Second Time Around Radar Return Suppression Using PRI Modulation

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A technique for suppressing second time around radar returns using pulse-repetition interval (PRI) modulation is presented and analyzed. It is shown that a staggered PRI radar system can offer considerable improvement over a nonstaggered radar system in rejecting second time around returns which cause false alarms. This improvement is a function of detector implementation (noncoherent integrator or binary integrator), the number of staggered PRI, the quiescent false alarm number, the Swerling number of the false return, the transmitted signal power, the second time around noise power, and the quiescent noise power of the radar. Small changes in transmitted signal power can be traded-off with the quiescent false alarm number to significantly suppress the bogus return. In addition, for a noncoherent integrator all other parameters being equal, if the second time around return is a Swerling case II or IV target, then there is an optimum number of staggered PRI that can be chosen to minimize the likelihood of detection of the second time around return. It was also shown that the binary integrator significantly reduces second time around return detections when compared with the noncoherent integrator. However there is an accompanying loss of detection by using the binary integrator.

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I. INTRODUCTION

A second time around return (Fig. 1) is a class of radar interference that does not necessarily interfere with the detection of a desired target but rather generates false targets (undesirable detections). A low pulse-repetition frequency (PRF) radar transmits a series of uniform pulses spaced $T_0$ seconds apart. If there is a large object (an island or mountain) located beyond the operating range $cT_0/2$ of the radar, where $c$ is the speed of light, then it is possible for this large object to create a substantial return (called a second time around return) at the front end of the radar receiver. In addition, the return will appear in a fixed range bin that is much closer than the actual range of the false target as seen in Fig. 1.

If we assume that we are attempting to detect slow moving targets and thus a moving target indicator (MTI) is not used and we process the radar returns using a constant false alarm rate (CFAR) threshold detector, there is a good probability that the second time around return will be detected and as a result generate a false target. Also, no amount of integration of noncoherent or coherent pulses improves the rejection of the undesired detections.

However, we show in this paper that modulating the pulse-repetition interval (PRI) of the transmitted pulses and then integrating the returns over the respective range bins substantially improves the suppression of the second time around returns and thus reduces the false alarm rate.

Fig. 2(a) illustrates the modulated PRI concept where the PRI is changed linearly. We transmit $n$ pulses with the pulse separation reduced by $\Delta T$ seconds each time a pulse is transmitted. The smallest PRI possible is $T_0$ and the largest is $T_0 + (n - 1)\Delta T$.

We assume that $\Delta T \geq \tau$ where $\tau$ is the pulsewidth ($c\tau/2$ is the range bin size, $cT_0/2$ is the operating range of the radar, and $T_0/\tau$ is the approximate number of range bins).

Fig. 2(b) shows the effect of modulating the PRI on the second time around return. After the second pulse, the undesired return is in the first range bin; after the third pulse, it is in the second range bin and so on. Hence if $\Delta T \geq \tau$, the second time around return will slide from one range bin to another and yet never be in the same range bin twice if $n\Delta T < T_0$. If we integrate incoherently over the $n$ pulses for a given range bin, then the effect of the second time around return in a given range bin can be diminished. This occurs because as $n$ increases, the bias threshold also increases while the input power of the second time around return remains constant for that given range bin. Thus, the detection likelihood of that return decreases for the given range bin. However, we must remember that as $n$ increases, the possible number of range bins that the second time around return can appear in also increases. Therefore, it will have more
chances to be detected at least once in one of the \( n - 1 \) possible range bins. The next sections present an analysis and discussion of the tradeoffs of using a modulated PRI radar to suppress second time around returns. The noncoherent integrator in combination with the staggered PRF is considered in Sections II-IV. The binary integrator is discussed in Section V. The results of the following sections also apply if the PRI is randomly jittered so long as same PRI is not repeated over the \( n \) transmitted pulses. In addition, we assume that the range extent of the second time around return is less than the radar range bin size, \( c\tau/2 \), and that the PRF of the radar is low enough so that only second time around and not third, fourth, . . . , time around returns are detectable.

II. ANALYSIS

We begin by making the following parameter definitions:

- \( S_d \) is the average single-pulse power of the desired signal;
- \( N_q \) is the average single-pulse power of the quiescent noise;
- \( S_2 \) is the average single-pulse power of the second time return.

