In the example selected, the received data samples are
\[ x(n) = A_1 \cos(\omega_1 n) + A_2 \cos(\omega_2 n) + w(n) \]
where \( w(n) \) is a white noise sequence of unit variance. The amplitudes \( A_1 \) and \( A_2 \) provide 15- and 20-dB SNR, respectively. The signal frequencies took the values \( \omega_1 = 1.16 \text{ rad/s}, \omega_2 = 2.75 \text{ rad/s} \) for the first 300 samples of the data. Then both frequencies were reduced by \( \Delta \omega = 0.375 \text{ rad/s} \) for the following 200 samples. This behavior is shown by solid lines in Fig. 2. The frequency estimates were the result of locating the highest two peaks of the spectral estimate \( S_1(n, \omega) \) evaluated every five samples. The input parameters used for the spectral calculations are \( L = 8, \gamma = 0.9, \) and \( k = 1, 5, 10 \). It is evident, from Fig. 2, that the convergence time increases as \( k \) increases. On the other hand, the frequency estimates exhibit smaller variance for higher value of \( k \).

To show the dependence of the estimator's spectral resolution on \( k \), we derived the spectral windows associated with \( S_1(n, \omega) \), \( S_2(n, \omega) \), and \( S_5(n, \omega) \) for \( L = 30 \) and \( n = 50 \). These windows were calculated by assuming stationarity and taking the expected value of the estimate. As shown in Fig. 3, the bandwidth of the normalized spectral window becomes narrower, i.e., resolution gets improved, for higher value of \( k \).

IV. CONCLUSIONS

The use of multiple pole filter in the autocorrelation function estimation provides a recursive mean to update the power spectrum estimate in both time and the order of multiplicity of the filter pole. Since the increase of the filter order changes the tradeoff between the estimator's convergence and resolution, recursive generation of higher filter order, spectrum estimates can be performed until an acceptable compromise is reached.

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Introducing a Third-Order Wigner Distribution

NEIL L. GERR

The purpose of this letter is to introduce a third-order Wigner distribution, which we call the Wigner bispectrum. This new mixed time-frequency-frequency representation extends the standard Wigner distribution in the same way that the bispectrum extends the power spectrum.

I. INTRODUCTION

Power spectrum analysis is a powerful tool for the study of random and deterministic signals. It has been successfully applied to a wide range of signal characterization, detection, and classification problems. In cases where the power spectrum is time-varying, spectrograms generated via short-time Fourier transform methods are often used. We have recently seen a growing interest in high-order cumulant polyspectra, and, in particular, the bispectrum, as a means for extracting more detailed spectral information about a signal, and in the Wigner distribution as a means for obtaining greater time resolution than that provided by conventional spectrograms. In this letter we will combine these two approaches and derive a third-order Wigner distribution, or Wigner bispectrum.

The power spectrum, also called the power spectral density, and the covariance function of a second-order stationary stochastic process are a Fourier transform pair. Thus they contain equivalent information about the second-order structure of the process. A zero-mean Gaussian process is completely characterized by its second-order structure, i.e., by its covariance. Hence, power spectrum analysis of a zero-mean Gaussian process provides all available information about its structure.

While many processes encountered in engineering applications are approximately Gaussian, few, and one might argue none, are exactly Gaussian. The high-order structure of a zero-mean stationary non-Gaussian process is not determined by the second-order structure. Thus for such a process, the power spectrum pro-

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provides only partial information. This observation has motivated a growing interest in high-order cumulant polyspectra. The bispectrum, which provides third-order information, is particularly useful for skewed processes and signals that exhibit phase coherence at sum and difference frequencies. For more on the subject see the tutorial by Nikias and Raghavan [1] and the book by Rosenblatt [2].

