Control of a Large Space Structure Using MMAE/MMAC Techniques

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The application of moving-bank multiple model adaptive estimation and control (MMAE/MMAC) algorithms to an actual space structure (Space Integrated Controls Experiment (SPICE)) being examined at Phillips Laboratory at Kirtland AFB, NM, is presented. The structure consists of a large platform and a smaller platform connected by three legs in a tripod fashion. Kalman filtering and LQG (linear system, quadratic cost, Gaussian noise) control techniques are utilized as the primary design tools for the components of the MMAE/MMAC. Implementing a bank of filters or controllers increases the robustness of the algorithms when uncertainties exist in the system model, whereas the moving bank is utilized to reduce the computational load. Several reduced-order models are developed from the truth model using modal analysis and modal cost analysis. The MMAE/MMAC design with a substantially reduced-order filter model provides an excellent method to estimate a wide range of parameter variations and to quell oscillations in the structure.

I. INTRODUCTION

This work presents the use of a moving-bank multiple model adaptive estimator (MMAE) and LQG-based (linear system, quadratic cost, Gaussian noise), multiple model adaptive controller (MMAC) to control a large flexible space structure (Space Integrated Controls Experiment, or SPICE), with the primary goal being to quell unwanted vibrations induced in the structure (to yield a maximum rms line-of-sight (LOS) angular level of one micro-radian). Utilizing such an adaptive control law is motivated by the inability of nonadaptive controllers to maintain closed loop stability and performance in the presence of rather small variations in some uncertain parameters of the system [5, 11]. First, a brief overview of the physical structure is presented, along with the corresponding system model. Next, the fundamental concepts and design of the moving-bank MMAE/MMAC algorithm are discussed. Finally, results from the system model simulation analysis are presented.

II. STRUCTURE

The SPICE structure [5, 11] is divided into three major structural sections. The hexagonal base referred to as the bulkhead forms the support for the entire structure and is 6.19 m in diameter. The primary mirror (PM) assembly is mounted on top of the bulkhead. Three legs (tripod) connect the bulkhead to the secondary mirror (SM) assembly. The SM Assembly is 1.32 m in diameter. The overall height of the structure is 8.14 m. Maintaining the alignment of the SM Assembly and the bulkhead is the primary concern of this research. An exaggerated example of the SPICE structure exhibiting a misalignment due to its flexible bending modes is illustrated in Fig. 1. Note that (for the purposes of this research) the alignment is not altered by a pure torsion force about the LOS axis.

Actuators provide the control force necessary to quell the structural vibrations. The specific actuator utilized is referred to as a proof mass actuator (PMA). The PMA uses a proof mass that is electromagnetically moved to counteract the bending motion of the structure at the location of the PMA. Eighteen PMAs are mounted on the structure. There are 6 PMAs located such that there is one on the vertical spar at each of the hexagonal corners of the bulkhead, each pointing in the Z direction. The tripod legs house the remaining 12 PMAs. Each leg has two sets of two PMAs mounted along the local orthogonal coordinate axes and optimally located approximately one-third and two-thirds up the length of the leg, respectively. Three different types of sensors are being used to provide measurement information. First, there are a total of 54 accelerometers, separated...
into 18 essentially colocated sets of 3 (one set per PMA), which measure the bending motion of the structure. Each set contains 2 Wilcoxin high frequency accelerometers and 1 Sundestrand low frequency accelerometer. One Wilcoxin accelerometer is physically mounted on each PMA proof mass. The remaining two are colocated on the physical structure at the point of attachment and along the reference axis of each PMA. The second type of sensor is the linear variable differential transformer (LVDT), which provides a differential position measurement of the PMA proof mass with respect to the structure. The third type of sensors are the elements of the optical scoring system (OSS) which provide LOS measurements between the two mirror assemblies. This system uses precisely placed laser and sensor pairs to determine a change in position of a laser with respect to the corresponding sensor. The first of two disturbance types (used for ground testing of the SPICE structure) enters at the base of each tripod leg at the attachment to the bulkhead (actually two highly correlated disturbances enter each tripod leg). The second type of disturbance enters each tripod leg at the SM assembly (one input per tripod leg).

