Applications of Highly Stable Oscillators to Scientific Measurements

ROBERT F. C. VESSOT

Invited Paper

I. INTRODUCTION

A. The Technological Basis for Experiments with Clocks

The frequency stability of highly stable oscillators has improved by a factor of about 10 every decade since the 1960 era, when atomic clocks were first introduced. While the frequency accuracy and precision of replication of the most stable oscillators has also improved, the emphasis in this discussion will be on applications of highly stable oscillators.

Most experimental procedures involving oscillators involve measurements of a time varying, or of a modulated, parameter that affects the frequency of the signal from an oscillator. The most useful measures of the error in the measurement process resulting from oscillator instability is the Allan standard deviation, $\sigma_y(\tau)$, and mod $\sigma_y(\tau)$. This is the one-sigma expectation of the fractional frequency difference (designated by the subscript, $y$) between time-adjacent frequency measurements, each made over time intervals of duration, $\tau$. The functional relationship of $\sigma_y(\tau)$ versus $\tau$ depends on the Fourier spectrum of the phase variations [1].

The statistical representation of oscillator performance given by $\sigma_y(\tau)$ or mod $\sigma_y(\tau)$ can provide estimates of the limits imposed on the precision of measurements when comparisons are made of data of some time dependent phenomenon averaged sequentially every $\tau$ s. Figure 1 shows sigma versus tau plots for recently developed stored ion devices [2], atomic hydrogen masers [3], and for the binary pulsar [4].

This figure also includes the Allan standard deviation of the disturbance caused by fluctuations in the earth's troposphere and ionosphere on a signal traversing vertically. In the discussion of experimental techniques, the $H$ maser performance data in Fig. 1 will be used as a basis for numerical examples.

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The author is with the Smithsonian Astrophysical Observatory, Cambridge MA 02138.

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B. Applications of Long- and Short-Term Stability

The Long and the Short of It

1) Binary pulsar measurements referenced to terrestrial time scales: Rapidly rotating neutron stars emit bursts of microwave signals that can be detected by radio telescopes [4]. During the past 14 years the frequencies of a number of these pulsars have been monitored. These celestial oscillators have become a new astrophysical laboratory. Measurements of their frequency variations with time have provided information to test the predictions of general relativity. Results from eight years of timing a millisecond pulsar have shown that, once the drift rate has been determined and removed, the frequency stability of this object challenges the long-term stability of the best available ensemble of atomic clocks, as shown in Fig. 1. It is now evident that terrestrial clocks having better frequency stability for intervals beyond one year are needed to resolve a number of cosmological question such as the existence of a Cosmic Gravitational-Wave Background [5] and the possibility of a variability of the gravitational constant [6]. Progress in the new technology of atomic clocks based on trapped and cooled ions, and of levitated atoms, is well under way to fulfill these needs for long term frequency stability.

2) Cryogenic oscillators and short-term performance: The other end of the stability plot, where intervals less than about an hour are of interest, is becoming the domain of cryogenic oscillators. These devices include superconductive cavity stabilized oscillators [7], sapphire dielectric cavity resonators with superconductive coatings [8] and uncoated sapphire disc resonators operating in the "whispering gallery" mode [9]. Operating at cryogenic temperatures, resonator quality factors in the $10^9$ domain have been achieved. These resonators, when combined with a low-noise amplifier such as a ruby laser [10], [11] provide high-level signals of extraordinary spectral purity.

Another oscillator that shows promise is the cryogenically cooled atomic hydrogen maser that operates at about
0.5 K and uses films of superfluid helium as the coating for the storage volume that confines the oscillating hydrogen atoms [12]. This oscillator is expected to provide signals with stability in the $10^{-17}$ to $10^{-18}$ domain for intervals longer than 10 seconds.

A test of the possible variability in the rate of oscillators that depends on different physical processes was conducted in 1982 between a pair of atomic hydrogen masers and an ensemble of three superconducting cavity stabilized oscillators [13]. The frequency of the hydrogen maser signal depends on (among many other things) the nature of the proton–electron magnetic hyperfine interaction. The frequency of the SCSO depends on the resonators dimensions. The hypotheses for the test is that these two frequency-determining properties might not vary in the same manner in a varying gravitational potential, $\Delta \phi$ [14]. The varying gravitational potential for this test was that of the sun, owing to the motion of the laboratory on the rotating earth and of the earth’s eccentricity in its orbit about the sun. Stability data between the SCSO and the H-maser yielded a null result at the 1% level in the variation of $\Delta f / f$ divided by $\Delta \phi / c^2$ over periods of several hours.

