Spatially Distributed Target Detection in Non-Gaussian Clutter

KARL GERLACH, Member, IEEE
Naval Research Laboratory

Two detection schemes for the detection of a spatially distributed, Doppler-shifted target in non-Gaussian clutter are developed. The non-Gaussian clutter is modeled as a spherically invariant random vector (SIRV) distribution. For the first detector, called the non-scatterer density dependent generalized likelihood ratio test (NSDD-GLRT), the detector takes the form of a sum of logarithms of identical functions of data from each individual range cell. It is shown under the clutter only hypothesis, that the detection statistic has the chi-square distribution so that the detector threshold is easily calculated for a given probability of false alarm \( P_F \). The detection probability \( P_D \) is shown to be only a function of the signal-to-clutter power ratio \((S/C)_{\text{opt}}\) of the matched filter, the number of pulses \( N \), the number of target range resolution cells \( J \), the spikiness of the clutter determined by a parameter of an assumed underlying mixing distribution, and \( P_F \). For representative examples, it is shown that as \( N, J, \) or the clutter spikiness increases, detection performance improves. A second detector is developed which incorporates a priori knowledge of the spatial scatterer density. This detector is called the scatterer density dependent GLRT (SDD-GLRT) and is shown for a representative case to improve significantly the detection performance of a sparsely distributed target relative to the performance of the NSDD-GLRT and to be robust for a moderate mismatch of the expected number of scatterers. For both the NSDD-GLRT and SDD-GLRT, the detectors have the constant false-alarm rate (CFAR) property that \( P_D \) is independent of the underlying mixing distribution of the clutter, the clutter covariance matrix, and the steering vector of the desired signal.

I. INTRODUCTION

A high resolution radar (HRR) can spatially resolve a target into a number of scattering centers depending on the range extent of the target and the range resolution capability of the radar [1]. Range resolution is normally inversely proportional to the bandwidth of the transmitted waveform of the radar. Radars that are designated as wideband (WB) or ultrawideband (UWB) inherently have a range resolution capability such that a range profile (or spatial distribution) of the amplitude returns from a target can be formed. Normally one tries first to detect the presence of a target using all of the energy of the target found in the range profile. Thereafter, classification of the target can be attempted using the range profile of the target.

If a target moves during a coherent processing interval (CPI) then it is possible that the scattering centers translate through contiguous range cells (whose width is determined by the range resolution of the radar). This is called range walking. Range walking significantly increases the complexity of the detector of a spatially distributed moving target. We consider here the simpler problem of detecting a spatially distributed moving target that does not range walk but which however is Doppler-shifted during a CPI. The radar transmits a multiple pulse waveform consisting of a train of identical pulses with a constant pulse repetition interval (PRI). This type of waveform enhances the detectability of targets in clutter since the clutter is correlated from pulse-to-pulse and hence can be canceled using a matched Doppler filter bank.

It is known that as the radar range resolution increases the background clutter may no longer be modeled accurately as a Gaussian random variable (RV). At the higher range resolution, the radar system receives target-like spikes that give rise to non-Gaussian observations. In many cases, it has been found that the spiky clutter returns can be suitably modeled by a multidimensional Gaussian mixture distribution or more specifically a spherically invariant random vector (SIRV) [2–9]. Such well-known clutter distributions as the \( K \) and Weibull (with shape parameter \( \leq 2 \)) can be expressed as Rayleigh mixtures [2] (the associated amplitude distribution of a SIRV).

In [2] it was shown that from a reasonable and simple phenomenological model of clutter, using an extension of the Central Limit Theorem (CLT), the clutter returns from multiple pulses of a given range can be modeled as a SIRV distribution.

There have been several papers published previously which examine different aspects of the detection problem of a spatially distributed moving target. These papers make different assumptions regarding either the target or the noise model which results in different detector forms. Reference [10] examines several detector forms for the detection of...
range spread targets. These techniques and others were further examined in [11] for applicability to UWB radar. In [12], the detection of a range spread target in white noise was analyzed. In [13] the target was also considered spread over many range cells. Two detectors were considered: a 1 out of \( M \) detector and a noncoherent integration detector which noncoherently integrates across the range cells. It was shown that when few scatterers are present, the 1 out of \( M \) detector performance dominates the noncoherent integrator whereas when there are many scatterers, the noncoherent integrator gives the best performance. Similar results were obtained by Rose in [14]. In [15–16] Farina, et al. examined more specific techniques for UWB detection and identification. In regards to detection, they derived a likelihood ratio test for the detector under the assumption that the target is a Gaussian-distributed stochastic process with zero mean and a known covariance matrix. The derived likelihood ratio test was a quadratic form with respect to a received signal vector which we represent here by \( x \); i.e., the test takes the form \( x^H Q x \) which is compared with a threshold. The matrix \( Q \) is a function of the covariance matrix of the target and interference. In [17] a detector form was derived using the generalized likelihood ratio test (GLRT). They assumed that the target consisted of a sum of point scatterers with unknown amplitudes and the target could range walk. A maximum likelihood (ML) estimator was used to estimate the scatterer amplitudes. The positions of the scatterers were assumed known in the derivation although simulations were presented that show performance when the scatterer locations are not known exactly. The form of their GLRT was also a quadratic form, with matrix \( Q \) that was different from that given in [15]. A major difference in assumptions between those made in [15] and in [17] was that the target returns were modeled by a stochastic model in the latter paper and were assumed unknown in the former. In both, clutter was modeled as having a Gaussian distribution.