Nonstationarity has motivated the development of a number of time-frequency representations that describe the frequency content of a signal as a function of time. As the most popular of these, the spectrogram, is typically generated by computing short-time periodograms of successive data segments. It is implicitly assumed that the signal is (approximately) stationary over each segment, but possibly nonstationary across segments. Such a signal is said to be “quasi-stationary.” Clearly, long segments yield poor temporal resolution. Short segments yield greater temporal resolution, but at the expense of spectral resolution.

The Wigner distribution is defined somewhat differently from the spectrogram, but also provides information regarding the frequency content of the signal versus time. Like the spectrogram, it reduces to the power spectrum when the signal is stationary. The temptation is to regard it as an instantaneous power spectrum except for the following unfortunate property: The Wigner distribution, though real, can take negative values. Additional properties of the Wigner distribution, its connection to the spectrogram and to the frequency-frequency spectrum of a harmonic process, and its use in analyzing nonstationary signals are given by Martin and Flandrin in [3].

Throughout the following, we will let \( X(t) \), \( t \in (-\infty, \infty) \), denote a zero-mean, real-valued, scalar stochastic process. A number of issues, including the problem of aliasing, arise in adapting the Wigner distribution to discrete time signals. These have been addressed by Peyrin and Prost in [4].

II. THE WIGNER DISTRIBUTION

Several variations on the definition of the Wigner distribution have been employed by different authors. We will employ the Boudevaux-Bartels and Parks [5]

\[
W_x(t, w) = \int X(t - u/2) X(t + u/2) \exp(-iwt) du. \tag{1}
\]

There are three essential properties of the Wigner distribution that we will attempt to preserve in its extension to a higher order:

1. Linearity: If \( X(t) \) is second-order stationary and real, then

\[
E[W_x(t, w)] = \int C_x(u) \exp(-iwt) du = P_x(w)
\]

where \( C_x(u) \) is the covariance function and, equivalently, the second-order cumulant function of \( X(t) \), and \( P_x(w) \) is the power spectral density of \( X(t) \).

2. The “lag functions,” which for this second-order case are given by (from (1))

\[
a(u) = -u/2 \tag{2a}
\]

\[
b(u) = u/2 \tag{2b}
\]

satisfy

\[
a(u) + b(u) = 0 \tag{3a}
\]

\[
b(u) - a(u) = u. \tag{3b}
\]

We refer to (3a) as the “centering constraint” because it centers the Wigner distribution (1) over \( t \), and to (3b) as the “lag constraint” for the obvious reason.

3. The Wigner distributions of both sinusoidal and impulsive signals are characteristic. In the first case, for a single real sinusoid

\[
X(t) = \exp(iwt_0) + \exp(-iwt_0)/2
\]

where \( w_0 > 0 \), we have, for \( t > 0 \),

\[
W_x(t, w_0) \propto \delta(w - w_0)
\]

where \( \delta(t) \) is the Dirac delta function and the calculation is made in the sense of a principal value integral. In the second case, for \( X(t) \) a single impulse at time \( t_0 \) which we model as

\[
X(t) = \delta(t - t_0)
\]

we have that

\[
W_x(t, w) = \begin{cases} 1, & t = t_0 \\ 0, & \text{otherwise.} \end{cases}
\]

III. THE WIGNER BISPECTRUM

Analogous to (1), we define the third-order (cumulant) Wigner distribution, or Wigner bispectrum, of a real-valued signal \( X(t) \) as follows:

\[
W_x(t, w_1, w_2) = \int \int [X(t + a(u_1, u_2)) X(t + b(u_1, u_2)) X(t + c(u_1, u_2)) \exp(-i[w_1 u_1 + w_2 u_2]) du_1 du_2 
\]

(4)

where the lag functions are given by

\[
a(u_1, u_2) = -((2/3)u_1 - (1/3)u_2) \tag{5a}
\]

\[
b(u_1, u_2) = (1/3)u_1 - (1/3)u_2 \tag{5b}
\]

\[
c(u_1, u_2) = (1/3)u_1 + (2/3)u_2 \tag{5c}
\]