III. SYSTEM MODEL

The full-order system model (a highly complex and accurate representation of the actual physical structure) is composed of an extremely high number of states (~ 1000). The complexity and excessive size of this model would prohibit its use as a “truth” model for simulation. Thus, reducing the total number of states becomes a necessity. Yet, the dominant structure modes and system components that will still qualify the model as a “true” representation of the physical structure must be maintained. In the following paragraphs, the original system model is addressed briefly, along with the underlying assumptions and justifications that result in the truth model selection depicted in Fig. 2. The PMA fcmd (force command) refers to the control inputs to the PMAs from the combined feedback loops. In the feedback loop portion, the PMA LAC (low authority control) damping refers to simple rate feedback for the structure, whereas the PMA local damping refers to localized damping for the PMAs.

**Disturbances:** The shaping filter [7] that forms the disturbance inputs (same for both types except for gain changes) is composed of a fourth-order bandpass model that passes six scalar white noise inputs ($w_n$) with equivalent statistics over the 5–10 Hz frequency band. The noise strengths are such that the structure achieves a 100 micro-rad open-loop rms LOS error (this assumes clamping of the proof masses in the PMAs). This complete model was maintained and contributes 24 states to the truth model.

**Structure:** The structure portion of Fig. 2 refers to the dynamic response of the passive flexible bending modes of the system and of the active control imparted by the PMAs. The flexible body portion of the structure model was developed from finite element analysis and modal test data [1, 2]. The full-order flexible body portion is composed of 194 modes (modal coordinate form), with undamped natural frequencies that range from 7 to 150 Hz. Truncating this portion of the system model to 100 Hz eliminates 86 flexible body modes, resulting in 216 states in the truth model. The underlying assumption is that any modes at higher frequencies will be passively quelled very rapidly and essentially have no effect on the overall system. The resulting frequency range (5–100 Hz) forms the total frequency range of interest to be considered in this research. Since the PMAs are essentially spring-mass systems, they are appropriately modeled as second-order systems, each with a damping ratio of ~ 0.01 and natural frequency of ~ 5 Hz. These 36 states are maintained in the
truth model. Additionally, the $X$ and $Y$ LOS outputs result from a transformation of the outputs of the LOS optical elements (OSS).

**Measurements and Feedback Loops:** In the original system model, a large proportionate number of overall system states are contributed by the sensor, noise, and feedback loop models. A thorough description of these models can be found in [11, ch. 3]. Elimination of these models based on several assumptions would decrease the state order significantly. The MMAC feedback loop utilizes three sensors: the Sundestrnad accelerometer, the Wilcoxin accelerometer, and the OSS. The Sundestrnad model is essentially a low pass filter with a break frequency well above the frequency range of interest ($\geq 100$ Hz). The Wilcoxin accelerometer model is essentially a high pass filter with a break frequency below the frequency range of interest ($< 5$ Hz). Viewed together, the combination of their outputs can be treated as a unity gain bandpass encompassing the entire frequency range of interest, where the attenuation outside of this range is ignored. Thus, they can be thought of as providing “perfect” measurements in the sense that there is no attenuation in the frequency range of interest. The OSS requires no states for modeling purposes. This same justification is made for each of the other feedback loop input measurement devices [11, ch. 3]. Additionally, this same situation loosely applies for the colored noise added to each of the respective sensors. Thus their replacement by white noise is justified for the same reasons as above, where the strength of the white noise is based on an averaged power spectral density value over the frequency band of interest. In the physical system, acceleration and position measurements are taken, which must be integrated and differentiated respectively to gain the required rate feedback. However, in the mathematical representation, direct “velocity measurements” are possible for each of the sensor outputs, thereby eliminating the need for integrators or differentiators in the truth model. Finally, other various models and noise shaping filters are assumed to have break frequencies well beyond the frequency range of interest or to have negligible impact to system performance and are eliminated from the truth model [11, ch. 3].

The feedback loop structure illustrated in Fig. 2 is the design approach adopted by Lockheed LMSC (the primary controls contractor on the SPICE project) with the exception of a single robust Kalman filter/controller in place of the MMAC block [1, 2]. The first and second tiers are maintained in this research based on the advice from Lockheed [1]. The first tier, PMA local damping force, provides for localized damping feedback for the PMAs. The second tier, PMA LAC damping force, provides simple rate feedback for the entire structure. The final tier is the MMAC design adopted in this research which provides feedback for attaining the desired performance specifications. Of special note is that only the outputs of the structure accelerometers and OSS are utilized by the MMAC loop. The existence of the first-order low pass filter on the control inputs from the MMAC block is based on the advice of Lockheed [1]. This filter provides for a 10 Hz rolloff on the MMAC loop, which is needed for stability [1] and contributes 18 states to the truth model.