The quiescent noise is the receiver input noise that does not include the second time around return. The quiescent noise power and the desired false alarm rate determine the detector threshold. We assume that second time around returns do not occur often enough to affect this threshold.

If we use a modulated PRI radar and consider a given range bin that contains only the second time around return and quiescent noise, then after envelope detection and integration the received signal voltage \( r \) will have the form

\[
r = S_2(j) + \sum_{k=1}^{n} n_q(k)
\]

where \( S_2(j) \) is the second time around return voltage which we assume occurs on just the \( j \)th pulse and \( n_q(k), k = 1, 2, \ldots, n \) is the quiescent noise voltages that occur on every pulse. If we consider the detection of the second time around return in noise, then from (1), we see that the integrated signal-to-noise power ratio for the second time around return \( (S/N) \) can be written as

\[
\left( \frac{S}{N} \right) = \frac{1}{n} \left( \frac{S_2}{N_q} \right)
\]

assuming no integration losses. Hence we see from (2) that as \( n \) increases the integrated \( (S/N) \) decreases.
The integrated $(S/N)$ is plotted in Fig. 3 for various values of $(S_2/N_0)$. This graph also shows the lossless integration gain or improvement of the desired signal-to-noise ratio as a function of $n$.

Let $P_D^{(a)}$ be the quiescent false alarm probability where $P_D^{(a)}$ is the probability that a false alarm is obtained each time there is an opportunity under the condition that the second time around return is not present. The quiescent false alarm number $r^{(a)}_f$ is related to the quiescent false alarm probability by the relationship [1]

$$P_D^{(a)} = \frac{0.693}{r^{(a)}_f}. \quad (1)$$

The probability $P_D$ of detecting the second time around return for a given range bin is a function of $(S/N)$, $r^{(a)}_f$, and the statistical characteristics of the second time around return which we characterize by its Swerling number $M$ (see [1] for an explanation of the various Swerling cases), where $M = 0$, I, II, III, IV (0 indicates a nonfluctuating target). We must be careful when calculating this probability of detection due to the nature of the received signal $r$ as seen in (1). Since the second time around return appears in this equation as a single random variable and not as a sum of random variables, the Swerling cases II and IV reduce to case I and III, respectively. To see this, we rewrite (1) as

$$r = \sum_{k=1}^{n} \left[ \frac{1}{n} s_2(j) + n_2(k) \right]. \quad (3)$$

Even though $s_2(j)$ may be varying statistically from pulse-to-pulse, only one of these random pulses appears in a given range bin. This pulse can be modelled for the purposes of analysis as $n$ pulses of identical amplitude, $s_2(j)/n$ in that range bin. Hence Swerling cases II and IV reduce to case I and III, respectively. Therefore for each case:

$$P_D^{(case = 0)} = P_D^{(S_2/nN_0, r^{(a)}_f, n, M = 0)} \quad (4)$$

$$P_D^{(case = I)} = P_D^{(S_2/nN_0, r^{(a)}_f, n, M = I)} \quad (5)$$

$$P_D^{(case = II)} = P_D^{(S_2/nN_0, r^{(a)}_f, n, M = II)} \quad (6)$$

$$P_D^{(case = III)} = P_D^{(S_2/nN_0, r^{(a)}_f, n, M = III)} \quad (7)$$

$$P_D^{(case = IV)} = P_D^{(S_2/nN_0, r^{(a)}_f, n, M = IV)} \quad (8)$$

The difference in performance due to the difference in Swerling cases becomes apparent if we define the performance measure $P_2$ as the probability that the second time around return will be detected in at least one range bin out of a possible $n - 1$ range bins. For Swerling cases 0, I, and III, $P_2$ is simply equal to $P_D$ since the second time around returns do not vary from pulse-to-pulse and the threshold in each range bin is equal. Hence if one return exceeds this threshold, all of the returns in each range bin exceed this threshold. However, for Swerling cases II and IV, the probabilities of detection for each range bin are independent. Thus for each Swerling case we can express $P_2$ as

$$P_2^{(case = 0)} = P_2^{(case = 0)} \quad (9)$$

$$P_2^{(case = I)} = P_2^{(case = I)} \quad (10)$$

$$P_2^{(case = II)} = 1 - (1 - P_D^{(case = II)})^{n-1} \quad (11)$$

$$P_2^{(case = III)} = P_2^{(case = III)} \quad (12)$$

$$P_2^{(case = IV)} = 1 - (1 - P_D^{(case = IV)})^{n-1}. \quad (13)$$

It is possible using well-known formulas [1] and existing computer programs [2] to calculate $P_D$ as expressed by the parameters seen in (4)–(8). Using these results, we can calculate $P_2$ for each Swerling case by using (9)–(13). We plot in Figs. 4–9, $P_2$ versus...