C. Measurements using Electromagnetic Signals

1) The effects of oscillator instability on measurements of distance and of Doppler range-rate data:

An estimate of the time dispersion of a clock or oscillator predicted for a future time interval, $\tau$, can be obtained from the relation $\sigma_{\Delta \tau}(\tau) \sim \tau \sigma_{\Delta \tau}(\tau)$. In the case of distance measurements made with the one-way propagation of light we can obtain an estimate of range dispersion by writing

$$\sigma_{\Delta r}(\tau) = c \tau \sigma_{\Delta \tau}(\tau)$$  \hspace{1cm} (1)

where $c$ is the velocity of light.

The one-way Doppler frequency shift expression of an oscillator transmitting at a frequency, $f$, moving with velocity $v_r$ toward the receiver is

$$\Delta f = f v_r / c.$$  \hspace{1cm} (2)

The contribution of the oscillator to the imprecision of determining range rate, $v_r$, during a measurement interval, $\tau$, is given by

$$\sigma_{v_r}(\tau) = c \sigma_{\Delta \tau}(\tau).$$  \hspace{1cm} (3)

Figure 2 is a nomograph of range-rate error and range distance error based on the H-maser data in Fig. 1. On the right hand axes are the scales for time dispersion, $\sigma_{\Delta \tau}(\tau)$, and the corresponding one-way range measurement error.

D. Systems for Cancelling First-Order Doppler and Signal Propagation

1) The three-link system used in the NASA/SAO gravitational redshift experiment: Precise frequency comparisons can be made between widely-separated clocks in relative motion with a Doppler cancelling system [15], [16]. This technique involves continuously measuring the cycles of phase of a signal that has been transmitted to, and phase-coherently transponded from, a space vehicle. By subtracting one-half the number of these cycles from the phase of the received signal in the one-way microwave link connecting the space...
vehicle clock to the earth station, the propagation effects can be systematically removed.

The cartoon shown as Fig. 3, describes the system that was used in the 1976 SAO-NASA test of the gravitational redshift. The fractional output frequency variations obtained by subtracting one-half of the two-way Doppler cycles from the one-way cycles received by the earth station is

$$\frac{f_s - f_e}{f_0} = \frac{\phi_s - \phi_e}{c^2} - \frac{\phi_s^2 - \phi_e^2}{2c^2} - \frac{r_{se} \cdot \alpha_e}{c^2}$$.

(3)

This expression is accurate to order $c^2$.

Here $(f_s - f_e)/f_0$ is the total frequency shift divided by the clock downlink frequency, $f_0$. The term $(\phi_s - \phi_e)$ is the Newtonian potential difference between the spacecraft and earth station, $\vec{v}_s$ and $\vec{v}_e$ are the velocities of the earth station and the spacecraft, $r_{se}$ is the distance between the spacecraft and earth station and $\vec{a}_e$ is the acceleration of the earth station in an inertial frame. For the 1976 test, an earth centered-frame with axes aimed at the fixed stars was sufficiently "inertial" to satisfy the requirements of the the two hour experiment.

The first term is the gravitational redshift resulting from the difference in the Newtonian gravitational potential between the two clocks, the second term is the second-order Doppler effect of special relativity, and the third term is the result of the acceleration of the earth station, owing to the earth’s rotation. (This term would be zero with an earth station at the earth’s poles—a chilling prospect!) During the two-hour near-vertical flight the first-order Doppler shifts were as large as $\pm 2 \times 10^{-5}$ and the noise from ionospheric and tropospheric propagation effects was at a level of about $1\times10^{-12}$ at $\tau \sim 100$ s, as shown in the top left curve of Fig. 1. After the frequency variations predicted in (3) were fitted to the data, we concluded that the error in the fit of the data was within $(+2.5\pm70) \times 10^{-6}$ of Einstein’s prediction [17]. When the predicted frequency variation over the time of the mission was subtracted from the data and the residuals were analyzed, the resulting Allan standard deviation shown in Fig. 4 was obtained. Here we see that the stability of the frequency comparison made through the 3-link system over signal paths of 10 000 km, in the presence of Doppler shifts of magnitude $\pm 2 \times 10^{-3}$ of the carrier frequency, including ionospheric and tropospheric noise as shown in Fig. 1, is comparable to frequency comparison made between the two reference masers in the same room, reaching $6 \times 10^{-15}$ stability at about $10^4$ s.

A closer look at the system in Fig. 3 is provided in Fig. 5. Phase coherence throughout the system was provided by phase coherent ratio synthesizers. We see that there was a considerable difference in the frequencies owing to the transponders turn around frequency ratio 240/221 that could have caused serious problems from the dispersion caused by the ionosphere.

2) Removal of ionospheric Doppler shifts: The total Doppler frequency shift, $f_D$, due to the signal path, including the refractive indexes of the propagation medium, is given by

$$f_D = \frac{1}{f_{zc}} \frac{d}{dt} \int n(t) \, dt$$

(4)

where $n(t) = n_A(t) + n_I(t)$ is the sum of the time-varying refractive indexes of the atmosphere and the ionosphere, respectively. The refractive index of the atmosphere, at S-band, has no significant frequency dependence and has no effect on the 3-link Doppler cancellation system when the propagation time is short compared to the measurement interval.