**Truth Model:** The final version of the truth model (for simulation of the real-world system) has 294 total states: 24 states from the disturbances, 36 states from the PMAs, 18 states from the MMAC output filter, and 216 states from the structure flexible bending modes. Additionally, a “truth model-based MMAC” was implemented to provide a benchmark of performance for evaluating MMACs based on reduced-order models.

**Reduced-Order Design Models:** Since the state order magnitude is dominated by the number of flexible body modes, this is the primary focus of order reduction efforts. The disturbance states are maintained since the output colored noise is shaped mainly in the frequency range of the lowest (considered dominant) flexible bending modes [1]. Two order reduction techniques were investigated for reducing the number of flexible body modes: modal reduction [4] and Skelton’s component (modal) cost analysis [12]. Only the resulting most effective reduced-order filter models are addressed in this work. The results from other models utilizing lesser orders of magnitude in the structure block can be found in [11, ch. 5]. For both order reduction techniques, filter models were developed encompassing only 26 structural bending modes and are referred to as the 26-mode modal model and 26-mode modal-cost model, respectively. Thus, the resulting state order of the controller design model is 130.

**IV. MULTIPLE MODEL ADAPTIVE ESTIMATION AND CONTROL**

The need for any type of adaptive algorithm arises from uncertainties in the system under consideration. In this application, a space structure is subject to severe variations in day and night temperatures, possible damage, physical aging, and fatigue [13]. Thus, the undamped natural frequencies parameter ($\omega_n$, a scalar by which all modal natural frequencies are assumed to be multiplied) of the flexible bending modes of the structure is likely to take on a range of values. The combined effects are assumed to cause uncertainties in the natural frequencies well beyond the level of robustness inherent in a single Kalman filter/LQG controller. Initial simulations showed that an increase of 1% and a decrease of 2% in the natural frequencies in the truth model caused a single LQG controller (developed for the nominal $\omega_n$ parameter) to
yield an unstable closed-loop system [11, ch. 5]. The possibility of a large range of \( \omega_n \) parameters and lack of robustness indicates the necessity for an adaptive system.

MMAE/MMAC has been presented as a technique that may prove effective in these situations [8, p. 129]. In principle, each value that the uncertain parameter \( \omega_n \) may take is incorporated into the system model of a single Kalman filter and cascaded LQ full-state-feedback controller of an LQG control law. Many such filter/controllers can be generated and arrayed in a parallel “bank,” each based on a different system model, as illustrated in Fig. 3. The output state estimate \( \hat{x}_t \) from each Kalman filter is based on the parameter value \( a_k \) used in its system model. The state estimate from the filter using the parameter value closest to the true value should be determined to be the most correct. Thus, the magnitude of the output residual, \( r_k \) (or perhaps of the scaled residual, scaled according to the filter-computed residual covariance) should be the smallest for this filter, whereas the magnitudes from other filters should be relatively larger [8, p. 133]. The residuals are used in conjunction with a hypothesis conditional probability to determine a weighting factor for the corresponding controller output. This hypothesis probability is the probability that the discrete parameter value used in the system model of a specific filter is closest to the true parameter, conditioned on the history of measurements observed until the current time. Consequently, the highest conditional probability should be assigned to the most correct filter (with smallest residual), and lower relative conditional probabilities assigned to the other filters [8, p. 133]. Each controller output is appropriately weighted based on the filter conditional probability \( p_k \), and summed with the others to create a probability-weighted average final controller output. Four control methods were investigated in this research (MMAC, modified MMAC, maximum a posteriori (MAP) form of MMAC, modified MAP form [11, ch. 2]), but only the standard MMAC method is presented here due to the negligible difference between the results from the separate methods.