In this work, our contribution is the development of detectors of a spatially distributed, moving (Doppler-shifted but not range walking) target in the presence of additive non-Gaussian clutter which is modeled as a SIRV. Quite often some knowledge of the spatial distribution of the desired target is known \textit{a priori}. For example, it may be known that over the range extent of the target only a small specified number of scattering centers occur. We also develop a detection scheme whereby this \textit{a priori} knowledge is incorporated. It turns out that a simple detector form results which significantly enhances the detection of a sparsely distributed target. This work is an extension of our work in [20], where we derived an effective detector of a nonmoving range-distributed target in white Gaussian noise such that \textit{a priori} knowledge of the spatial distribution is given.

II. PRELIMINARIES

For this analysis, the desired target return is spatially distributed with respect to the transmitted waveform range resolution \( \Delta r \) of the target. Thus the time duration of a target return is longer than the transmitted waveform time resolution \( \Delta t \), where \( \Delta r = c \Delta t \) and \( c \) is the speed of light. A target return consists of one or more scattering centers or for short, scatterers. Let \( \tilde{s}(t) \) denote the transmitted waveform and \( \tilde{S}(\omega) \) its spectrum (the Fourier transform of \( \tilde{s}(t) \)). If \( B \) is the bandwidth of \( \tilde{s}(t) \) then \( 1/B \) is the time resolution of this waveform and \( c/2B = \Delta r \) is the range resolution. We transmit a pulse train consisting of \( N \) identical pulses. We assume that a given target returns to be functions in the range cell (whose extent is determined by \( \Delta r \)) during the time duration of the \( N \) pulses which we call the CPI. Hence the target does not range walk during a CPI. If the speed of the target is \( v_T \), then the condition that ensures that there is no range walking is

\[
\nu_T \cdot \text{CPI} \ll \Delta r. \tag{1}
\]

In Fig. 1, we show curves of the equation \( \nu_T \cdot \text{CPI} = 0.2 \Delta r \). Here we plot \( \Delta r \) (in meters) versus \( \nu_T \) (in meters per sec) for various values of CPI (in ms). Values of \( \Delta r \) and \( \nu_T \) that satisfy (1) for a given CPI are above the straight line curve of \( \Delta r \) versus \( \nu_T \) for a given CPI.

We model the Doppler frequency shift of a moving target. We can represent this effect by phase shifting the target return by a linearly increasing phase shift \( \phi \) on each pulse return. In our development, radar returns have the complex representation. For example, consider the \( N \) returns from a moving point target that satisfies (1). The \( n \)th pulse return from this target is represented as \( e^{j(\omega_0 n - \phi)} \tilde{s}(t) \) where \( j = \sqrt{-1} \).

For notational convenience, we consider the radar returns to be functions in the range domain \( r \) and not the time domain. Assume a given target has a spatial domain transfer function \( h(r) \), so that on a
given pulse the received return from a stationary target has the form \( h(r) \ast \tilde{s}(r) = \tilde{a}(r) \) where \( \ast \) denotes linear convolution and we have written the transmitted waveform as a function of range, i.e., \( \tilde{a}(r) \). For a moving target the \( N \) pulse returns are represented by the vector \( (\tilde{a}(r), e^{j\phi_1} \tilde{a}(r), e^{j2\phi_1} \tilde{a}(r), \ldots, e^{j(N-1)\phi_1} \tilde{a}(r))^T \) where \( T \) denotes transpose. If we assume that the spatial spectrum of the target is given by \( H(\omega) \) and define the spatial spectrum of the waveform to be given by \( \tilde{S}_0(\omega) \), then \( H(\omega)\tilde{S}_0(\omega) \) represents the single pulse spatial spectrum of the target return. If a multiple point scattering model of the target (or clutter) is assumed such that

\[
h(r) = \sum_{j=1}^{J} d_j \delta(r - r_j)
\]

where \( \delta \) denotes the impulse function, \( d_j \) and \( r_j \) are independent zero-mean RVs indicating the complex amplitude and spatial position of the \( j \)th scatterer, respectively, and \( J \) is the number of point scatterers, then it is straightforward to show that \( E\{|H(\omega)|^2\} \) is a constant where \( E\{\cdot\} \) denotes expectation. Thus if the spatial power spectrum of the target is white, i.e., \( E\{|H(\omega)|^2\} = 1 \), then the power spectrum (in the statistical sense) of the single pulse target return is given by \( |\tilde{S}_0(\omega)|^2 \).