However, propagation through the time varying ionosphere can cause large frequency variations in a received signal. In the 1976 test these shifts were estimated to

\[ \sigma_{f_D}(T) \approx \sigma_{r/I}(T) \]
Fig. 3. The 1976 SAO/NASA gravitational redshift experiment.

Fig. 4. Allan standard deviation of frequency stability of the two ground station hydrogen masers, P-6 and P-7 made simultaneously with the comparison with the GP-A space maser during the two hour near-vertical mission flown on June 18, 1976. The space maser data are from the frequency residuals after removal of the predicted relativistic and gravitational effects.
be several parts in \(10^{-10}\) and would have completely overwhelmed the data from the combined redshift and second-order Doppler effect.

Frequency shifts caused by variations in ionospheric electron density have been analyzed by Tucker and Fannin [18] who write the ionospheric refractive index as follows:

\[
n_{I} = \left[1 - \frac{f_m^2}{f^2}(1 - \frac{f_m}{f})\right]^{1/2}
\]

(5)

where \(f_m = \mu H e/2\pi m = 2.8\) MHz/Oersted, is the effect of the electrons’ magnetic interaction (which for S-band signals near earth is small, and in our case could be neglected). The quantity \(f_m^2 = \frac{pe^2}{(2\pi)^2}\) is the square of the electron plasma frequency. Here \(p\) is the electron density, \(e\) and \(m\) are the electrons charge and mass, and \(\varepsilon_0\) is the permittivity of free space.

Following Tucker and Fannin, the frequency shift owing to the change in the amount of ionization in the propagation path \(\Delta f_{DI}\), is given by including (5) in (4) and expanding, neglecting the magnetic term:

\[
\Delta f_{DI} = \frac{1}{c_{fc}} \int_{P} \frac{1}{dt} \left(1 - \frac{pe^2}{8\pi^2\varepsilon_0 m}ight) dt
\]

\[
= \frac{e^2}{8\pi^2 c_{fc} \varepsilon_0 m} \int_{P} \rho(t) dt
\]

\[
= \frac{40.5}{c_{fc}} \int_{P} \rho(t) dt.
\]

(6)

Here, \(p\) is the propagation path over which the integral is taken. The integral represents the columnar electron density in the path. When we follow the ionospheric Doppler frequency shifts through the two transponder links and the clock downlink, we find that the ionosphere contributes a frequency error at the output from mixer M3 of Fig. 5 in the amount

\[
|\Delta f_{\text{error}}| = \frac{4.05}{c_{fc}} \int_{P} \rho(t) dt \cdot \left[\frac{Q}{P} - \frac{1 + N^2/M^2}{S^2P/2R^2Q}\right].
\]

(7)

This error can be removed by choosing the ratios so as to make the quantity in the square bracket equal to zero [19]. In the 1976 redshift experiment the transponder ratio, \(N/M\), was 221/240. We chose \(P/Q = 76/49\) and \(R/S = 82/55\), so that the resulting error was \(2.5 \times 10^{-5}\) of the ionosphere Doppler shift in the one-way link. We estimated that [20] under typical ionospheric conditions \(\Delta f_{DI}/f\) could have been as large as \(3 \times 10^{-10}\), which is comparable to the relativistic and gravitational effects we measured. Without cancellation of the ionospheric effect the experiment would not have been possible.

E. A Symmetrical Four-Link Doppler Cancelling System

By transponding the clock downlink back to the space-
Fig. 6. A four-link Doppler cancelling system that allows time correlated data to be obtained both at the earth and space terminal.

In addition to providing Doppler cancellation for intervals \( \tau < R/c \), the 4-link system also makes possible high precision measurements of Doppler signals over very long distances. By time-correlating the Doppler responses we can systematically cancel a strong localized noise source such as the earth’s troposphere and ionosphere [22]. For example, let us consider the frequency variations in the \( E_1(t), E_2(t), S_1(t), \) and \( S_2(t) \) Doppler outputs shown in Fig. 4, assuming the complete removal of the smoothly varying Doppler shifts owing to relative motion. We see that the iono-tropo noise pattern received from the spacecraft transmitted at time \( t_i \) and received at earth at time \( t_i + R/c \) is the same as the noise received at the spacecraft at time \( t_i + 2R/c \). By advancing \( E_i(t) \) by time \( R/c \) with respect to \( S_i(t) \) and subtracting the two data sets we can systematically remove the noise in the \( S_i(t_i + R/c) - E_i(t_i + R/c) \) combined data set at the small expense of increasing the random noise in the data by \( \sqrt{2} \). In situations where the localized dominant noise is substantially larger than the nonlocalized random noise, this process can be highly effective.
II. GRAVITATIONAL AND RELATIVISTIC MEASUREMENTS WITH THE 4-LINK DOPPLER SYSTEM