**Bayesian Estimation:** A Kalman filter is optimal when the system model matrices are known exactly. However, an uncertain parameter \( a \), with scalar components that affect one or more of the matrices, may cause a substantial loss of precision in the state estimation. Additionally, \( a \) may be defined over a continuous range; thus an infinite number of Kalman filters would be required in an algorithm as depicted in Fig. 3, each based on a specific realization \( a = \alpha \) in this range. The infeasibility of this result dictates that the continuous range be discretized such that the parameter \( a \) will take on a reasonable set of values \( a_1, a_2, \ldots, a_K \), where \( K \) is the total number of Kalman filters. The goal of a Bayesian estimator is to produce the conditional density of \( x(t) \) and \( a \), given the measurement history \( z(t) \) (where \( z(t) = [z(t_1); z(t_2); \ldots; z(t_r)]^T \) terms of measurements \( z(t) \) at each sample time \( t_i \)). As follows:

\[
f_{a|z(t)}(\alpha | z_t) = f_{a|z(t)}(\xi | \alpha, z_t) f_{a}(\alpha | z_t).
\]

The first term on the right is nothing more than the Gaussian density function totally defined by the outputs \( \hat{x}(t^+) \) and state estimation error covariance matrix \( P(t^+) \) of the Kalman filter based on \( a \) assuming a specific realization \( \alpha \). The second term can be described by the following equation:

\[
f_a(z(t)|a) = \sum_{k=1}^K p_k(t_i) \delta(\alpha - a_k)
\]

where \( p_k(t_i) = \text{prob}(a = a_k | z(t_i) = z_t) \) is the hypothesis conditional probability associated with the parameter value \( a_k \). This probability is determined by the recursion:

\[
p_k(t_i) = \frac{f_{a|z(t_i)}(z_t | a_k, z_{t-1}) p_k(t_{i-1})}{\sum_{j=1}^K f_{a|z(t_i)}(z_t | a_j, z_{t-1}) p_j(t_{i-1})}
\]

where the first term in the numerator is the probability density of the current measurement, conditioned on the particular assumed parameter value and the observed past measurement history. This density function is determined by

\[
f_{a|z(t)|a}(z_t | a_k, z_{t-1}) = \frac{1}{(2\pi)^{m/2} |A_k(t_i)|^{1/2}}\exp\{ - \frac{1}{2} (z_t - H_k(t_i) \hat{x}_k(t_i))^T A_k^{-1}(t_i) (z_t - H_k(t_i) \hat{x}_k(t_i)) \}
\]

where \( r_k(t_i) = [z(t_i) - H_k(t_i) \hat{x}_k(t_i)]^T \) (filter residual), \( A_k(t_i) = [H_k(t_i) P_k(t_i) H_k(t_i)^T + R_k(t_i)] \) (filter-computed residual covariance) and \( \hat{x}_k(t_i) \), \( P_k(t_i) \), measurement matrix \( H_k(t_i) \), and measurement noise covariance...
matrix $R_k(t_i)$ are quantities in the $k$th Kalman filter which is based on the assumption that $a = a_k$. The resulting conditional mean state estimate from the preceding development is given by [8, p. 131]:

$$\hat{x}(t_i^+) = E\{x(t_i) | Z(t_i) = Z_i\}$$

$$= \int_{-\infty}^{\infty} \xi \left[ \sum_{k=1}^{K} f_{x(t_i)|a_kZ(t_i)}(\xi | a_k, Z_i)p_k(t_i) \right] d\xi$$

$$= \sum_{k=1}^{K} \hat{x}_k(t_i^+)p_k(t_i). \quad (5)$$

Similarly, the MMAC control is generated in an analogous manner, as illustrated in Fig. 3. It is useful to compute the conditional mean value for the parameter $a$ at any given time, and this is given by the following [8, p. 132]:

$$\hat{a}(t_i) = E\{a(t_i) | Z(t_i) = Z_i\}$$

$$= \int_{-\infty}^{\infty} \alpha f_{a|Z(t_i)}(\alpha | Z_i)d\alpha$$

$$= \sum_{k=1}^{K} a_k p_k(t_i). \quad (6)$$

The corresponding covariance indicates the level of precision in the estimate of the parameter [8, p. 131]:

$$P_a = E\{(a - \hat{a}(t_i))(a - \hat{a}(t_i))^T | Z(t_i) = Z_i\}$$

$$= \sum_{k=1}^{K} [a_k - \hat{a}(t_i)][a_k - \hat{a}(t_i))^T p_k(t_i). \quad (7)$$

The additional calculation for the covariance of the estimated state vector can be generated [8, pp. 131–133], but is not necessary for the actual online MMAC algorithm.

