Broadside Resonance Scattering From Elastic Spheroids

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Abstract—We study the scattering of monochromatic plane acoustic waves incident at 90° angles relative to the axis of symmetry (i.e., broadside or beam aspect) on solid elastic spheroids. In this analysis, the aspect ratios of the spheroids vary in the range: 2 ≤ L/D ≤ 5 in steps of one. The nondimensional frequency kL/2 is kept within the band: 2 ≤ kL/2 ≤ 24. As we have repeatedly stated in the past, no closed-form analytic solution can be found for this problem, since the method of separation of variables fails. We thus generate a numerical solution based on a modification of the T-matrix method. We generate predictions for the backscattered echoes (i.e., form functions whose squares are the monostatic cross sections) and graphically display their frequency dependence in order to be able to study the resonance features present within them. In this three-dimensional study we identify the (leaky) Rayleigh-type resonances consistent with those present in infinite cylinders. We have explained these in terms of our earlier standing-wave method to interpret these resonances. Rayleigh resonances, caused by the corresponding Rayleigh surface waves, get excited by the circumnavigations of these standing waves along the longest possible complete geodesic path around the spheroid for any angle of incidence. The (2, 1) Rayleigh resonances excited from the end-on direction for L/D = 1 are also present for the L/D-values we have considered. They are also consistent with our earlier surface-wave interpretation of end-on resonances. We also identify the presence of flexural resonances at oblique incidence angles and two types of whispering gallery resonances that are excited along the longest and the shortest geodesical paths around the spheroid. A comparison with the (broadside) resonances of an infinite cylinder is also presented.

I. INTRODUCTION

We predicted and computed form functions for elastic shells insonified by plane monochromatic sound waves incident from all directions [1]. This study included the broadside incidence case, but only for low aspect-ratio spheroids (i.e., L/D ≤ 2). The broadside incidence case is always the hardest because there the convergence of the T-matrix method requires lengthy computations. These convergence difficulties increase as L/D increases. We have found [1] that for low aspect ratios, two classes of resonances are always excited. These resonances are caused by surface standing waves that travel along the longest and shortest geodesic paths around the spheroid. Incidences along the end-on or the broadside directions, respectively, generate the surface waves along the longest or the shortest path lengths on the spheroid. Waves incident at an arbitrary oblique angle excite both families of resonances with different weights. Another type of resonance appears when plane waves are incident on a spheroid about an angle of 45° with its symmetry axis. We recently noticed [2] that in that case the backscattered echoes are dominated by resonances of a flexural nature, which have amplitudes several orders of magnitude greater than those due to either Rayleigh or whispering gallery resonances. We extend that analysis here to include the previously unreported case of broadside incidences (i.e., θ = 90°), where the convergence difficulties were greatest, particularly for high aspect-ratio spheroids such that L/D ≥ 2. The spheroids are made of steel with a compressional speed c_d = 5.95 × 10^3 cm/s, a shear speed c_s = 3.24 × 10^3, and a density ρ = 7.7 g/cm^3, and they are submerged in water, of speed c_1 = 1.4825 × 10^3 cm/s and density ρ_1 = 1 g/cm^3.

We remark in closing that we have already studied [3] the end-on scattering from spheroids of aspect ratios in the range 2 ≤ L/D ≤ 10 and in frequency bands as wide as 5 ≤ kL/2 ≤ 18. The form functions for these progressively more prolate spheroids were compared to those for a sphere. We concluded that the (leaky) Rayleigh-type resonances studied time and again [4]–[7] for the case of spheres were also present in the end-on return from spheroids, even for those with L/D = 10. This conclusion emerged from the observation that all the resonance locations were consistent with our surface-wave interpretation [1], [8], [9] of resonances, provided that the phase velocity of the standing wave that generates it coincides with that of a Rayleigh surface wave. We note that, consistent with critical-angle arguments and the surface-wave interpretation of resonances, the resonances are excited by the circumnavigations of standing waves around the longest and shortest geodesic paths around the spheroid, especially in the pertinent case of incident waves along the broadside direction. Fig. 1 illustrates this point.