We assume the desired signal is corrupted by additive clutter. With no knowledge of the spatial distribution of the clutter, it is reasonable to assume that the single pulse clutter spatial power spectrum is also given by \( |\tilde{S}_0(\omega)|^2 \). The \( N \) pulse clutter returns are modeled as a SIRV distribution. In recent years this has become a standard model for representing non-Gaussian clutter [2–9]. Using this model, the \( n \)th pulse clutter return at range \( r \) can be represented as

\[
\tilde{c}_n(r) = \sqrt{\tau_n(r)} \cdot \eta_n(r)
\]

where \( \eta_n(r) \) is a zero-mean complex circular Gaussian RV with variance equal to 1 and \( \tau_n(r) \) (called the texture RV) is a semipositive real RV with probability distribution \( P_\tau \) (called the mixing distribution). The RV \( \tau_n \) is used to model the large power fluctuations associated with the various clutter levels found in different range cells. Standard clutter models of amplitude distribution, such as K and Weibull, are included in the class of Rayleigh mixture distributions which are related to the class of SIRV distribution [2].

To further characterize the \( N \) RVs \( \tilde{c}_n(r), n = 1, 2, \ldots, N \), an \( N \times N \) normalized clutter covariance matrix \( R_0 \) associated with \( \eta_n(r) \) is defined as

\[
R_0 = E\{\eta \eta^H\}
\]

where \( \eta = (\eta_1(r), \eta_2(r), \ldots, \eta_N(r))^T \) and \( H \) denotes the conjugate transpose. The diagonal elements of \( R_0 \) are equal to one. We assume the \( R_0 \) is known. If \( R_0 \) is unknown, then the derivation of the GLRT is quite complicated, and at present a closed-form solution for the ML estimate of \( R_0 \) under either hypothesis is not possible. Thus, we leave this as a topic of future research. The assumption that \( R_0 \) is known is often made when deriving moving target indicators (MTIs) or matched Doppler processors of desired signals in clutter. The spectrum of the clutter is generally assumed to be Gaussian with a given mean and variance. Hence the elements of \( R_0 \) are readily derived for the associated autocorrelation function of the clutter spectrum.

We desire to devise a detector to distinguish between the two hypotheses

\[
H_1 : x_n(r) = e^{j(n-1)\phi_1} a(r) + \tilde{c}_n(r) \quad n = 1, 2, \ldots, N
\]

\[
H_0 : x_n(r) = \tilde{c}_n(r)
\]

where \( H_1 \) is the signal-plus-clutter hypothesis and \( H_0 \) is the clutter-only hypothesis.

Finally, we mention that the effects of internal noise are not analyzed in this development. A standard method of processing a signal in clutter is to reduce the effects of clutter by match filtering the \( N \) input pulses: one multiplies the \( N \)-length input vector at a given range by an \( N \)-length weighing vector given by \( R_0^{-1} s_0 \) where \( s_0 \) is the steering vector of the desired signal and \( \ast \) denotes conjugation. If the resultant clutter in the matched filtered output is small with respect to the internal noise contribution, then the detection schemes that are described in the following sections are not needed. One merely match filters and threshold detects on the output magnitude. However, if the resultant clutter residue power is significantly greater (approximately 10 dB) than the internal noise contribution after match filtering, then it was found that the internal noise contribution had little effect on detector performance and hence the subsequent detection schemes have application. The detector which results from the case when the resultant clutter residue is not significantly greater than the internal noise (in the range of –3 dB to 10 dB) is left to a topic of future investigation.

III. GENERALIZED LIKELIHOOD RATIO TEST

For a single pulse, the clutter component of the samples in range is correlated and has a spatial correlation function given by the inverse Fourier transform of \( |\tilde{S}_0(\omega)|^2 \). Since the input data is sampled at times that are uniformly spaced, the spatial covariance matrix of the data can be found via the spatial correlation function. We assume that the individual pulses are passed through a spatial whitening filter and thereafter detection takes place as depicted in Fig. 2. Hence the clutter component of the samples in range is statistically decorrelated. The spatial whitening filter essentially removes the correlation from range sample to range sample due
to overlapping of the waveform returns from the individual scatterers. We assume that the underlying texture component in range is independent from range cell to range cell. Hence, after spatial whitening the texture component on each range sample is an independent RV with respect to the other range samples. The reason behind spatially whitening the input clutter is so that we can conveniently write down its space/time dependent multidimensional probability density function (pdf). Because all of the RVs are conditionally Gaussian (conditioned on the texture RV), spatially whitening the clutter allows us to write down the joint space/time pdf conditioned on the texture RV as a product of the pdfs conditioned on the texture RV at each range. This will be seen in the subsequent development. It is pointed out that matched filtering in range on receive (i.e. convolving \( s(r) \) with \( s^*(r) \)) is not used because the target and clutter have the same power spectrum.

For digital processing, this spatial whitening takes the form of a whitening matrix derived from the Cholesky decomposition of the covariance matrix (associated with the spatial correlation function) of the input samples in range on a single pulse. Assuming that this matrix is nonsingular, then the transformation of the input data is reversible and the likelihood ratio test (no unknown parameters) is invariant to this transformation. We point out that for this development, the width of the range cell is restricted to be approximately equal to the range resolution of the radar. If heavy oversampling were permitted, then even though the whitening matrix decorrelates the clutter and signal in the sampled range cells, the internal noise, which has been ignored for this development, could become quite large when transformed through the whitening matrix.