The denominator in (3) is the sum of the numerator terms from all $K$ filters at time $t_i$. This can be considered a scaling factor that ensures that the sum of all the individual conditional probabilities will equal one. However, there is a subtle problem since this is an iterative equation. Should the conditional probability, $p_k(t_{i-1})$, become zero at any time, it will remain at zero, unaffected by the real world from that time forth. This condition is termed “lockout.” Thus, even if the elemental filter-based parameter value $a_k$ were to match the real value closely, which would also imply a precise state estimate, its contribution to the weighted average would be zero. An ad hoc method [8, p. 135] for resolving this situation is to add a lower bound artificially to the probability calculation, which would eliminate the possibility of it ever going to zero. The remaining probabilities are rescaled to maintain the summation equal to one. The lower bound of 0.01 is implemented in this research.

**LQG Controller Development:** Since the desire is to quell the vibration in the structure, which is equivalent to driving all the bending mode states to zero, a regulator form of the LQG controller is used [9, p. 19]. A steady-state constant-gain law is used under the assumptions that initial gain transients in the Kalman filter and the final transients in the deterministic optimal controller gain are negligible. The resulting controller gains (based on one parameter value $a_k$) are cascaded with elemental Kalman filters (based on the same parameter value $a_k$) and combined as discussed previously to form the final controller output. The quadratic cost function for each LQG controller development is defined by

$$J = E\left\{ \sum_{t=0}^{\infty} \frac{1}{2} [x^T(t)Xx(t) + u^T(t)Uu(t) + 2x^T(t)Su(t)] \right\} \quad (8)$$

where the state weighting matrix $X$ is defined to place a quadratic penalty on the two LOS deviations ($X$ and $Y$). The state weighting matrix $X$ and control weighting matrix $U$ are chosen so as to provide rapid quelling of the structural vibrations yet not to saturate the actuators. Previous research results indicated that the cross weighting matrix $S$ had a negligible magnitude for this application, and it was ignored with no appreciable performance impact [6].

Moving Bank Development: The concept of the moving-bank MMAE/MMAC arose from the need to reduce the computational burden created by a full bank of Kalman filters. Therefore only a subset or “bank” of filter/controllers is maintained active at any given time. The concept is to allow the bank to “move” in an attempt to center itself around the best estimate of the parameter in the parameter space. As the parameter varies outside the confines of the current bank, the bank would have to move by dynamic redeclaration of which elemental filter/controllers are to be implemented. Initialization of new elemental filters/controllers involves redistributing the $p_k$ probability values last obtained from the deactivated filters equally among the newly activated filters. Bank movement can be accomplished in a finely discretized manner should the parameter drift slightly, or in a coarsely discretized manner should the parameter make a large discrete jump or during an initial acquisition phase. This concept as applied to the one-dimensional parameter space utilized in this research is depicted in Fig. 4. A full bank of filter/controllers is indicated in Fig. 4 (1); each of the ten discrete parameter values shown would be the basis of an elemental filter/controller within the MMAC. The moving-bank approach is presented in Fig. 4 (2), where the move logic is implemented for a small jump in the parameter value; only the three discrete parameter points closest to the current parameter value (estimate) are used as the basis of elemental filter/controllers. A bank expansion for a large jump is presented in Fig. 4 (3); once the new
estimated parameter is well established, the coarse discretization can be returned to a finer discretization level, as in Fig. 4 (2).

There are four methods for deciding to expand, contract, move, or not move the bank. These decision-making techniques are: 1) residual monitoring, 2) parameter position estimate monitoring, 3) probability monitoring, and 4) parameter estimation error covariance monitoring [3, 10]. The second and third techniques are not presented here but can be found in [11, ch. 2]. In the following discussions, it is necessary to determine the specific decision logic threshold values empirically, which is analogous to filter “tuning.”