II. THEORETICAL BACKGROUND

Scattering of monochromatic plane waves from an elastic 3-D object submerged in water has been studied by means of many numerical techniques [10]. Analytic, closed-form solutions are not possible as soon as the elastic body is not a sphere or a cylinder, since the method of separation of variables then fails. A particularly suitable calculational approach for elastic spheroids (i.e., "cigar-shaped" bodies) is the T-matrix method. This approach has evolved considerably [11] since it was first introduced into acoustics [12] and elastodynamics.
BROADSIDE RESONANCE SCATTERING FROM ELASTIC SPHEROIDS

Fig. 1. Greatly exaggerated view of the surface distortion of a prolate spheroid in water excited along the broadside (I) and end-on (11) directions. We show the standing (surface) waves that travel along the longest and shortest complete geodesic paths around the spheroid. Note that both standing wave patterns for the Rayleigh resonance can be excited by broadside incidences.

We have used it, described it, and modified it repeatedly in the past [13]. A brief synopsis of its main results follows.

Consider a partial-wave expansion of an incident and scattered field; i.e.,

\[ U^i = \sum_n A_n \text{Re} \psi_n, \quad U^s = \sum_n F_n \psi_n \]

where \( \psi_n \) is the nth partial wave, \( \text{Re} \psi_n \) is the regular part of \( \psi_n \), and \( A_n \) and \( F_n \) are the expansion coefficients of the incident and scattered fields \( U^i, U^s \), respectively. The set of known quantities \( A_n \) is related to the set of unknown quantities \( F_n \) by

\[ F_n = \sum_m T_{nm} A_m \quad \text{or} \quad F = TA \]

where \( T \) is the \( T \) matrix. The basis states \( \psi_n \) are arbitrarily selected for the problem in question. For 3-D problems, \( \psi_n = h^{(1)}_m(r) Y^m_n(\theta, \phi) \), where the \( Y^m_n \) are the spherical harmonics of order \( n \) and azimuthal index \( m \), and \( h^{(1)}_m \) are the spherical Hankel functions [15]. We never found any need for a nonspherical basis formulation.

For scattering from elastic targets, the boundary conditions for the displacement \( \vec{U} \) and traction \( \vec{t} \) are

\[ \vec{n} \cdot \vec{U} = \vec{n} \cdot \vec{U}^-, \quad \vec{n} \cdot \vec{t}^+ = \vec{n} \cdot \vec{t}^- , \quad \vec{n} \times \vec{t} = 0 \]

which are applied on the spheroid's surface.

The next step in the derivation is to use the integral representations for both the exterior and interior regions in partial-wave space, and to use the boundary conditions in the process, after expanding \( \vec{U} \) and \( \vec{t} \) in partial waves. Eventually we arrive at the expression that relates incident to scattered fields. After much algebraic detail, the resulting \( T \) matrix takes the form:

\[ T = -(Q_{RR} + Q_{R0} T_2) M^{-1} P^* ((Q_{OR} + Q_{O0} T_2) M^{-1} P)^{-1} \]

where

\[ M = R_0 + RT_2 + iT_2. \]

Here \( Q \) is an \( n \times 3n \) matrix, \( R_0 \) is a real, symmetric \( 3n \times 3n \) matrix, and \( P \) is a \( 3n \times n \) matrix.

The matrices \( Q, R, P, \) and \( T_2 \) are given by the expressions

\[ Q_{ij}(\omega) = \frac{k_0^2}{\rho_0 \omega^2} \int_S \left\{ \lambda_i \nabla \cdot (\text{Re} \psi^{(Q)}_n \hat{\vec{n}} \cdot (\text{Re} \psi^{(Q)}_n) \psi^{(Q)}_n \right\} dS \]

\[ R_{ij}(\omega) = \frac{k_0^2}{\rho_0 \omega^2} \int_S \left\{ \lambda_i \nabla \cdot (\text{Re} \psi^{(R)}_n \hat{\vec{n}} \cdot \psi^{(R)}_n) \right\} dS \]

\[ P_{ij} = \frac{k_0^2}{\rho_0 \omega^2} \int_S \left\{ \lambda_i \nabla \cdot (\text{Re} \psi^{(P)}_n \hat{\vec{n}} \cdot \psi^{(P)}_n) \right\} dS \]

\[ T_2 = -\text{Re} [Q_2(2C_2)^{-1}]. \]