After the spatial whitening transformation, let the \( N \) pulse returns from the moving target be represented by \( (a(r), e^{j0}a(r), e^{j2\theta}a(r), \ldots, e^{j(N-1)\theta}a(r))^T \) where \( a(r) \) is the spatially whitened output of the first pulse. Also after this transformation, let \( c_\theta(r) \) be the spatially whitened clutter on the \( n \)th pulse. Again, because this is a linear transformation, the \( N \) components of clutter can be modeled as a SIRV distribution with a representation given by (3). In this case though, \( \tau_\theta(r) \) and \( \eta_\theta(r), (n = 1,2,\ldots,N) \) are assumed to be independent RVs from range cell to range cell. Obviously, this assumption becomes invalid when the range resolution becomes extremely small since the texture RVs will eventually become significantly correlated. At this time we cannot give a range resolution value for when the texture RV becomes moderately correlated. The covariance matrix of \( \eta_\theta(r), (n = 1,2,\ldots,N) \) is given by \( R_0 \).

Let \( z_n(r) \) be the output of the spatial whitening transformation for the \( n \)th pulse. Set \( z(r) = (z_1(r), z_2(r), \ldots, z_N(r))^T \), \( c(r) = (c_1(r), c_2(r), \ldots, c_N(r))^T \) and \( s_\theta = (1, e^{j0}, e^{j2\theta}, \ldots, e^{j(N-1)\theta})^T \). After the spatial whitening transformation, we desire to devise a detector to distinguish between the two hypotheses

\[
H_1 : z(r) = a(r)s_\theta + c(r) \\
H_0 : z(r) = c(r)
\]

For each pulse, we assume that there are \( J \) range resolution cells which the target scatterers can occupy. Many more range cells than \( J \) are formed by the radar. However, we assume that our target is completely contained within a range window equal to \( J \) resolution cells. We denote a given range cell by \( r \) and the set of range indices by \( \Omega_r \). It is assumed that the underlying mixing distribution \( P_z \) is unknown. Hence as the detector designers, we assume that the clutter is Gaussian with an unknown variance (or power) for each range cell. Note however that the input clutter samples are still non-Gaussian SIRVs. It is only because \( P_z \) is unknown that forces us as the detector designers to model the input range variances as nonrandom variables and the input clutter as conditionally Gaussian. We also assume the \( a(r) \) is unknown but that \( s_\theta \) is known. The joint pdf of the elements of \( z(r) \) under each hypothesis is given by

\[
p(z(r) | H_0) = \prod_{\Omega_r} \frac{c}{\tau^r(r) \det(R_0)} \times \exp \left[ -\frac{1}{\tau} \tau^H(r) R_0^{-1} z(r) \right] \\
p(z(r) | H_1) = \prod_{\Omega_r} \frac{c}{\tau^r(r) \det(R_0)} \times \exp \left[ -\frac{1}{\tau} (z(r) - a(r)s_\theta)^H R_0^{-1} (z(r) - a(r)s_\theta) \right]
\]

where \( \det() \) denotes determinant and \( c \) is the pdf normalization constant.

For the GLRT [10], the unknown parameters under each hypothesis are found via ML estimation. Thereafter the ML estimates under each hypothesis are substituted into (7) or (8) and the ratio of the resultant expressions given by (8) and (7) is formed. The resultant ratio is called the GLRT.

It is straightforward to show that the ML estimate of \( \tau(r) \) under each hypothesis is given by

\[
H_0 : \tau_{\text{ML}}(r) = \frac{1}{N} z^H(r) R_0^{-1} z(r) \\
H_1 : \tau_{\text{ML}}(r) = \frac{1}{N} (z(r) - a(r)s_\theta)^H R_0^{-1} (z(r) - a(r)s_\theta)
\]

Fig. 2. Spatial whitener-detector configuration.
The ML estimate of \( a(r) \) under \( H_i \) is given by

\[
d_{\text{ML}}(r) = \frac{s_{\text{ML}}^HR_0^{-1}z(r)}{s_{\text{ML}}^HR_0^{-1}s_o^*}. \tag{11}
\]

Substituting (10) and (11) into (8), and (9) into (7), it can be shown that an equivalent form (any strictly monotonically increasing function of the detection statistic, [10]) for the GLRT is

\[
\lambda = -2(N-1)\sum_{\Omega_s} \ln \left( 1 - \frac{|s_{\text{ML}}^HR_0^{-1}z(r)|^2}{(z^H(r)R_0^{-1}z(r))(s_{\text{ML}}^HR_0^{-1}s_o^*)} \right). \tag{12}
\]

The GLRT takes the form

\[
\frac{H_0}{H_i} \lambda \geq T. \tag{13}
\]

It is interesting to note that the argument of the \( \ln \) function seen in (12) takes the form of the detector derived in [18 and 21]. For the developments in [18 and 21], the target was not spatially distributed.