A. Relativistic Doppler Shifts and Redshifts

The Doppler cancelled signal outputs $S_0(t)$ and $E_0(t)$ in Fig. 6 contain relativistic and gravitational information that can be time correlated. Equation 3 is repeated below as $E_0(t)$, along with its counterpart expression at the spacecraft, $S_0(t)$:

$$E_0(t) = \frac{\phi_0 - \phi_\infty}{c^2} - \frac{[\vec{v}_e - \vec{v}_s]^2}{2c^2} - \frac{\vec{r}_e \cdot \vec{a}_e}{c^2}$$

(8)

$$S_0(t) = \frac{\phi_0 - \phi_\infty}{c^2} - \frac{[\vec{v}_s - \vec{v}_e]^2}{2c^2} - \frac{\vec{r}_e \cdot \vec{a}_e}{c^2}.$$  

(9)

By adding the two time ordered data sets we cancel the first term and double the magnitude of the second term, conversely if we subtract the data sets, we double the first-term and cancel the second term. In both instances we must account for the components of acceleration of the earth and space stations along the line of sight in a suitable inertial frame.

B. A Test of Relativistic Gravitation with a Clock in a 24 Hour Eccentric Earth Orbit

The original concept for testing the Gravitational Redshift called for operating a spaceborne clock in a spacecraft placed in a highly eccentric twenty four hour earth orbit [16], [23]. Low inclination orbits with eccentricities as high as 0.6 can produce apogee-to-perigee redshifts of about $4.8 \times 10^{-10}$, and still keep the spacecraft in view of an earth station with a lowest elevation angle of about 15 degrees. Accompanying the redshift there is a second-order Doppler shift of comparable magnitude, which produces a combined frequency variation of $9.6 \times 10^{-10}$ in the Doppler cancelled data described in (8).

Table 1 shows the error analysis of the combined gravitational redshift and second-order Doppler test made in 1976 using a near vertical trajectory. This can be compared with the predicted results of a proposed orbital test. The improvement from 70 parts per million to 2 parts per million results partly from improved clocks, but mostly from the longer averaging intervals and lower estimates of bias errors made available by having time for adjusting and tuning the space maser before taking data.

C. An Extension of the GP-A Experiment to Test General Relativity with a Solar Probe

A series of studies has been conducted [24] of tests of relativistic gravitation using a clock in a space probe going over the poles of the sun within 4 solar radii of the sun’s center. The time of travel from pole-to-pole is roughly 14 hours. This test is intended to reveal the behavior of the second order in the redshift, $[\Delta \phi/c^2]^2$, in a varying gravitational potential.

During the 10 hours before and after the perihelion, the value of $\Phi/c^2$ varies from $5.3 \times 10^{-7}$ at perihelion to $2.0 \times 10^{-7}$ at 10 hours from perihelion. During the same time interval, the second order redshift $[\Delta \phi/c^2]^2$, varies from $2.81 \times 10^{-13}$ to $4 \times 10^{-14}$, as shown in Figs. 8 and 9. Taking the Allan standard deviation of today’s H-masers over 10 hours averaging time as $6 \times 10^{-18}$, the inaccuracy of measurement imposed by the maser instability for the
Table 1  Error Analysis of a Proposed Orbiting Redshift Test and Comparison with the 1976 NASA-SA0 GP-A Rocket Probe Experiment