Thus we see from (14) that if the false target can be characterized by one large reflector together with a number of small reflectors, then performance improves ($P_2$ becomes smaller). However, there is not a large change in performance.

The plots in Figs. 6 and 8 also obviously indicate that performance degrades as the second time around return to quiescent noise power ratio ($S_2/N_q$) increases. We see from these figures that $P_2$ is very sensitive to ($S_2/N_q$). For example, if the Swerling case is II or IV and the number of PRIs, $n$, is approximately 50, then decreasing ($S_2/N_q$) by 5 dB results in a hundredfold decrease in probability of at least one false alarm. Also we see for ($S_2/N_q$) > 10 dB and the
quiescent false alarm number equal to $10^6$, that for most practical purposes even a modulated PRI system does not effectively suppress the second time around return. These plots also indicate for $(S/N_q) < 5$ dB that the modulated PRI system can offer significant improvement if the number of PRIs is chosen properly.

Let us examine how $P_2$ varies with $n$, the number of modulated PRIs. We see that for small $n$, in most cases $P_2$ rises to a local maximum then decreases to an absolute minimum and finally increases. In fact, it can be shown that as $n$ approaches infinity, $P_2$ approaches one. Intuitively, this occurs because as $n \to \infty$, the integrated signal-to-noise ratio as expressed by (2) goes to zero. Thus for a given range bin, the probability of detecting the second time around return will approach the quiescent false alarm probability or $P_D \to P_q$. Hence from (11) or (13), we see that $P_2 \to 1$ as $n \to \infty$ and $P_D \to P_q$. In practice, $P_2$ will not approach one because $n$ is upper bounded by the number of possible range bins.

The local maximum exists in most cases because as $n$ initially increases from two, there are more opportunities for the second return to be detected whereas the decrease in integrated input $(S/N)$ as expressed by (2) does not offset this until after the local maximum.

If we asked what is the improvement of using modulated PRI over a nonmodulated PRI system, then
we can show that

$$P_2(\text{nonmodulated}, n) \geq P_2(2 \text{ pulse modulated PRI}).$$

Equation (15) is true under the assumption that we are integrating more than one pulse for the nonmodulated system. The inequality becomes larger as the number of integrated pulses increases for the nonmodulated PRI system. In addition, we can show that if the number of pulses integrated is larger than the number of PRIs, then performance degrades.

We see an interesting phenomena if we compare the curves of Figs. 6 and 9. In these figures, all parameters are the same except for the quiescent false alarm number. In Fig. 6, the false alarm number is $10^6$ and in Fig. 9, the false alarm number is $10^{10}$. We see that by increasing the false alarm number to $10^{10}$, tremendous improvement is possible. For example, if $n = 50$, $(S/N_q) = 5 \text{ dB}$, and $N_{sa} = 10^6$, then $P_2 = 1$ percent. However, if we raise the false alarm number to $10^{10}$ while holding the other parameters constant, then $P_2$ falls to 0.01 percent.

The price we pay by decreasing the quiescent false alarm probability is that the detection probability of the desired signal decreases. This is because the threshold of the detector must be raised in order to decrease the probability of a false alarm.

Let us examine what occurs to the modulated PRI system performance if we increase the desired signal-to-quiescent-noise ratio in order to maintain a
constant probability of detection for desired targets.

In Fig. 10 we have plotted the required signal-to-noise ratio versus the number of integrated pulses necessary to obtain a probability of detection of 0.5 for a Swerling case II target using the false alarm number as a parameter. We see from the figure that in order to maintain \( P_D = 0.5 \) for all \( n \), we need only to increase our transmitter power by approximately 1.5 dB when going from a false alarm number of \( 10^6 \) to \( 10^{10} \). However by increasing the transmitter power by a given amount also increases the power of the second time around return by that same amount. Hence \( P_2 \) will increase.