IV. GLRT FALSE-ALARM PROBABILITY

Under the \( H_0 \) hypothesis, it is straightforward to show that the GLRT statistic \( \lambda \) is independent of \( s_o \) and \( R_0 \). Furthermore, \( \lambda \) is independent of the underlying mixing distribution of \( r : P_r \). This is because the multiplier \( \tau(r) \) cancels itself out of the numerator and denominator of the second term of the \( \ln \) argument seen in (12).

There exists an \( N \times N \) matrix \( A \) such that 1) if \( z(r) \) is multiplied by this matrix then the elements of \( z(r) \) are statistically uncorrelated and equi-powered with \( \sigma = 1 \), and 2) transforms \( s_o \) into \((s_{\text{ML}}^HR_0^{-1}s_o)^{1/2}I_0 \) where \( I_0 = (1 \ 0 \ \cdots \ 0)^T \), [19]. Let \( y(r) = Az(r) \). It can be shown that \( \lambda \) given by (12) has the same pdf as

\[
\lambda = -2(N-1)\sum_{\Omega_s} \ln \left( 1 - \frac{|y_1(r)|^2}{\sum_{n=1}^N |y_n(r)|^2} \right) \tag{14}
\]

where \( y_1(r), y_2(r), \ldots, y_N(r) \) are independent and identically distributed (IID) zero-mean complex circular Gaussian RVs with variance equal to one. Also the \( y_n(r) \) over \( \Omega_s \) are IID RVs. Equation (14) can be simplified to

\[
\lambda = -2(N-1)\sum_{\Omega_s} \ln \left[ 1 + \frac{|y_1(r)|^2}{\sum_{n=2}^N |y_n(r)|^2} \right]^{-1}. \tag{15}
\]

Set

\[
u(r) = \left( 1 + \frac{|y_1(r)|^2}{\sum_{n=2}^N |y_n(r)|^2} \right)^{-1}. \tag{16}
\]

and

\[
v(r) = -2(N-1)\ln u(r). \tag{17}
\]

The second term inside the parenthesis in (16) is recognized to have the well-known \( F \) distribution with numerator/denominator parameters 1 and \( N-2 \), respectively. It is easily shown that the pdfs of \( u(r) \) and \( v(r) \) are

\[
p_u(u) = \begin{cases} 
(N-1)u^{N-2} & 0 \leq u \leq 1, \\
0 & \text{otherwise},
\end{cases} \tag{18}
\]

\[
p_v(v) = \begin{cases} 
\frac{1}{2}e^{-v/2} & v \geq 0, \\
0 & \text{otherwise}.
\end{cases} \tag{19}
\]

We recognize \( u \) as having a Beta distribution and \( v(r) \) as a chi-square, order 2 RV with \( \sigma = 1 \) (i.e., \( v = x_1^2 + x_2^2 \) where \( x_1, x_2 \) are IID zero-mean Gaussian RVs with \( \sigma = 1 \)). Hence

\[
\lambda = \sum_{\Omega_s} v(r) \tag{20}
\]

is a chi-square, order \( 2J \) RV with the \( \sigma \) parameter equal to one. Thus for a chosen false-alarm probability \( P_f \) the threshold is easily computable.

V. SCATTERER DENSITY DEPENDENT GLRT

For the detector developed in the previous sections, it was implicitly assumed that the target scatterers occupied all of the \( J \) range cells. However, in many cases of target scattering, the scatterers may occupy only a fraction of the \( J \) range cells. There are what is commonly called “hot spots” on a target, i.e., range cells along the spatial extent of a target whose amplitudes are significantly greater than the other range cells. If we process and detect the input data which has sparse target scattering using the previously derived detector (non-scatterer density dependent (NSDD) GLRT) which assumes that target scattering occurs in each range cell, then we expect some loss of detection performance. This loss of detection performance due to no target scattering in some of the processed range cells, is called “collapsing loss” [22]. For detectors which take the form of an integrator (or sum) over a specified number of range cells (the NSDD-GLRT has this form), range cells which have no target scatterers in them contribute just noise to the integration test statistic. Hence, there is a loss of detection performance which is a function of the percentage of range cells that have no scattering. In this section we develop a detector of spatially distributed, Doppler-shifted target in clutter which incorporates \textit{a priori} knowledge about the spatial scattering density of the target scatterers. We find that the new detector has superior performance to the NSDD-GLRT. We point out that it was observed via simulation that the spatial whitening that precedes the
detector tends to localize the hot spots in range even further since the spatial whitening approximates the continuous-time spatial whitening filter.

There are J possible range cells that the scatterers of the target can occupy. The probability that there are j target scatterers where j = 1, 2, ..., J is denoted by P_j. If there are j target scatterers, each scatterer occupies only one of the J possible range cells. Any combination of the j scatterers occupying the J range cells is equally likely. Let \( \Omega_j \) denote the set of combinations of range indices where only j out of the J range cells have a scatterer present. For example if j = 2, J = 3, then \( \Omega_2 = \{(1,2),(1,3),(2,3)\} \). Let \( q_j \) be a member of \( \Omega_j \). If \( \lambda(q_j) \) is the GLRT of the hypothesis test of (6) given that target scattering is present only in the range indices indicated by \( q_j \), then a composite GLRT over all scattering combinations is given by

\[
\lambda = \sum_{j=1}^{J} P_j \sum_{q_j} \binom{J}{j}^{-1} \lambda(q_j) \tag{21}
\]

where \( \binom{J}{j} \) denotes the binomial coefficient \( J! / [(J-j)!j!] \).