<table>
<thead>
<tr>
<th>Error Source</th>
<th>GP-A Test</th>
<th>Proposed Orbiting Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-Hours</td>
<td>1 Orbit</td>
</tr>
<tr>
<td></td>
<td>30 Orbits</td>
<td>30 Orbits</td>
</tr>
<tr>
<td>Random Noise—$\Delta f/f$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H$-Maser and Microwave System Noise</td>
<td>$13 \times 10^{-15}$</td>
<td>$&lt; 1 \times 10^{-16}$</td>
</tr>
<tr>
<td>Uncancelled Atmospheric Noise</td>
<td>$2.5 \times 10^{-15}$</td>
<td>$2.5 \times 10^{-15}$</td>
</tr>
<tr>
<td>Uncancelled Ionospheric Noise</td>
<td>$5.0 \times 10^{-15}$</td>
<td>$2.0 \times 10^{-15}$</td>
</tr>
<tr>
<td>RSS</td>
<td>$14.2 \times 10^{-15}$</td>
<td>$3.2 \times 10^{-15}$</td>
</tr>
<tr>
<td>Possible Systematic Bias Errors—$\Delta f/f$</td>
<td></td>
<td></td>
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<tr>
<td>Ground Station Clocks</td>
<td>$3.0 \times 10^{-15}$</td>
<td>$1.0 \times 10^{-15}$</td>
</tr>
<tr>
<td>Space Clock</td>
<td>$4.5 \times 10^{-15}$</td>
<td>$1.0 \times 10^{-15}$</td>
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<tr>
<td>Spacecraft Tracking Error</td>
<td>$3.3 \times 10^{-15}$</td>
<td>$1.0 \times 10^{-15}$</td>
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<tr>
<td>RSS</td>
<td>$6.3 \times 10^{-15}$</td>
<td>$1.7 \times 10^{-15}$</td>
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<td>RSS Combined Random and Systematics—$\Delta f/f$</td>
<td>$15.5 \times 10^{-15}$</td>
<td>$3.6 \times 10^{-15}$</td>
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<tr>
<td>Variation of $\Delta f/f = \Delta f/c^2 + 1/2v_{rel}^2/c^2$</td>
<td>$2.2 \times 10^{-10}$</td>
<td>$9.6 \times 10^{-10}$</td>
</tr>
<tr>
<td>Fractional Accuracy of Test $\Delta f/(c^2 + 1/2v_{rel}^2/c^2)$</td>
<td>$70 \times 10^{-6}$</td>
<td>$3.8 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Fig. 8. The gravitational redshift signature at first order in $\Delta f/c^2$ for a clock passing over the poles of the sun with 4-solar radius distance from the sun’s center when over the equator.

Fig. 9. Second-order redshift signature for a clock passing over the poles of the sun with 4-solar radius distance from the sun’s center when over the equator.
first-order measurement is $1.8 \times 10^{-9}$, the corresponding inaccuracy for the second-order measurement is $2.5 \times 10^{-3}$.

The sun’s gravitational potential is complicated by having a number of multipole components. The largest of these is the solar quadrupole moment, $J_2$, which must be accounted for in an accurate measurement of the second order term in the redshift. Measurements of $J_2$ have been made from solar oscillations [25], and we can confirm the effect of the uncertainty in these measurements from the behavior of the $J_2$ signature in the data during the 14 hour pole-to-pole passage.

At order $c^{-2}$ the first-order redshift has the following behavior:

$$\Phi/c^2 = \mu/r + \mu/r^3 J_2 R^2 \sin(3\cos^2 \vartheta - 1)/2$$

$$= \Phi_1/c^2 + \Phi_2/c^2$$

where $\mu = GM_{\text{sun}}/c^2$. Assuming $J_2 = 1.7 \times 10^{-7} \pm 0.17 \times 10^{-7}$, then at perihelion,

$$r = 4R_{\text{sun}}, \quad \vartheta = \pi/2$$

and

$$\Phi_2/c^2 = -2.8 \times 10^{-15}.$$ 

At about $\pi/7$ hours from perihelion,

$$\vartheta = 0, \quad r = 8R_{\text{sun}}$$

and

$$\Phi_2/c^2 = +7.0 \times 10^{-16}.$$  

(12)

The frequency variation caused by the $J_2$ contribution to the sun’s redshift is shown in Fig. 10 over the time 7 hours before and after perihelion. Its peak-to-peak magnitude is $3.5 \times 10^{-15}$. If we have an error of 10% in the estimate of $J_2$, the uncertainty in its contribution to the redshift over this interval is about $3.5 \times 10^{-16}$, comparable to the instability of the clock over the 14-hour passage. The error contribution will have a distinctive $(3\cos^2 \vartheta - 1)/r^3$ signature in contrast to the very smooth $1/r$ dependence of the first-order redshift and of the $1/r^2$ dependence of the second-order redshift, as shown in Figs. 8 and 9.

An important feature that makes this experiment possible is the ability to take Doppler cancelled data at the probe. In Fig. 7, the earth’s tropo-iono noise having magnitude $N(t_i + R/c)$ occurring at $t = t_i + R/c$ is reported in the spacecraft’s two-way Doppler signal at $t = t_i + 2R/c$ with magnitude $2N(t_i + R/c)$. The one-way data, $S_1$ that is received at the probe at $t_i + 2R/c$ and reported with magnitude $N(t_i + R/c)$ passes through the same tropo-iono conditions as did $S_2$. The spacecraft Doppler cancellation system can thus systematically remove the effect of the tropo-iono noise when it produces the $S_0(t_i + 2R/c)$ data. Note that this is not the case for the earth station Doppler cancelled output $E_0$. In this case there would be about 1000-s delay between the uplink transmission and reception from the transponder at a time 2R/c later. The combined atmospheric and ionicospheric delay could have varied considerably during this time.

D. The Search for Gravitational Radiations using Doppler Techniques

During the long travel time to Jupiter, the NASA Galileo mission is expected to provide an opportunity to search for gravitational radiation using Doppler techniques with the spacecraft transponder in conjunction with the Deep Space Network Tracking Stations that are equipped with H-maser oscillators [26], [27].