For example, using Fig. 6, if \( n = 100 \), \( (S_2/N_q) = 5 \) dB, \( r_{th}^{(4)} = 10^6 \), and the radar returns are Swerling case II, then \( P_2 = 0.35 \) percent. If we raise \( r_{th}^{(4)} \) to \( 10^{10} \) and also increase our transmitter power by 1.5 dB to maintain a constant probability of detection, then \( (S_2/N_q) = 6.5 \) dB. For this case, we can show using Fig. 9 that if \( n = 100 \), then \( P_2 = 0.035 \) percent. Therefore by slightly increasing the transmitter power and increasing the detector threshold, we have decreased the probability of at least one second time around return being detected by tenfold. Hence, it would seem that modulated PRI systems work best when the quiescent false alarm number of the detector is large. Additional curves similar to those seen in Fig. 10 are found in [3] for various Swerling cases and false alarm numbers.
Plots similar to those seen in Figs. 6 and 8 are possible whereby we vary the desired signal-to-second-time-around return power ratio, \( S_d/S_2 \), while holding the quiescent signal-to-noise ratio, \( S_d/N_q \), a constant. These results because we can write
\[
S_d/S_2 = (S_d/N_q)/(S_2/N_q). \quad (16)
\]
We plot \( P_2 \) versus \( n \) with \( S_d/S_2 \) as a parameter for a Swerling case II target, \( S_d/S_2 = 10^6 \), and the quiescent signal-to-noise ratio equal to 0 dB in Fig. 11. Not unexpectedly, we see that performance degrades as \( S_d/S_2 \) decreases or equivalently as the power of the second time around return increases.

B. Swerling Cases 0, 1, and III

We can order the performance of the modulated PRI radar system by Swerling number and find that
\[
P_2(0) < P_2(III) < P_2(I) \quad (17)
\]
where we hold all other parameters equal. Thus a nonfluctuating second time around return (case 0) is suppressed to a greater extent than fluctuating returns. In addition, if the false target can be characterized by one large reflector together with a number of small reflectors, then performance improves.

Similar to Swerling cases II and IV, performance degrades as \( S_2/N_q \) increases and is very sensitive to this parameter as indicated by the curves seen in Figs. 4, 5, and 7. However, unlike cases II and IV, \( P_2 \) has no local extrema when the number of PRIs is varied. For Swerling cases 0, I, and III, \( P_2 \) is a monotonically decreasing function of \( n \). Its maximum occurs at \( n = 2 \) and its minimum at \( n = \infty \). In fact, it is possible to show (see the discussion on cases II and IV) that \( P_2 \rightarrow p_{k}^{00} \) as \( n \rightarrow \infty \). Also similar to cases II and IV, significant improvement in performance is possible by raising the detection threshold (and hence the false alarm rate) as indicated by the curves seen in Fig. 4.

IV. A DESIGN EXAMPLE

Let us determine the number of modulated PRIs necessary such that the probability of at least one false alarm due to the second time around return is less than 1 percent. We do this under the following conditions.

1) The second time around return is located just beyond the maximum operating range of the radar (i.e., at least as far away as a desired target at the maximum operating range).
2) The radar cross section of the second time around return is three times larger than the desired target.
3) The quiescent false alarm number is \( 10^6 \).
4) The signal-to-quiescent-noise ratio of the desired signal is 0 dB at the maximum operating range.
5) The second time around return pulses are independent from pulse-to-pulse and consist of many uniformly distributed scatterers (Swerling case II).

From conditions 1, 2 and 4, we can show that \( (S_2/N_q)_m = 4.8 \text{ dB} \).

To find \( n \), the required number of modulated PRI, we use Fig. 6 and the above given parameters. From this figure, we see that \( n \) is approximately 45. Note that we can reduce the number of modulated PRI significantly by raising the quiescent false alarm number and increasing our transmitter power slightly in order to maintain a constant probability of detection for the desired target (see Fig. 10). Hence we see that the processing complexity can be reduced by using more transmitter power. Also note that we placed the second time around return at the best possible range for its detection. In most situations the bogus return will be located much farther away than the maximum operating range of the radar so that its cross section can increase considerably while still maintaining \( P_2 \) less than 1 percent.