In the calculations shown here, an improvement of the above formulation was used by setting

\[ T = (\bar{U} \bar{G}^* + I)/2 \]

where

\[ G = (Q_{0R} + Q_{R0} T_2) M^{-1} P \]

\[ U = (Q_{RR} + Q_{R0} T_2) M^{-1} P - 2G \]

\[ T_2 = -\text{Re} [Q_2(2C_2)^{-1}] \]

with \( M \) defined as in (5). The quantities \( G, \bar{U} \) are obtained from \( G \) and \( U \) by performing unitary transformations. The monostatic angular distribution is generated from the above, by

\[ \left| f_m(\theta, \phi, kL/2) \right| = \frac{2\pi L}{kL/2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} (i^* - l^* - m^* - n^*) \frac{1}{T_{n^*}} \cdot \cos m\theta \sin \epsilon_{m} |P|^m \left( \cos \theta \right) P|^m \left( \cos \theta \right) A_m \]

where

\[ \epsilon_{m} = \left( \epsilon_m (2\ell + 1)(\ell - m)! \right)^{1/2} \]

\[ \epsilon_m = 2 - \delta_m. \]
If the scattering angle \( \theta_s \) is fixed in the direction opposite to \( \theta_r \), i.e., \( \theta_s = \theta_r + 180^\circ \) and we vary \( \theta_r \) in \( 0 \leq \theta_r \leq 360^\circ \), we obtain the monostatic angular patterns to be displayed here. Bistatic patterns would result if we fixed \( \theta_r \) and then let \( \theta_s \) vary in \( 0 \leq \theta_s \leq 360^\circ \). For the monostatic case, we find

\[
|f_n(\theta, kL/2)| = \left| \frac{4}{kL} \sum_{l=0}^{\infty} \sum_{m=-\infty}^{\infty} (-1)^{l+m} \right. \\
\left. \cdot e^{i \theta} T_{lm} T_{l}^{*} m P_I(\cos \theta) \right|^{1/2} \tag{13a}
\]

where we have dropped the subindex \( l \). This expression coincides with earlier [16] results and further simplifies in the backscattering direction \( \theta = \pi \). This contains almost all the required \( T \)-matrix formulation we have used in the past. Here, \( \Omega / \Re \) means the "out-going/regular" portions of the solution following these symbols. The above formulation also holds for spheroidal shells. For solid spheroidal bodies we have \( T_2 = 0 \) and \( M = R_0 \), and then the \( T \) matrix in (4) reduces to

\[
T = Q_{RR} R_0^{-1} P Q_{OR} R_0^{-1} P \tag{13b}
\]

Furthermore, in this case, (8)-(10) reduce to

\[
G = Q_{RR} R_0^{-1} P \tag{13c}
\]

\[
U = (Q_{RR} - 2Q_{RR}) R_0^{-1} P \tag{13d}
\]

\[
T_2 = 0 \tag{13e}
\]

while everything else remains as above.

We have also found [1] that the resonance locations for the case of end-on incidences on the spheroid are approximately given by

\[
\left( \frac{kL}{2} \right)_n = \frac{\pi L N}{P} \left( \frac{v_0^p}{c_1} \right)_n \tag{14}
\]

where \( P \) was the length of the path traveled by the standing wave around the spheroid, i.e.,

\[
P \equiv \frac{\pi D}{\sqrt{2}} \sqrt{A_s^2 + 1}, \quad (A_s \leq 2) \tag{15}
\]

where \( A_s = L/D \), \( c_1 \) is the speed of sound in the outer medium, numerically given above for water, \( v_0^p \) is the phase velocity of the \( n \)th surface wave traveling in the end-on direction, and \( N = n + 1/2 \), where \( n \) is the integer mode-order of the resonance. For broadside incidences, the equivalent expression is

\[
\left( \frac{kL}{2} \right)_n = A_s N \left( \frac{v_0^p}{c_1} \right)_n. \tag{16}
\]