In Einstein’s General Theory of Relativity (GRT), gravitational radiation results whenever a massive body is accelerated. Rotating binary stars radiate energy and will eventually collapse together. While evidence for such radiation has not been observed directly, the orbital decay of a binary pulsar has been observed since 1975 and its rate continues to follow very closely the predicted behavior for loss of energy by gravitational radiation [28].

According to GRT, gravitational radiation is described as a wavelike distortion of space-time traveling at the speed of light. When a gravitational wave intercepts an electromagnetic wave, it distorts the frequency of the wave by an amount $h = \Delta f/f$. It should be possible to detect gravitational waves by observing Doppler shifts of a signal that is transmitted by a highly stable microwave (or laser) transmitter and detected by a receiver located at a distance that is greater than about one-half the wavelength of the gravitational wave.

An example of the possible Doppler detection [29] of a pulsed gravitational wave using the 4-link Doppler measurement system is shown in Fig. 11. Here the wavefront of the gravitational pulse is assumed to intercept the earth-probe line at an angle, $\theta = 60$ degrees. With the spacecraft transponder, as in the Galileo experiment, the effect of the pulse would be observed three times in the Earth 2-way Doppler trace:

1) By a Doppler shift of the gravitational-wave disturbing the earth station at $t = t_1$, while it is receiving a signal transmitted earlier by the spacecraft;
2) By its “echo” when the earth station receives a transponded signal at \( t = t_1 + 2R/c \).

3) By the disturbance when the gravitational wave arrives at the spacecraft at time \( t_2 \) and reported at earth at \( t = t_2 + R/c \).

The spacing of the pulses, their time signature designated by the parameter \( \Psi(t) \), and the relative magnitude and sign of the signature are described by a single parameter \( \mu = \cos \theta \) [30], [31]. Here \( t_R \) signifies arrival time at the first station:

\[
\frac{df}{f} = \frac{(1 - \mu)}{2} \Psi(t_R) - \mu \Psi \left[ t_R - L \left( \frac{1 + \mu}{c} \right) \right] + (1 + \mu) \Psi \left( t_R - \frac{2L}{c} \right).
\]

The one-way transmission from the spacecraft would only show the pulses at \( t_1 \) and \( t_2 + 2R/c \).

A similar set of five observations of the gravitational pulses is available at the spacecraft. In this case, \( \mu = \cos(\pi + \theta) \) and another set of five manifestations of the pulse appears in the spacecraft data. While only four of the ten pulses, i.e., those from the two one-way Doppler signals, are unique, the other six are obtained from other paths through the electronics system and offer redundancy to avoid system aberrations masquerading as gravitational wave signals.

If one of the stations is on earth, noise from the earth’s troposphere and ionosphere would be the main limitation to the sensitivity of detection. Because of the \( f^2 \) frequency dispersion of the ionosphere, reduction of its noise is possible by operating at higher frequencies than the presently used S-band (2 GHz) and X-band (10 GHz) systems. Future tracking systems are planned to operate at 33 GHz and even higher.

However, tropospheric noise cannot be reduced by such techniques and will substantially degrade the stability of a signal. Studies [32] show that the Allan deviation of the tropospheric noise for signals passing vertically has a \( \tau^{-2.5} \) behavior for intervals between 20 and 200 s, with \( \sigma_\Delta(100 \text{ s}) = 8 \times 10^{-14} \), as shown in Fig. 1. While it is possible to model the tropospheric frequency shifts using other data, such as the columnar water vapor content and the local barometric pressure, tropospheric propagation variations will nevertheless severely limit the detection of gravitational radiation with transponded two-way Doppler signals. Estabrook [22] estimates that the sensitivity with ideal tropospheric conditions at night in the desert will be at a level of \( h = \Delta f/f \sim 3 \times 10^{-15} \) for gravitational waves in the millihertz region, which is one to two orders of magnitude above the levels estimated by astrophysicists [31].

As mentioned earlier, in the discussion of Fig. 7, the tropospheric noise can be removed systematically by simultaneously recording of Doppler data from a clock in a spacecraft and at the earth station and combining these data. Computer simulations of this process [22] show nearly complete rejection of such spatially localized sources as the near-earth tropospheric and ionospheric variation and the earth station antenna motion noises. In a future situation where both clock systems are in space, and operating at frequencies where the noise from the solar corona ionization is not significant compared to clock stability, the principal nongravitational noise sources will likely be from the buffeting of the space vehicles by nongravitational forces such as light pressure, particle collisions and sporadic outgassing of the spacecraft. Here, again, since these
disturbances are localized at the ends of the system, the time signatures of the noises are separated by $R/c$, and can be distinguished from the patterns expected from pulsed gravitational waves, which have signatures that depend on the parameter $\mu$ of [13].