As the lag functions (2) satisfy the constraints (3), the choice (5) is obtained as the unique linear solution to the analogous centering and lag constraints

\[
a(u_1, u_2) + b(u_1, u_2) + c(u_1, u_2) = 0 \tag{6a}
\]

\[
b(u_1, u_2) - a(u_1, u_2) = u_1 \tag{6b}
\]

\[
c(u_1, u_2) - b(u_1, u_2) = u_2. \tag{6c}
\]

It is easy to see that with this definition (4), if \( X(t) \) is third-order stationary, then

\[
E[W_x(t, w_1, w_2)] = \int \int C_x(u_1, w_1) \cdot \exp(-i[w_1 u_1 + w_2 u_2]) du_1 du_2 = \int \int C_x(v_1, v_2) \exp(-i[w_1 v_1 - w_2 v_2]) dv_1 dv_2 = P_x(w_1, w_2) \tag{7}
\]

where \( C_x(u_1, u_2) \) is the third-order cumulant function of \( X(t) \) and \( P_x(w_1, w_2) \) is the bispectrum of \( X(t) \). In addition, for a real sinusoid

\[
X(t) = \exp(iwt_0) + \exp(-iwt_0)/2
\]

where \( w_0 > 0 \), (4) yields

\[
W_x(t, w_1, w_2) = \delta(w_1 - (2/3)w_0) + \delta(w_2 - (2/3)w_0)
\]

on the bispectrum principal domain \( 0 \leq w_2 \leq w_1 \leq w_2 \leq \pi \). It is somewhat strange that the energy appears as a spike at the frequency pair \((w_1, w_2) \) = \((2w_0/3, 2w_0/3)\). Of course, we would prefer it at the frequency pair \((w_1, w_0)\). This can be arranged by rescaling the frequency axes, but at the expense of (7). Finally, for an impulse

\[
X(t) = \delta(t - t_0)
\]

we have that

\[
W_x(t, w_1, w_2) = \begin{cases} 1, & t = t_0 \\ 0, & \text{otherwise.} \end{cases}
\]

IV. CONCLUSION AND FINAL REMARKS

Using the bispectrum as a model, we have extended the Wigner distribution to third order in a way that preserves many of its essential features and appealing properties. It is premature to speculate as to whether or not the Wigner bispectrum will ever be usefully applied. Perhaps the time-frequency marginal \( W_x(t, w_1, w_2) \) at \( w_2 \) can provide information about the time-varying phase coupling of frequency components at \( w_1 \) and \( w_3 \). The Wigner bispectrum may also prove useful for extracting time-varying phase information, as in [6] for stationary linear signals.

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Computing Image Texture Features in Parallel Computers

JORGE L. C. SANZ

In this letter, the problem of computing image texture features in parallel computers is addressed. Specifically, it will be shown that Haralick's texture measures [1] are amenable to efficient implementation in certain fine-grained architectures. The main operation that will be used to compute these features is the SEND, also called Random Access Write, command. This command is efficiently implemented in a number of today's computers such as binary n-cubes, mesh-arrays, and some shared memory systems. In particular, it will be shown that the computation of gray-level dependency matrices requires random global communication patterns. This feature and the need for other standard local processing make the classification measures proposed by Haralick and his associates good candidates as benchmarks for parallel computer vision architectures.

I. INTRODUCTION

Parallel computing for image processing and computer vision has received considerable attention during the last few years [2]-[6]. Technological advances have been made possible new processor interconnection topologies [7], [8], and fine-grained architectures [9]-[12]. Many practical algorithms have been implemented in fine-grained arrays [13]-[18], and more recently, also in coarse-grained meshes [19], [20].