Since (14) is an approximation that holds only for small aspect-ratio spheroids, the ratio of the various [(\( n, l \)] Rayleigh resonances in the end-on (i.e., \( \parallel \) direction to those of the corresponding resonances in the broadside (i.e., \( \perp \) direction is given approximately by

\[
\left( \frac{kL}{2} \right)_n \left( \frac{kL}{2} \right)^{1/2} \approx \sqrt{\frac{\pi^2}{2}} \left( \frac{v_0^p}{c_1} \right)_n \tag{17}
\]

which no longer depends explicitly on \( n \)--as long as we use the same one for both—and where \( v_0^p \) is the phase velocity of the Rayleigh-wave traveling along the maximum-length geodesic path normal to the spheroid's symmetry axis. For low aspect-ratio spheroids \( v_0^p \approx u_0^p \), and then (17) could be solved for the value of the aspect ratio; i.e.,

\[
A_s = 2 \sqrt{\left( \frac{(kL/2)^{1/2}}{(kL/2)_n^{1/2}} \right)^2 - 1} \tag{18}
\]

in terms of the locations of pairs of \( \parallel \) and \( \perp \) resonances of the same order \( n \). We note that for \( A_s \geq 2 \), the approximation in (15) becomes progressively poorer and we must then use the exact expression [1], which contains an elliptic integral.

### III. Analysis of Results

The form functions of low aspect-ratio spheroids in the broadside incidence direction should exhibit the main resonance features excited at any arbitrary incidence direction, including the end-on (i.e., the parallel) direction. This follows since the angular region for which both types (i.e., broadside and end-on) of resonances can be excited is broadest for small spheroids, the ratio of the various [\( (n, l) \)] resonances of the lower broadside Rayleigh resonances (\( n = 2 \) to 7, plus the end-on whispering gallery (WG) resonances. At the low-end of the spectrum, near \( kL/2 = 5 \), we notice the presence of the \( m = 2 \) flexural resonance, which we have analyzed at length elsewhere [2].

This resonance shifts to lower frequencies as \( A_s \) increases, as we can see in Fig. 3(a), including the lowest six Rayleigh (R) resonances (\( n = 2 \) to 7), plus the end-on whispering gallery (WG) resonances. At the low-end of the spectrum, near \( kL/2 = 5 \), we notice the presence of the \( m = 2 \) flexural resonance, which we have analyzed at length elsewhere [2].

We have found [8] that the phase velocity of the (2, 1) end-on resonance for steel was \( v_0^p = 3.24 \times 10^4 \text{ cm/s} \), while for the \( n = 3 \) and \( n = 4 \) resonances that value increases by a multiplicative factor [8] of 1.14 and 1.08, respectively. These findings can be used in (16) to predict the location of the three lowest broadside Rayleigh resonances \( n = 2, 3, \) and \( 4 \). For \( A_s = 2 \), we find these shifted values to be \( (kL/2)^{1/2} = 10.93, 17.44, \) and 21.24. These values are quite close to the dips
illustrated in Fig. 3(a). This figure also displays a deep dip at $kL/2 \approx 13$, which we suggest is the lowest broadside WG resonance $(1, 2)\parallel$.

Our earlier studies [16], [8], [1], [3] of end-on scattering from elastic spheroids were easier to compute. They showed us that as $A_e$ increased, the $(2, 1)\parallel$ Rayleigh resonance is very strongly excited for steel, while the higher-order Rayleigh resonances $(n, 1)$ for $n > 2$ are substantially weaker. At the same time, the WG resonances are shifted-up in frequency to the extent that most of them go out of the band displayed in our figures. This is illustrated in Figs. 3(b) and 2(b). These figures show the $(2, 1)\parallel$ Rayleigh resonance strongly excited at both broadside (i.e., Fig. 3(b)) and end-on (i.e., Fig. 2(b)) at the same $kL/2$-value of ~7.0. The other pertinent resonances are indicated in these figures. The two lowest (i.e., $n = 2, 3$) broadside Rayleigh $(n, 1)\perp$ resonances are still accurately predicted by (16), even though now $A_e = 3$. Their values are $(2, 1)\perp = 16.39$ and $(3, 1)\perp = 22.95$, which clearly check with the dips seen in Fig. 3(b). The higher-order ones fall outside the displayed band.