It is readily shown that the GLRT conditioned on the knowledge of \( q_j \) is given by

\[
\lambda(q_j) = c_0 \exp \left[ -N \sum_{s \in q_j} \ln \left( 1 - \frac{|s^H R_0^{-1} z_j|^2}{(\bar{z}^H R_0^{-1} z)(s^H R_0^{-1} s_0)} \right) \right] \tag{22}
\]

where the summation given above is taken over the range indices of \( q_j \), and \( c_0 \) is a constant (this result is derived in the same manner that (12) was derived).

We now assume a scattering density for \( P_j \), j = 1, ..., J, which has utility in modeling a small number of scatterers and which greatly simplifies the form of the GLRT given by (21) and (22). Let

\[
P_j = c_1 \binom{J}{j} (1-\alpha)^{J-j} \alpha^j, \quad j = 1, ..., J \tag{23}
\]

where \( c_1 = (1-(1-\alpha)^J)^{-1} \) is a constant which satisfies the normalization condition \( \sum_{j=1}^{J} P_j = 1 \) and \( \alpha \) is a parameter chosen to control the spatial density of the scatterers. We observe that \( P_j \) is approximately equal to the binomial distribution with parameter \( \alpha \).

We choose the parameter \( \alpha \) based on the expected number of scatterers \( j_0 \). For the distribution given by (23), the average number of scatterers is given by \( J\alpha[1-(1-\alpha)^J]^{-1} \). Thus we set

\[
j_0 = J\alpha[1-(1-\alpha)^J]^{-1} \tag{24}
\]

and solve for \( \alpha \). For \( j_0/J \ll 1, \alpha \approx j_0/J \). Hence for a small number of scatterers, \( \alpha \) is small. If any scattering combination is equally likely, \( \alpha = 0.5 \) and for a large number of scatterers where \( j_0 \approx J, \) then \( \alpha \approx 1 \).

If \( P_j \) given by (23) is substituted into (21), we can show that (21) simplifies to

\[
\lambda = c_0 c_1 \prod_{j=1}^{J} (1 - \alpha + \alpha e^{j_f}) - c_0 c_1 \tag{25}
\]

where

\[
f_j = -N \ln \left( 1 - \frac{|s^H R_0^{-1} z_j|^2}{(\bar{z}^H R_0^{-1} z)(s^H R_0^{-1} s_0)} \right) \tag{26}
\]

and \( z_j \) is the N-length vector return from the jth range cell. The expression for \( \lambda \) can be further simplified resulting in the final equivalent detection statistic:

\[
\lambda = \sum_{j=1}^{J} \ln \left( 1 + \beta \left( 1 - \frac{|s^H R_0^{-1} z_j|^2}{(\bar{z}^H R_0^{-1} z)(s^H R_0^{-1} s_0)} \right)^{-N} \right) \tag{27}
\]

where

\[
\beta = \alpha/(1-\alpha). \tag{28}
\]

From (27), we can show that the false alarm probability \( P_f \) is independent of the underlying mixing distribution, the clutter covariance matrix \( R_0 \), and the steering vector \( s_0 \) of the desired signal. \( P_f \) does depend on \( \beta, J, \) and \( N \). Unlike the NSDD-GLRT derived in the previous sections, the test statistic \( \lambda \) of this detector does not have a well-known distribution. As a result, the \( P_f \) must be estimated for a given threshold using Monte Carlo methods. Finally, we note that as \( \beta \rightarrow \infty \), the detection form of (27) reduces to an equivalent form of (12). This is expected because as \( \beta \rightarrow \infty \), every range cell is occupied which was the implicit condition under which (12) was derived.

VI. RESULTS

In this section, we present some performance results for the NSDD-GLRT developed in Section III and the scatterer density dependent GLRT (SDD-GLRT) developed in Section V. For the NSDD-GLRT results, we assume that target scattering occurs in every range cell. The target scattering in every range cell is modeled as a zero-mean complex circular Gaussian RV with power equal \( \sigma^2_t \). These J RVs are IID. For a given range cell, the target scatterer RV is constant over the N received pulses. For the NSDD-GLRT, it is elementary to show that the probability of detection \( P_d \) depends on \( (\sigma^2_t/\sigma^2_c)s^H R_0^{-1} s_0 \) where \( \sigma^2_c \) is equal to the average clutter power per range cell, the number of range cells J, the number of pulses N, the false alarm probability \( P_f \), and the underlying mixing distribution \( P_j \). Thus \( P_d \) depends on the steering vector \( s_0 \) and the normalized clutter covariance \( R_0 \) through the single parameter \( (\sigma^2_t/\sigma^2_c)s^H R_0^{-1} s_0 \). The quantity \( (\sigma^2_t/\sigma^2_c)s^H R_0^{-1} s_0 \) is actually equal to the
output signal-to-clutter power ratio of the matched filter weighting of the $N$ pulses: $R_0^{-1}s^Hs$. This is the optimal linear weighting which maximizes the output signal-to-clutter power ratio. Thus we set $(S/C)_{\text{opt}} = (\sigma_s^2/\sigma_c^2)s^H R_0^{-1}s$.