V. OTHER TECHNIQUES

There are other techniques which reduce the effects of second time around returns. In this section we consider two. One of the simplest is to limit the video amplifier and follow this by a noncoherent integrator. The limit level is set perhaps 10 dB above the quiescent noise level. If the PRFs are again staggered, the large interfering signals are limited to \( S_d/N_q = 10 \text{ dB} \), and hence false alarms are reduced. However, there is also a loss of desired signal detection.

Another technique is to use an \( m \) out of \( n \) detector [1, 4] (also called a binary integrator) in combination with staggering \( n \) PRFs. We assume that \( m \) is chosen so as to optimize detection of first time around returns [4]. A second time around return will be detected if it is detected on a single pulse in one of the range bins and if at least \( m - 1 \) out of the remaining \( n - 2 \) returns are false alarms in that range bin.

Let \( D_k \) be the event that the second time around return is detected on a single pulse on the \( k \)th PRI (or equivalently the \( k \)th range bin out of a possible \( n - 1 \)) and \( F_k \) be the event that at least \( m - 1 \) out of the remaining \( n - 2 \) returns associated with the \( k \)th range bin are false alarms. It then follows that
\[
P_2 = \text{Pr} \left\{ \bigcup_{k=2}^{n} (D_k \cap F_k) \right\} \quad (18)
\]
where \( \cup \) and \( \cap \) denotes the union and intersection of events operations. This probability can be bounded as
\[
P_2 \leq \sum_{k=2}^{n} \text{Pr} \{ D_k \cap F_k \}. \quad (19)
\]
The probabilities in the above summation are identical and $D_k$ is independent of $F_k$ so that

$$P_k \leq (n-1) \Pr\{D_2\} \cdot \Pr\{F_2\}. \quad (20)$$

It is straightforward to show that

$$\Pr\{F_2\} = \sum_{j=m-1}^{n-1} \binom{n-2}{j} P_k^{(j)} (1-P_k^{(j)})^{n-1-j} \quad (21)$$

where $\binom{\cdot}{\cdot}$ denotes the binomial coefficient. Typically, $P_k^{(0)} \ll 1$ so that the above can be approximated as

$$\Pr\{F_2\} \approx \binom{n-2}{m-1} [P_k^{(0)}]^{m-1} \cdot \quad (22)$$

The $\Pr\{D_2\}$ was given for the various Swerling cases by (4)-(8). Thus

$$P_2(\text{case } = M) < \approx (n-1) \binom{n-2}{m-1} \times [P_k^{(0)}]^{m-1} P_d(\text{case } = M). \quad (23)$$

In almost all practical instances, the factor on $P_d(\text{case } = M)$ given above is much less than one and $P_d(\text{case } = M) \ll 1$. Hence for $M = 0, 1, III$, it is seen by comparing (23) with (9), (10), and (12), that using the binary integrator instead of the noncoherent integrator reduces $P_2$ significantly. Furthermore, for $M = II, IV$, and $P_d(\text{case } = M) \ll 1$,

$$1 - (1 - P_d(\text{case } = M))^n \approx (n-1)P_d(\text{case } = M). \quad (24)$$

Hence comparing (23) with (11) and (13) in lieu of (24) again demonstrates that using the binary integrator instead of the noncoherent integrator reduces $P_2$ significantly.

The price to be paid for this significant improvement in reducing the effects of second time around returns is a loss of detection performance for first time around returns. For example, for a nonfluuctuating target model, the binary integrator requires a signal-to-noise ratio 0.8 to 1.4 dB larger than the incoherent integrator to achieve the same probabilities of detection and false alarm [1].

VI. CONCLUSIONS

We have shown that a staggered PRI radar system can offer considerable improvement over a nonstaggered radar system in rejecting second time around returns which cause false alarms. This improvement is a function of detector implementation (noncoherent integrator or binary integrator), the number of staggered PRI, the quiescent false alarm number, the Swerling number of the false return, the transmitted signal power, the second time around noise power, and the quiescent noise power of the radar. Small changes in transmitted signal power can be traded-off with the quiescent false alarm number to significantly suppress the bogus return. In addition for a noncoherent integrator all other parameters being equal, if the second time around return is a Swerling case II or IV target, then there is an optimum number of staggered PRI that can be chosen to minimize the likelihood of detection of the second time around return. It was also shown that the binary integrator significantly reduces second time around return detections when compared with the noncoherent integrator. However there is an accompanying loss of detection by using the binary integrator.

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