**Residual Monitoring:** This method is formed on the basis of the likelihood quotient, \( L_j(t_i) \), which is defined as

\[
L_j(t_i) = r_j^T(t_i)A_j(t_i)^{-1}r_j(t_i)
\]

and can be seen to be the major contributor to the exponential term in (4). The decision to move the bank is based on the value of this scalar quadratic function. The smaller the value (assumed as the result of smaller residuals), the closer the filter-based parameter value is to the actual parameter position. The threshold for the likelihood quotient is determined such that, if all the filters quotients are above this value, then this is an indication that the true parameter value is to the actual parameter position. However, unanticipated problems occurred that led to the decision to modify this approach [11, ch. 5]. The resulting parameter space was determined simply by discretizing every 0.5 percent between the range of minus 2 to plus 10 percent from nominal. This results in a 21-point one-dimensional parameter space. The individual points in this space will be addressed as ranging from \( \omega_1 \) to \( \omega_2 \), where \( \omega_1 \) is the minus 2 percent point. Once this step is complete, the elemental system models for filter/controller development can be created.

V. RESULTS

**Tuning Procedures:** This section discusses the specific method by which the filters and controllers are tuned. In this application, the tuning process is accomplished by trial and error, guided by engineering insight, rather than more sophisticated methods. For all tuning efforts and subsequent performance analysis, a ten-run Monte Carlo analysis of 10 s duration (except where noted) is being utilized. Tuning the filters [7, 11] is accomplished by visually examining the plots of the true mean estimation error and mean estimation error \( \pm \) one standard deviation \( (m_j \pm \sigma_j) \) calculated from the Monte Carlo simulation runs, versus the filter-predicted mean error (namely, zero) \( \pm \) one standard deviation \( (\pm \sigma_j) \). The objective is to have the filter-predicted \( \pm \sigma_j \) match the mean estimation error \( m_j \pm \sigma_j \) as much as possible, which will result in an optimally tuned filter. The filter-predicted \( \pm \sigma_j \) bounds are modified by alternately adjusting (tuning) the dynamics driving noise strength of the assumed model, \( Q_j \), of the
filter, and the measurement noise covariance $R_f$. Since there were no changes made between the truth model and filter models for dynamics driving noise or the measurement noise inputs to represent actual disturbances in the real world system, the primary method of tuning $Q_f$ and $R_f$ (to address order reduction of the filter design model) was by adjusting each of the respective values by a scalar multiplier. This procedure worked well, although there is a noticeable difference in the tuning of the $X$ and $Y$ axis, respectively, due to the geometrical differences in the two axes on the actual SPICE tripod structure (reference Fig. 1).

For controller tuning [9, 11], only the scalar weight affecting the state weighting matrix was adjusted, since the value for the control weighting matrix was assumed to be at its limiting admissible value [1]. Tuning the controller was a simple matter of adjusting the scalar state weighting value until the point of closed-loop system instability was reached (determined visually by a divergence in the error estimation plots or the LOS plots). This point implies the “tightest” possible state values will be obtained with slightly smaller state weightings. The weighting value was then tuned back 10 percent. Once again, this value was used initially for all of the controllers in the discretized parameter space.

Model Analysis: This section presents the results of the tuned truth-model-based and reduced-order steady-state filters and LQG controllers. For this analysis, a single filter/controller combination based on the nominal truth model is being utilized. Fig. 5 displays the LOS deviations of the truth model with no LQG control applied. Recall from Fig. 2, that there is closed-loop control in the form of simple rate feedback (LAC) applied to the structure. When references are made to control or no control, this is strictly referring to the application of LQG control, since the LAC feedback loop is active in all simulations. For no control, the temporal average rms values for the truth model $X$- and $Y$-axis LOS deviations are 9.32 micro-rad and 15.74 micro-rad, respectively. This clearly indicates the need for an additional type of control to quell the vibrations of the structure to within the one micro-rad specification.

Fig. 6 illustrates the dramatic reduction in the $X$-axis LOS deviations when the truth-model-based filter/LQG controller is applied at the 1 s mark. The resulting rms values are 0.47 micro-rad for the $X$-axis LOS and 0.66 micro-rad for the $Y$-axis LOS. Thus the controller based on the truth model indicates the capability to control the structure to well within the 1 micro-rad specification. Again, the discrepancy between the $X$- and $Y$-axis values is due to the differences inherent in the geometry of the tripod structure. Of course, implementing a full-order filter/controller such as this would be computationally infeasible. The underlying anticipation is to have a much reduced-order filter/controller performance closely compare with this full-order filter/controller performance.