III. HIGH RESOLUTION ANGULAR MEASUREMENTS BY VERY LONG BASELINE INTERFEROMETRY (VLBI)

A. The Effect of Oscillator Instability on the Measurement of Angles

High resolution angular measurements are of interest in astrometry and light deflection tests of relativistic gravitation [33]. Measurement of the angle between the propagation vector of a signal and the direction of a baseline, defined as the line between the phase centers of two widely-separated antenna, can be made with VLBI techniques [34]. In Fig. 12, two radio telescopes (or spacecraft tracking stations), separated by a distance $L$, each detect the arrival of radio noise signals from a distant radiostar (or a modulated signal from a spacecraft). After heterodyning to a lower frequency the noise signals are recorded as a function of time. This recording is usually made on magnetic tape and the two sets of noise data are subsequently brought to a computer facility to be time correlated. The observable quantities from the correlation process are the correlated amplitude and the relative phase of the signals detected at the widely separated points on the wavefront.

The stability limit on the successive measurements of angle imposed by the oscillator instability on successive measurements of angle taken $\tau$ seconds apart is

$$\sigma_{\Delta \theta}(\tau) \sim \frac{\sigma_{\mu}(\tau)}{L \sin \theta},$$

where $\theta$ is the angle between the propagation vector and the baseline. The result of correlating the noise data obtained from a common source by the two stations is the production of fringes analogous to those observed from two-slit optical diffraction. The spacing between the fringes is $\lambda/L \sin \theta$, where $\lambda$ is the average wavelength of the signals arriving at the antennas. The visibility of the fringes depends on the extent to which the signals arriving at the antennas are correlated. The angular resolution of the interferometer in the direction of the source is given by the change of fringe phase, $\phi$, with source angle, $\theta$,

$$\frac{d\phi}{d\theta} = \frac{2\pi L}{\lambda}.$$

The error in successive angular measurements owing to the instability of the clocks in a terrestrial system with $L = 6000$ km, assuming $\sigma_{\mu}(10^3 \text{ s}) = 1 \times 10^{-15}$, and $\theta = \pi/2$, is given by

$$\sigma_{\Delta \theta}(10^3 \text{ s}) = 5 \times 10^{-11} \text{ radians or } 2\mu \text{ arc s.}$$

However, this is far smaller than the realistic limit on angular measurement with terrestrial stations. The effect of tropospheric and ionospheric fluctuations impose limits that are far more serious than clock instability.

To date (December 1990) the best resolution observed has been at the 100 micro-arc-second level over an 8000
Fig. 13. An array of four spaceborne radio telescopes, each connected to the other by the system shown in Fig. 6.

km baseline in a VLBI experiment operating at 7 mm wavelength [35].

Tropospheric and ionospheric propagation limits and the limits imposed by the size of the earth on the baseline distances can be overcome by operating VLBI stations in space. A successful demonstration of a spaceborne radio telescope operating as a VLBI terminal was made in 1986 [36] using NASA’s orbital Tracking and Data Relay Satellite System (TDRSS) system as a spaceborne radiotelescope in conjunction with a number of radiotelescopes on earth.

As an example of the limits that a spaceborne system could achieve, let us consider a spaceborne system where \( L = 5 \times 10^6 \text{ km} \), \( \sigma_\Delta(10^4) = 4 \times 10^{16} \), and \( \theta = \pi/2 \). In this case,

\[
\sigma_\Delta(10^4) = 2 \times 10^{-13} \text{ radians or 0.05 arc s}.
\]

For \( \lambda = 1 \text{ mm} \) we have \( \lambda/D = 2 \times 10^{-13} \) radians and we see that the limit imposed by clock stability with \( 10^4 \text{ s} \) integration time is capable of resolving fringes at wavelengths, \( \lambda \), as short as 1 mm in a spaceborne system with baseline distances of \( 5 \times 10^6 \text{ km} \). The numbers in this example are chosen to be close to present estimates of the limits for having reasonably well correlated flux at the two stations [37] at the distances chosen.

In addition to making astronomical and astrophysical observations, terrestrial VLBI systems are used to record polar motion and rotation of the earth and to monitor the movements of the earth’s tectonic plates. Relative positions of radio stars and features of their brightness distribution can be made with a precision of a few tenths of a milli-arc-second, however the absolute directions in space of the baselines between VLBI stations depend on the choice of a frame of reference which is usually taken from the position of very distant radio sources.