We point out that we could have defined as an input parameter, $(S/C)_{\text{opt}} = (\sigma_s^2/\sigma_c^2)s^H R_0^{-1}s$, where $S/C$ equals the ratio of the total input signal power to the total input clutter power over the $J$ range cells. However, since $S/C = J \sigma_s^2/(J \sigma_c^2) = \sigma_s^2/\sigma_c^2$, we chose to define $(\sigma_s^2/\sigma_c^2)s^H R_0^{-1}s$ as an input parameter.

For the underlying mixing distribution, we choose the gamma distribution with pdf
\[
p_s(\tau) = \frac{(L/b)^L}{\Gamma(L)} \tau^{L-1} e^{-(L/b)\tau}, \quad \tau \geq 0
\]
where $\Gamma$ is the gamma function, $b$ determines the mean of the distribution, and $L$ controls the deviation from Gaussian statistics. As $L$ goes to infinity, $p_s(\tau)$ approaches an impulse function centered at $b$. Hence the resultant multidimensional Gaussian mixture distribution is a pure multidimensional Gaussian distribution. The smaller $L$ is, the larger the tails of the distribution are and the more spiky the clutter will appear. Without loss of generality, $b$ is set equal to one, which merely normalizes the elemental clutter power to a specific value. With this mixing distribution, the statistics of the univariate amplitude $x$ of a clutter cell are described by the $K$ distribution:
\[
p_s(x) = \frac{2Lx}{\Gamma(L)} (x\sqrt{L/2})^{L-1} K_{L-1}(x/\sqrt{2L})
\]
where $K_L$ is a modified Bessel function of the second kind. The $K$ distribution is often used to model non-Gaussian sea clutter [7].

We present detection results for the NSDD-GLRT in Figs. 3–5. For all results, the number of Monte Carlos used to estimate each $P_D$ was 1000. In Fig. 3, we show plots of $P_D$ versus $(S/C)_{\text{opt}}$ for various numbers of range cells ($J = 2, 10$, and $50$), $N = 5$, $L = 1$, and $P_F = 10^{-6}$. Here we observe that there is significant improvement in detection performance by increasing the number of cells that a target is resolved. There is also a large detection performance improvement when the number of pulses used is increased as exemplified by the curves seen in Fig. 4 where we present curves of $P_D$ versus $(S/C)_{\text{opt}}$, $N = 2, 5$ and $10$, $J = 10$, $L = 1$, and $P_F = 10^{-6}$. In Fig. 5, we examine the effects of the spikiness of the clutter by varying $L$ (the smaller $L$ is the higher the tails of the clutter distribution). In Fig. 5, we plot $P_D$ versus $(S/C)_{\text{opt}}$, $L = 1, 2, 10, 30$, $N = 5$, $J = 10$, and $P_F = 10^{-6}$. We observe that the detection performance improves for this detection scheme as $L$ becomes smaller (or the clutter becomes more spiky). This
For the NSDD-GLRT in this sparse scattering scenario, SDD-GLRT performance is significantly better than a single range cell. From Fig. 6, we observe that the probability of detection of SDD-GLRT and NSDD-GLRT is higher for the same signal-to-clutter power ratio taken over any given Monte Carlo. Only one scatterer can occupy a single range cell. From Fig. 6, we observe that the SDD-GLRT performance is significantly better than the NSDD-GLRT in this sparse scattering scenario. For $P_D \geq 0.5$, the required $(S/C)_{opt}$ for a given $P_D$

same result was seen in [18] for the single range cell ($J = 1$) detector.

For the SDD-GLRT performance results, we use the same clutter model as the NSDD-GLRT. For the target modeling, we specify $J_0$ which is the actual and expected number of range cells which have scatterers. We model each of the $J$ range cells as having a clutter component and only $J_0$ range cells as having a signal component. The signal-to-clutter power ratio in a cell that has a target scatterer is given by $(J_0/\bar{J}_0)\sigma^2_s/\sigma^2_c$ where $(\sigma^2_s/\sigma^2_c)$ is the average signal-to-clutter power ratio per range cell taken over $J$ range cells. The returns from cells that have target scattering are modeled as IID zero-mean complex circular Gaussian RVs with $\sigma^2 = (J/\bar{J}_0)\sigma^2_s$. Hence, the total target power is $J\sigma^2_s$ which was the case when we modeled the target for the NSDD-GLRT. For a given range cell, if target scattering is present, the target scatterer RV is constant over the $N$ received pulses.