The analysis with the filter/controllers based on the 26-mode modal reduced-order model and 26-mode modal-cost reduced-order model had results with negligible differences [11, ch. 5], so only the results from the modal model are presented in this work. Additionally, the modal-cost model had difficulties that precluded its use in the subsequent MMAC analysis [11, ch. 5]. Fig. 7 illustrates the $X$-axis results when control is applied. The resulting rms values are 0.682 micro-rad for the $X$-axis LOS and 1.012 micro-rad for the $Y$-axis LOS. It should be noted that, due to the relatively large difference in the performance results between the two axes, the performance of each axis is addressed separately. The 1 micro-rad specification is met to within an acceptable margin (+0.012 micro-rad) for the $Y$-axis, while the $X$-axis is well within the specification. There is some relative control degradation (as compared with the truth-model-based filter/controller)
of 45 percent and 53 percent in the LOS rms errors of the X and Y axes, respectively. This level of degradation is still highly encouraging, considering the significant level of order reduction involved.

Moving-Bank MMAC Analysis: This section presents the results obtained from simulations implementing the MMAC design with move logic based on residual monitoring as discussed previously. The ability of the moving bank of filters to identify the true parameter is of the utmost importance prior to applying MMAC control. Although it might be desirable not to apply control during periods of large parameter variations, this may be unavoidable. As such, this last situation is presented in this work.

For bank expansion, the three filters in the bank are relocated at the widest possible locations in the discretized space: \( \omega_5, \omega_{11}, \) and \( \omega_{21} \), respectively. The threshold determination for the likelihood quotient of (9) was obtained by trial and error. The final value of 60 was set, which allowed for expansion given a true parameter jump change greater than approximately 4.5 percent (e.g., a jump from \( \omega_5 \) to \( \omega_{14} \)). In determining the contraction threshold, it was found that two different levels of contraction, medium and fine, resulted in better performance than just one level. The medium bank discretization placed the outer two elemental filters at \( \pm 5 \) points from the center filter (e.g., \( \omega_1, \omega_6, \) and \( \omega_{11}, \) respectively). The specific thresholds for medium and fine contraction were set at 5.0 and 2.0, respectively. These values were set relatively high, which resulted in very quick contraction of the bank (typically within five sample periods).

Once the expansion/contraction thresholds were determined, the move threshold could be determined for the residual monitoring technique. There was a tradeoff between consistency (less variation of the parameter position estimate around the true parameter position value during a “tracking” phase in which the true parameter is constant) and response time (how quickly the parameter position estimate approaches the true parameter in an “acquisition” or “reacquisition” phase in which the true parameter changes). Setting the threshold too low resulted in unnecessary move decisions, whereas setting the threshold too high resulted in slow responses to actual parameter position changes. The resulting threshold was optimized at 10.

Two types of simulations are presented here; numerous additional types are depicted in [11, ch. 5]. The first type simulation initially centers the filter bank on the true parameter \( \omega_5 \), then allows the true parameter to make a discrete jump to a new location \( \omega_{17} \), (at the 5 s mark). This attempts to simulate a sudden temperature variation or, possibly, damage to the real world system. Fig. 8 depicts the MMAC performance in response to this type of situation. Second, the true parameter is allowed to vary slowly over a period of time, with the initial bank center and true parameter position at \( \omega_2 \), and then small discrete positive jumps in the true parameter position every 2 s for 30 s. This would simulate the possible temperature variations arising from a transition into- and out-of sunlight. Fig. 9 depicts the MMAC performance in response to this type of situation. In both figures, the true parameter position is indicated by the piecewise constant line and the parameter position estimate (mean value obtained from a ten-run Monte Carlo analysis) is indicated by the wavering line. As can be seen, the MMAC with the residual monitoring technique and bank expansion/contraction enabled is extremely effective in tracking the parameter position. This is true in terms of consistency (during parameter stationary periods) and speed of response (during parameter movement). In each of these MMAC simulations, the actual LOS deviations are essentially identical to that of the single filter/controller performance indicated in Fig. 7.
VI. SUMMARY

The necessity for adaptive control for this application was based on concerns that nonadaptive controller designs do not demonstrate enough inherent robustness to be effective in the presence of uncertain parameters in the system model. Here the uncertain parameter was assumed to be a scalar multiplier of the undamped natural frequencies in the flexible body modes of the physical structure. The design procedure began with the development of the new system model as the truth model and then progressed to reduced-order design models for implementation in the Kalman filters, LQG controllers, and moving-bank MMAE/MMAC algorithms. The parameter space was then discretized to allow for parameter variations. Finally, the ability of a moving bank of filter/controllers to estimate the uncertain parameter accurately and apply control effectively was examined.

