS AND DISCUSSION

In this section, we present representative simulation results that compare the CLSMI and FML convergence performances. We shall see that for representation interference scenarios that the CLSMI and FML convergence performances are almost identical. We also compare the CLSMI and FML convergence performance with a canceler that we call...
the pseudo sample matrix inversion (PSMI) canceler. For $K \geq N$, the PSMI is exactly the SMI algorithm. However for $K < N$, since the SMI algorithm is not applicable (the SCM is singular), we use an alternate procedure which we now describe to find the canceler weighting vector.

Let $Z_A$ be the $(N - 1) \times K$ data matrix associated with auxiliary channels ($z_2, \ldots, z_K$) and $z_M$ be the $K$-length column vector associated with the main channel ($z_1$). We can find a weighting vector of length $N - 1$, $w_A$, such that

$$Z_A^T w_A = z_M.$$  

(16)

In fact for $K < N - 1$, there are an infinite number of solutions for $w_A$. We solve the $w_A$ using the pseudoinverse:

$$w_A = Z_A^T (Z_A Z_A^T)^{-1} z_m.$$  

(17)

The pseudoinverse solution minimizes $\|w_A\|$ with respect to all possible solutions of (16). The $N$-length weight vector $(1, -w_1^T)^T$ is the solution that we use for the PSMI when $K \leq N - 1$. For $K \geq N$, the SMI solution is used for the PSMI technique. As with the FML and FCC the PSMI is implemented using nonconcurrent data.

Although there are many configurations that can be considered for interference cancellation problems, for simplicity, we assume that an $N$-element array of identical antenna elements exists such that the jammer vectors have the following form

$$(1, \exp(j\theta), \exp(2j\theta), \ldots, \exp((N - 1)j\theta))^T$$  

(18)

where $j = \sqrt{-1}$. The main channel is the left-most array element and the remaining $N - 1$ elements are the auxiliary channels. We model the external noise environment via its input covariance matrix.

The $N \times N$ covariance matrix associated with $J$ NB jammers can be represented as $R_0 = (r_{nm})$ where

$$r_{nm} = \sum_{i=1}^{J} \sigma_i^2 \exp[j(n-m)\phi_i] + \delta_{nm},$$  

(19)

$\phi_i$ ($i = 1, 2, \ldots, J$) is the phase angle associated with the $i$th jammer, and $\sigma_i^2$ is the jammer power of the $i$th jammer normalized by the internal noise level. The $\delta_{nm}$ contribution to the covariance matrix is associated with the internal thermal noise power.

In order to compare different techniques, we calculate the normalized average signal-to-interference ratio (SIR) $\overline{\text{SIR}}$, which is defined as the averaged output SIR ratio for a given technique divided by the optimal SIR that can be achieved when the optimal weight is used. Interference as defined here includes all unwanted interference including thermal noise. The optimal SIR is known to be $\text{SIR}_{\text{opt}} = 1^T R^{-1} 1$. Averaged over the Monte Carlos, $\overline{\text{SIR}}$ is defined by

$$\overline{\text{SIR}} = \frac{1}{N_s} \sum_{s=1}^{N_s} \left( \frac{|w_m^T \text{I}_s|^2}{w_m^T R_m \text{SIR}_{\text{opt}} w_m} \right),$$  

(20)

where $w_m$ is the random weight vector associated with a given canceler technique (FML, CLSMI, or PSMI) indexed by the Monte Carlo number (MC denotes the number of Monte Carlos). The kernel of the sum seen in (20) represents the normalized instantaneous SIR for a given MC. The expected value of the nonconcurrent data has been taken and is exemplified by the $R$ term in the denominator of the kernel.

For all of our simulation results, we fix the jammer angles and powers and generate MC realizations of input data. We now show a number of simulation results for representative interference scenarios.

In all cases the number of input channels to the canceler $N$ is 20 and the number of MC equal 100. In Figs. 3–6, the normalized average SIR (denoted in short as the normalized S/I in the figures) is plotted versus the number of independent snapshots $K$ per input sensor for the canceler configurations: CLSMI, FML, and PSMI and various interference scenarios. The maximum $K$ was chosen to be equal to 40 (= 2N) which is normally the $K$ chosen for “good” convergence performance (~ 3 dB on average from the optimal) for the SMI. We see from Figs. 3–6 that the LSMI and FML convergence performances are almost identical and that they converge on average ~ 3 dB below the optimum in about $2J$ independent sample vectors per sensor input where $J$ is the number of NB jammers. The PSMI canceler performance is notably inferior to the CLSMI and FML over much of the range of $K$. The PSMI performance in the $1 \leq K \leq N$ region is not readily explainable. It is interesting to note, however, that for $1 \leq K < N$, a maximum occurs in the PSMI convergence performance, which in a number of cases is not much less than the CLSMI and FML performance. In addition, a number of cases were run where the jamming was no longer NB. It was found also that the performances of CLSMI and FML tracked closely.