In Fig. 6, we compare the detection performance of the SDD-GLRT with the NSDD-GLRT for a single representative case. For the two detectors we plot $P_D$ versus $(S/C)_{opt}$ (equals $(\sigma^2_s/\sigma^2_c)(s^\dagger R_0 s)$) for $N = 2$, $J = 50$, $L = 1$, and $P_F = 10^{-4}$ and various values of $\beta$ corresponding to the expected number of scatterers: $J_0 = 3 (\beta = 0.64)$, $J_0 = 4 (\beta = 0.087)$, $J_0 = 5 (\beta = 0.111)$, and $J_0 = 10 (\beta = 0.25)$. For example, for the SDD-GLRT, $\beta$ is calculated using (24) and (28) and values of $J_0$ and $J$. The actual number of scatterers used in the simulation is 3. Any combination of the 3 scatterers across the 50 range cells is equally likely on any given Monte Carlo. Only one scatterer can occupy a single range cell. From Fig. 6, we observe that the SDD-GLRT performance is significantly better than the NSDD-GLRT in this sparse scattering scenario. For $P_D \geq 0.5$, the required $(S/C)_{opt}$ for a given $P_D$ level is 7–10 dB less for the SDD-GLRT than for the NSDD-GLRT. In addition, we have plotted the detection performance of the SDD-GLRT for various values of $\beta$ which are mismatched to the expected number of scatterers $J_0$ or equivalently the expected $\beta$. Here we see a graceful degradation in performance when there is a mismatch. A slight improvement in performance is observed when $\beta = 0.087$. Hence the SDD-GLRT is fairly robust even if we moderately err in our estimate of the expected number of scatterers.

VII. CONCLUSIONS

Two detection schemes for the detection of a spatially distributed, Doppler-shifted target in non-Gaussian clutter have been developed. The non-Gaussian clutter was modeled as a multidimensional Gaussian mixture distribution (more specifically a SIRV). For the first detector, called the NSDD-GLRT, the detector takes the form of a sum of logarithms of identical functions of data from each individual range cell. It was shown under the clutter-only hypothesis, that the detection statistic has the Chi-square distribution so that the detection threshold is easily calculated for a given probability of false alarm $P_F$. In addition, under the clutter-only hypothesis, $P_F$ is independent of the underlying mixing distribution of the clutter, the normalized clutter covariance matrix, and the desired signal vector. The detection probability $P_D$ was shown to be only a function of the signal-to-clutter power ratio $(S/C)_{opt}$ of the matched filter, the number of pulses received $N$, the number of target range resolution cells $J$, the spikiness of the clutter determined by a parameter of an assumed underlying mixing distribution, and $P_F$. For representative examples, it is shown that as $N$, $J$, or the clutter spikiness increase, detection performance improves.

A second detector was developed which incorporated a priori knowledge of the spatial scatterer density. This detector was called the SDD-GLRT and also has the form of a sum of logarithms of identical functions of data from each individual range cell. It was shown for a representative case to improve significantly the detection performance relative to the performance of the NSDD-GLRT. The performance SDD-GLRT was also shown to be robust if the expected number of scatterers is moderately misestimated. In addition, it was shown under the clutter-only hypothesis that $P_F$ is independent of the underlying mixing distribution of the clutter, the normalized covariance matrix, and the desired signal vector. Hence both the SDD-GLRT and NSDD-GLRT exhibit the CFAR capability with respect to these quantities.
REFERENCES

High-Resolution Radar.  

Coherent detection of radar targets in a non-Gaussian background.  

Radar properties of non-Rayleigh sea clutter.  

Non-Rayleigh sea clutter: Properties and detection of targets.  

A model for non-Rayleigh sea echo.  

Compound representation of high resolution sea clutter.  

Significance of K distributions in scattering experiments.  

Characterization of radar clutter as a spherically invariant random process.  

Computer generation of correlated non-Gaussian radar clutter.  


On the detection of ultrawideband radar signals.  

Ultrawideband radar detection in white noise.  
In L. Carin et al. (Eds.), Ultrawideband Short-Pulse Electromagnetics 3.  

A high-resolution radar detection strategy.  

A look at automatic detection algorithms for a X-band radar using a high resolution search mode.  

Radar detection of correlated targets in clutter.  

Detection with high resolution radar: Advanced topics and potential applications.  

Moving target detection performance potential of UWB radars.  

Asymptotically optimum radar detection in compound Gaussian clutter.  

Convergence properties of Gram–Schmidt and SMI adaptive algorithms.  

Detection of a spatially distributed target in white noise.  

Sub-optimum coherent radar detection in a mixture of K-distributed and Gaussian clutter.  

Radar Systems Analysis.  

Karl Gerlach (M’81) was born in Chicago, IL. He received his B.S. in 1972 from the University of Illinois, Urbana, and his M.S. and D.Sc. from George Washington University, Washington, DC, in 1975 and 1981, respectively. All degrees are in electrical engineering.

Since 1972, he has been employed by the Naval Research Laboratory in Washington, DC. From 1972 to 1976, he worked on experimental submarine communications systems and from 1976 to the present he has been with the Radar Division where his research interests include adaptive signal processing and space-based radar.

Dr. Gerlach was the 1986 recipient of the IEEE AESS Radar Systems Panel award.