It is likely that at the higher frequencies at which the broadside resonances occur, their phase velocities become less dispersive and approach values closer to the flat-plate limit. In Fig. 3(b) there is a deep dip at $\sim 18$, slightly above the lowest "parallel" Rayleigh resonance $(2, 1)\parallel$. This resonance is the $(1, 2)\parallel$ resonance, which is the lowest of the WG resonances.

Figs. 2(c) and 3(c) display the end-on $(\perp)$ and broadside $(\parallel)$ form functions for $A_e = 4$, respectively. At $kL/2 \approx 7.5$, both plots show the $(2, 1)\parallel$ resonance peak which, therefore, must be of the Rayleigh type, as in all the previous cases. It is unlikely that this resonance is a "bar-type" resonance excited by a bar wave. Bar waves are axial in the direction of propagation, compressional in nature, and could only be excited at end-on incidences. The fact that this resonance also appears for broadside incidences (cf. Fig. 3(c)) provides another argument to disqualify it as a possible longitudinal "bar" wave [8].

In the band $2 \leq kL/2 \leq 24$, and for $A_e = 4$, Fig. 3(c) shows relatively small contributions from the parallel resonances which in contrast are quite noticeable and strong in Fig. 2(c). The lowest $(n = 2)$ broadside resonance is estimated from (16) to occur at $(kL/2)_1 \approx 21.85$, and this is confirmed by the observation of Fig. 3(c). The lowest broadside Rayleigh resonance $(2, 1)\perp$ is again followed by a deep dip at $kL/2 \approx 23$. This dip is consistent with the WG interpretation of resonances, and is the $(1, 2)\parallel$ WG resonance.

Figs. 2(d) and 3(d) illustrate (again) the presence of the end-on $(2, 1)\parallel$ Rayleigh resonance at $(kL/2)_1 \approx 7.5$ for both
incidence directions. All the graphs in Figs. 2 and 3 show that the (2, 1)$^1$ resonance is weaker when it is excited on the broadside direction rather than along the end-on, and that it becomes progressively weaker as $A_t$ increases. This is consistent with the fact that the allowable angular span at which these broadside resonances can get excited diminishes as $A_t$ increases.

The slight dips respectively seen at $kL/2 = 17.5$ and 18.5 in Fig. 3(c) and (d) correspond to the (4, 1)$^1$ Rayleigh resonance. Fig. 3(d) exhibits no broadside resonance in the displayed band. The lowest such resonance (i.e., the (2, 1)$^1$ can be estimated to occur (from (16)) at $kL/2 \approx 27.3$, which falls out of the figure bounds. We will not see any more broadside resonances in broadside form functions such as those in Fig. 3 for any aspect ratio $A_t \geq 5$ unless we broaden the band that the figure displays. We note here that to broaden this band is a costly and computationally intensive operation, particularly for a larger $A_t$. The ones displayed in Fig. 3, which constitute the thrust of this paper, have never been obtained before and were generated at quite an expense. We point out that we have attempted to generate experimental data to validate these curves in Fig. 3, measuring them at the NSWC's Hydroacoustics Facility. However, due to experimental limitations not yet overcome, the data all lies at high frequencies (i.e., $kL/2 > 30$). We hope that in the future we will be able to lower the experimental least upper bound on the frequency and/or to be able to increase its computational greatest lower bound in order to show a theory/experiment comparison which at present is not possible.

Equation (18) could be used to extract the aspect ratio of a spheroidal target in terms of the ratios of normal and parallel Rayleigh resonances of like orders. We confirm the validity of (18) by using the $n = 2$ pairs (2, 1)$^1 \equiv 10.93$ and (2, 1)$^1 \equiv 7.0$, and also the $n = 3$ pairs (3, 1)$^1 \equiv 17.4$ and (3, 1)$^1 \equiv 11.1$, from Figs. 2(a) and 3(a), all given above. Inserting these values into (18) yields the aspect ratios $A_t = 1.97$ and 1.98, respectively. These values are quite close to the value of 2 used to generate the figure by the $T$-matrix method.