Among the many analysis tasks which are relevant to the understanding of images, the description of texture plays a key role. Although significant work has been done on texture analysis, not much of this progress has been used in actual systems. The reason is that texture analysis is computationally intensive, and its related image classification methods require extensive testing to verify their applicability to actual problems. A set of texture measures which have been useful in image classification are given in [1]. Haralick's textural features have recently found applicability to important industrial problems, such as carpet wear assessment [21]. However, high-speed requirements have always been an impediment for applications to automated inspection.

The advent of parallel computers for image processing and other general areas of computing brings new light to the above problem. Massively parallel architectures can provide the necessary power for sophisticated feature computation that would probably take hours of processing on conventional Von Neumann computers.

In this letter, the computation of Haralick's texture measures in parallel architectures will be addressed. Specifically, it will be shown that Haralick's textural features are suitable for fast implementation in message-passing parallel computers such as mesh arrays, hypercubes, and also in some shared-memory machines.

II. THE PARALLEL SEND (RANDOM-ACCESS WRITE) COMMAND

Let us assume a parallel architecture which comprises a set of processors and memories interconnected in some manner. Assume that the architecture works in Single Instruction Multiple Data (SIMD) mode for which a single controller broadcasts instructions to the processing elements (PES). Suppose that an M x M gray-level image is to be processed. We will defined "the grain of the architecture with respect to the image size" as M^2 divided by the total number of PEs. This ratio determines the amount of serial processing that may be needed in a problem. An architecture is called fine-grained respect to the image size when this number is low, typically 16 or less. For example, the MPP is fine-grained for a 256 x 256 image because the ratio is 4. Also, The Connection Machine is fine-grained for its ratio is 1. The concept of architecture grain is important because the time complexity of many problems is proportional to this ratio.

Parallel computers share data among processors using different methodologies, involving different resources. A common model for sharing data is via the SEND command, also known as Random Access Write (RAW). Suppose that each PE holds a record composed of a destination key and data. The key indicates where the PE wants to write the data contained in the record. The type of key depends on the communication technology. For example, in a shared-memory architecture the key is an address in the global memory of the system. In the other hand, in a message-passing processor interconnection the key is the identifier of the processor in the system. As is seen, the SEND command is a parallel write operation where every processor tries to send data to another processor, or to write to a memory location [23], [22]. The SEND command usually allows for many processors to have the same destination key. In this case, conflicts must be resolved since several PES want to make their data available to the same PE, or try to write to the same memory location. A common way to resolve these collisions is to store or receive for each key, the minimum value of the data, or the sum of all the data, or the value of any other binary, commutative, and associative operator. This methodology is useful for devising functions that support different languages, such as parallel versions of LISP [23]. In this way, parallel list traversing, and other graph-oriented algorithms can be easily programmed. Also, image operations such as component labeling are implemented using divide-and-conquer principles and SEND commands [22].

In certain applications, the size of the image may be large enough so that each PE has to handle a window of pixels. On the other hand, some algorithms require the execution of a SEND command for every pixel in the image. This means that each pixel has a message to be transmitted. Since the PEs have a set of pixels to process, the performance of a SEND command degrades with coarse architecture grain. The reason is that the PEs have more than one message to be sent and they will also be the final destination of several messages. However, for fine-grained architectures such as The Connection Machine [8], the performance of SEND commands is quite satisfactory [23].

As will be shown in the next section, the SEND command is the essential operation for computing Haralick's texture features in parallel architectures.

III. HARALICK'S TEXTURE MEASURES

The texture measures reported in [1] are based on the construction of several matrices called gray-tone spatial dependency. In this section, each dependency matrix will be built by using two SEND commands. After this matrix is computed, a number of texture features can also be obtained efficiently. Since several fast SEND commands are known for different message-passing architectures, and practical algorithms are available in current machines, the proposed methodology is expected to have significant practical use. In addition, it will be noted that the worst case asymptotic complexity of Haralick's measures is k - O(N) for an N x N PE mesh array architecture having grain k. In addition, in a hypercube or binary n-cube having 2^p PEs, the worst case complexity is k - O(2^p).