It was shown that the substantial order reduction required in the original system model supplied by the Phillips Laboratory resulted in a truth system model selection that was still a very good representation of the real-world system. One reduced-order model was presented, in which the only order reduction took place in the flexible body portion of the truth system model. The 21-point parameter space allowed for a full analysis of the moving-bank method, with expansion and contraction as well as motion of the filter bank. Allowing expansion and contraction of the filter bank with the residual monitoring move logic consistently resulted in very effective parameter identification. Upon applying LQG-based control in the MMAC simulations, the resulting performance was indistinguishable from that of the single artificially informed (based on the correct parameter value) LQG controller utilizing the same model-based filter/controller.

The final conclusion of the research indicates that the moving-bank MMAE/MMAC design method is extremely effective in quelling unwanted vibrations in the SPICE structure, even during periods of large parameter variations.

REFERENCES

Capt. Greg Schiller grew up in Las Vegas, NV. He received a Bachelor of Science in astronautical engineering at the Air Force Academy in 1985. In 1993, he received a Masters of Science in astronautical engineering at the Air Force Institute of Technology.

His first assignment was at Onizuka AFB, CA where he was a planner/analyst for the NATO Communications Satellite Program. His second assignment was at Falcon AFB, CO where he held positions as a planner/analyst, current operations lead and finally program lead engineer for the Fleet Satellite Communications Program. At AFIT, his areas of study included inertial navigation systems and stochastic guidance and control theory utilized in the application of Kalman filter and LQG control algorithms. Currently, he is assigned to the Phillips Laboratories, Lasers and Imaging Directorate, Optical Sensing Division at Kirtland AFB, NM.

Peter S. Maybeck (S’70—M’74—SM’84—F’87) was born in New York, NY on February 9, 1947. He received the B.S. and Ph.D. degrees in aeronautical and astronautical engineering from M.I.T., Cambridge, in 1968 and 1972, respectively.

In 1968, he was employed by the Apollo Digital Autopilot Group of The C. S. Draper Laboratory, Cambridge, MA. From 1972 to 1973, he served as a military control engineer for the Air Force Flight Dynamics Laboratory and then joined the faculty of the Air Force Institute of Technology in June of 1973, where he is currently Professor of Electrical Engineering. Current research interests concentrate on using optimal estimation techniques for guidance systems, tracking, adaptive systems, and failure detection purposes.

Dr. Maybeck is the author of numerous papers on applied optimal filtering as well as the book, *Stochastic Models, Estimation and Control* (Academic Press, Vol. 1, 1979, Vols. 2 and 3, 1982; republished by Navtech in 1994). He is a member of Tau Beta Pi, Sigma Gamma Tau, Eta Kappa Nu, and Sigma Xi. He was recipient of the DeFlorez Award (ingenuity and competence of research), the James Means Prize (excellence in systems engineering) and the Hertz Foundation Fellowship at M.I.T. in 1968. In all years from 1975 to 1995, he received commendation as outstanding Professor of Electrical Engineering at A.F.I.T. In December of 1978, he received an award from the Affiliate Societies Council of Dayton as one of the twelve outstanding scientists in the Dayton, Ohio area. In March of 1980, he was presented with the Eta Kappa Nu Association’s C. Holmes MacDonald Award, designating him as the outstanding electrical engineering professor in the United States under the age of 35 (he had placed second in this national competition for 1977 as well). In 1985, he received the Frederick Emmons Terman Award, the highest national award to a Professor of Electrical Engineering given by the American Society of Engineering Education. He is a member of the A.I.A.A., and is the current I.E.E.E. Dayton Section Student Activities Chairman and a member of the I.E.E.E. Executive Committee of Dayton. He previously served as Chairman of the local Automatic Control Group.