B. A Spaceborne Four Terminal (VLBI) Array that Establishes an Inertial Reference Frame, a "Gedanken" System to Exercise our Imaginations

Having already stretched the VLBI technique to baseline distances of 5 million kilometers let us go several steps further and postulate the existence of an array of four such stations in the form of a tetrahedron that defines a three dimensional figure in space that is in an orbit about the sun. (The station separations need not be equal) Fig. 13 shows such an array where each station contains a clock that is synchronized to a coordinate time scale and is connected to its three neighbors by the 4 link system shown in Fig. 6 [38]. The six baselines distances define an object in space whose shape is very precisely known as a function of time.
from distance measurements made by measuring the phase of the two-way signals at the stations defining the ends of each baseline.

The orientation of the array of six baselines poses an interesting problem. It should be possible to determine changes in the orientation of the array in terms of an inertial frame defined by the constancy and isotropy of the velocity of light by invoking the Sagnac effect [39]. This effect is the basis for today's laser gyroscopes. To describe the effect, we can visualize the arrival times of light signals sent in opposite senses about a closed path. If the transmitter and receiver are on a surface rotating at $\Omega$ rad/s, and their signals go about a path with projected area, $A$, on that surface, the difference in the arrival times of light signals going around its perimeter is $\Delta \tau = 4\Omega A/c^2$. If we measure the difference in arrival times of signals going in opposite senses about the triangle defining one face of the tetrahedron, we can obtain the component of rotation for that face. From the four triangles that define the tetrahedron we have four rotation components that allow us to measure the rotation vector, and, because the measurement is overdetermined, we can estimate the accuracy of its measurement.

The limit of precision in the determination of the rotation rate imposed by the maser frequency stability performance described in Fig. 1 depends on how the system is operated. If the signals originate from a single clock and are simultaneously transmitted about the three legs in opposite senses and arrive at the point of origin with time difference, $\tau$, the error in the determination of $\tau$, $\sigma_{\Delta \tau}(\tau)$, is given by the time dispersion plot shown in Fig. 2. For intervals up to about 20 s, the white phase noise of the maser oscillator dominates and $\sigma_{\Delta \tau}(\tau)$ is constant at about $2 \times 10^{-15}$ s. The corresponding error owing to oscillator instability in the determination of $\tau$, $\sigma_{\Delta \omega}(\tau) = \sigma_{\omega}(\tau)c^2/4A = 4 \times 10^{-16}$ rad/s. If the array is in a solar orbit with radius 1 AU, the measurement would have to account for the Einstein-deSitter precession of $2 \times 10^{-2}$ arc s/yr (3 $\times 10^{-15}$ rad/s) owing to the bending of space-time by the suns gravity.

This array offers possibilities for astrometric observations rather than imaging of radio sources. However, it may have other applications. Essentially it combines the frame of reference defined by the most distant radio sources using VLBI with an inertial frame defined by the local isotropy and constancy of the velocity of light. This comparison opens, in a modern context, the question of inertial frames raised long ago by Bishop Berkeley, Ernst Mach, and others. This system may provide a way to observe some aspects of the behavior of the missing matter in the universe, which is alleged to have mass overwhelmingly greater than the mass of celestial bodies observed by electromagnetic means. We look to theorists to provide scenarios of processes that could interact with this system by considering effects due to various hypothetical types of missing matter.

This system could be extended to detect pulsed gravitational radiation. Here the array of baselines could provide as many as 60 manifestations of the signature from a single pulse and give information about the speed of propagation, the direction, and the polarization of the gravitational wave.

IV. CONCLUSION

Since the mid-1960's the frequency stability of atomic clocks (or oscillators) has been improving with no end in sight. The units of time and frequency, and the now redefined unit of distance through the velocity of light, are solidly based on atomic frequency standards. This metrology, in a local sense, has been made consistent with the present concepts of gravitation and relativity. Measurements of astronomical and astrophysical quantities near the edges of our universe are now being made in terms of quantum phenomena that occur in particles whose physics encompass staggeringly smaller distance scales. As the performance of atomic clocks improves (and, hopefully, as our national willingness to support fundamental science in space) we should expect some surprises about the nature of our universe, including the relationship between gravitation and the three other known fundamental forces.

There are also new frontiers to explore in spectroscopy, especially in low temperature atomic spectroscopy with the advent of new techniques in cryogenics, laser cooling, and trapping, where the physics of ultralow energy particle interaction will undoubtedly also surprise us.

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REFERENCES

Robert F. C. Vessot received the Ph.D. degree in physics from McGill University, Montreal, Canada in 1956.

From 1951 to 1954 he served in the Royal Canadian Airforce as a Telecommunications Officer. From 1955 to 1960 he was a post-doctoral staff member at the Massachusetts Institute of Technology, Cambridge, MA, working on atomic clocks. He has published over 100 papers in refereed journals, and has made many contributions to books on science and technology. Vessot is a member of the American Physical Society, the American Association for the Advancement of Science, and the Sigma Xi Society. He has been awarded several medals and citations for his work on atomic clocks and gravitation, including the 1964 National Medal of Science.