Performance was also evaluated as a function of the loading factor $\alpha$. It was found that CLSMI performance degraded slightly (less than 0.25 dB) for $\alpha$ equal 1 to 5. For $\alpha = 8$, the CLSMI performance as seen in Fig. 7 was about 1 dB less than the FML performance for a given $K$. We also varied the loading matrix given in (14). Instead of using the actual internal noise covariance matrix at each $i,j$ 2-input MLSMI, we used the $2 \times 2$ identity matrix with $\alpha = 1$ as the loading matrix. Thus $R_{ij} = R_{ij} + I$, which is the standard form for the 2-input MLSMI. Using this form in the CLSMI, it was found that significant degradation (greater than 3 dB) occurred in the performance of the CLSMI. Hence, it would seem that
using the actual internal noise covariance matrix as the loading matrix is important in order for the CLSMI to function properly.

The sampled standard deviation (SD) of the normalized average S/I was also evaluated over a number of representative cases. A single plot of this measure is shown in Fig. 8 for the same interference scenario given in Fig. 5. Here we observe that the CLSMI’s SD is similar to the FML’s and that the CLSMI’s and FML’s SD is moderately better than the PSMI’s for $K$ approaching $2N$ (or 40). In fact one might argue that one reason for using the FML or the CLSMI rather than the SMI for $K = 2N$ is that the CLSMI or FML output SD of the output residue is noticeably better than the SMI.

The total number of floating point multiplication operations (FPMOP) associated with finding the canceler weight for the CLSMI is approximately $0.5N^2(3K + 3N)$. For the FML canceler implemented with full singular value decomposition (SVD) (all singular values and eigenvectors calculated via SVD), the number of FPMOPs is approximately $3N^2K +$
Fig. 5. Normalized S/I versus number of independent snapshots $K$; $N = 20$, $J = 5$, $\theta_J = 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ$, $P_J = 30$ dB, $\alpha = 1$, $MC = 100$, three canceler configurations: CLSMI, FML, PSMI.

Fig. 6. Normalized S/I versus number of independent snapshots $K$; $N = 20$, $J = 10$, $\theta_J = 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ$, $110^\circ$, $P_J = 30$ dB, $\alpha = 1$, $MC = 100$, three canceler configurations: CLSMI, FML, PSMI.

$4NK^2$. For the FML canceler implemented with partial SVD (only the eigenvalues greater than 1 and their associated eigenvectors need be calculated) the number of FPMOPs $= O(KNJ)$ (this assumes a NB jamming scenario). For the SMI canceler, $\text{FPMOP} \approx 0.33N^3 + KN^2$. Hence the number of FPMOPs needed to implement the CLSMI in software is roughly equivalent to either the FML or SMI implementations. However, as it was pointed out in the introduction, the advantage of the CLSMI is not its software numerical efficiency but its hardware numerical efficiency. The algorithm can be laid out functionally using a highly parallel/pipeline architecture. This structure is ideal for efficiently block processing input data blocks that are sequentially updated at each time step (or some other dimension).

VI. SUMMARY

A fast-converging, highly parallel/pipeline cascaded canceler which uses the 2-input loaded SMI as the fundamental building block has been
developed which has convergence performance almost identical to the FML canceler [8, 9]. The new algorithm is denoted as the CLSMI. For NB jamming scenarios, it has been shown that the CLSMI convergence performance is almost identical to the FML’s. For both the FML and CLSMI developments it is assumed that the unknown interference covariance matrix has the structure of an identity matrix plus an unknown PSDH matrix. The identity matrix component is associated with the known covariance matrix of the system noise and the unknown PSDH matrix is associated with the external noise environment. For NB jamming scenarios with \( J \) jammers it was shown via simulation and analysis that the CLSMI and FML converge on the average \(-3\) dB below the optimum in about \( 2J \) independent sample vectors per sensor input. Both the CLSMI and FML converged much faster than the SMI algorithm. A number of cases were run where the jamming was wideband. For all of these cases, the convergence performances of the CLSMI and FML were almost identical.
REFERENCES


Fuzzy-Logic-Based CLOS Guidance Law Design

A fuzzy-logic-based command to line-of-sight (CLOS) guidance law is proposed. In this design, the guidance problem is converted to the tracking problem. Then the fuzzy controller is design to achieve satisfactory tracking performance. Meanwhile, an on-line tuning factor is introduced to speed up the convergence of tracking error and to reduce the miss distance. Besides, a CLOS compensator is designed to eliminate the pseudo-CLOS effect. Simulation results for different engagement scenarios illustrate the validity of the proposed guidance law.

I. NOMENCLATURE

\[\psi_t\] Yaw angle of target  
\[\theta_t\] Pitch angle of target  
\[\psi_m\] Yaw angle of missile  
\[\theta_m\] Pitch angle of missile  
\[\phi_{mc}\] Roll angle command  
\[\sigma_t\] Azimuth angle of line-of-sight (LOS) to target  
\[\gamma_t\] Elevation angle of LOS to target  
\[\sigma_m\] Azimuth angle of LOS to missile  
\[\gamma_m\] Elevation angle of LOS to missile  
\[\Delta \sigma\]  
\[\sigma_m - \sigma_t\]  
\[\Delta \gamma\]  
\[\gamma_m - \gamma_t\]  
\[T\] Thrust force  
\[D\] Drag force

Manuscript received July 21, 1997; revised September 14, 1999 and March 31, 2000; released for publication February 5, 2001.  
IEEE Log No. T-AES/37/2/06346.  
Refereeing of this contribution was handled by X. R. Li.  
This work was supported by the National Science Council of the Republic of China under Grant NSC-86-2213-E-155-027.