We now compare the broadside resonances on a spheroid of $A_t = 5$ with those obtained in an infinite cylinder at broadside incidence. The infinite cylinder case is a real 2-D situation that admits no other incidence other than broadside. This case has been studied repeatedly in the past [17]-[19]. The cylinder's diameter is $D = 2a$ and its cross-sectional plots are displayed versus $ka$. To compare that scale to the present $kL/2$ we use, $kL/2 = A_t \cdot ka$ (in our case). For an aluminum cylinder, the first four resonances that we have observed [17] are listed in increasing order in Table I. For a steel cylinder the resonances are almost at the same locations as for aluminum. The correction to change steel data to aluminum is simply

$$\Delta(kL/2) = \frac{c_s^6}{c_t^6} \frac{c_t^4}{c_s^4} = \frac{3.10 \times 10^5}{1.476 \times 10^5} \cdot \frac{1.4825 \times 10^5}{3.24 \times 10^5} = 0.96$$

which shows that the resonances for this steel in a water of a certain speed are 4-percent larger than those of aluminum in water of a slightly different speed. The resonances will be broader for aluminum than for steel, but their location is almost the same.

Equation (16) for the broadside resonances is only valid for the Rayleigh resonances (2, 1)$^1$, (3, 1)$^1$, (4, 1)$^1$, etc. It does not work for WG resonances such as (1, 2) or for the anomalous Rayleigh resonance with $n = 1$ (i.e., (1, 1)). For (2, 1) and (3, 1), (16) yields $(kL/2)_t = 27.3$ and $(kL/2)_t = 38.2$. These values agree with the underlined values in Table I for the cylinder. The agreement is not exact because (16) becomes less accurate for spheroids of a large aspect ratio ($A_t > 2$) as we have here, but it is clear that the resonances agree within a few percent. Unfortunately, both these resonances fall outside the range of Fig. 3(d), which as we said displays no broadside resonances. The resonances seen in Fig. 3(d) are all end-on ($\hat{n}$). They are seen there in a weaker way than in the end-on form function of Fig. 2(d). In fact, (4, 1)$^1$ is a small dip, and (3, 1)$^1$ is so weak that it is not seen at all.

The main point emerging from the last comparison is that the broadside form function $|f_{\alpha}^{b}|$ of a spheroid of aspect-ratio 5 contains the broadside features of the 2-D infinite cylinder—which are the only ones the cylinder has—and that these

### Table I

<table>
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<tr>
<th>$n$, $l$</th>
<th>(1, 1)$^1$</th>
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<th>(3, 1)$^1$</th>
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### Table II

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<tr>
<td></td>
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<td>Rayleigh$^4$:</td>
<td>Rayleigh$^4$:</td>
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<tr>
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resonance features already almost coincide with those of the cylinder. In addition, $|f_{m}^{+}|$ also contains end-on (||) resonances which, although weak, are noticeable at the lower frequencies of Fig. 3(d). This is due to 3-D end effects that could have never been predicted with the earlier [17]–[19] 2-D analysis of infinite cylinders at normal incidence. As we mentioned earlier, the low-frequency region where these end-on effects are most noticeable is the hardest region to reach with experimental tank tests of the type we have conducted in the past [18], [20]. We note that the generation of helical or helicoidal surface waves on cylinders of either finite [21] or infinite [22] length has been studied in the past. These waves are analogous to those propagating along geodesics on spheroidal bodies, as we have discussed elsewhere [1], [2], [14]. Table II lists some of the numerical values of the broadband and end-on resonances that are graphically displayed in Figs. 3 and 2, respectively.

IV. SUMMARY

We have studied the end-on (||) and broadband (⊥) resonance features present in the form functions of an elastic prolate spheroid of increasing aspect ratio. We find that all the end-on features also manifest themselves in the broadband form functions of an elastic spheroidal bodies, as we have discussed elsewhere [1], [2], [14]. As mentioned above, the low-frequency region where these end-on effects are most noticeable is the hardest region to reach with experimental tank tests of the type we have conducted in the past [18], [20]. We note that the generation of helical or helicoidal surface waves on cylinders of either finite [21] or infinite [22] length has been studied in the past. These waves are analogous to those propagating along geodesics on spheroidal bodies, as we have discussed elsewhere [1], [2], [14]. Table II lists some of the numerical values of the broadband and end-on resonances that are graphically displayed in Figs. 3 and 2, respectively.

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