Observations of the von Karman Constant Over Open-Ocean Waves

PAT S. DeLEONIBUS AND LLOYD S. SIMPSON

Abstract—Forty-six open-ocean observations of the von Karman constant $k$, estimated from the momentum flux, wind velocity, and air temperature differences observed at the Argus Island tower, yielded an average value of $0.40 \pm 0.18$ for $|Ri| \leq 0.014$, where $Ri$ is the gradient Richardson number. This average value agrees with the recent determinations of $k$ over land by Hogstrom [13] and Zhang et al. [27]. Scatter in these oceanic data sets may be due in part to ocean-wave influence as well as to instrumental and statistical variability.

I. INTRODUCTION

DIFFICULTIES in conducting open-ocean field experiments to measure atmospheric surface-layer wind turbulence over wind-generated waves and swell are well recognized. Simultaneous observations of momentum flux, wind profiles, atmospheric stability, and wave spectra over the open ocean are still relatively scarce. For this reason, some of the older data are still useful in documenting characteristics of surface-layer wind flow over open-ocean wind-generated waves and swell, particularly in documenting the ocean-wave influence on turbulent wind flow, which has been observed a few meters above the highest waves. DeLeonibus and Simpson [4], for example, provided an interpretation of near-neutral drag coefficients in terms of wave phase velocities of dominant waves (ocean-wave spectral peak) as well as the drag coefficient dependence on the mean horizontal wind velocity. Their data illustrated differences in trends of near-neutral drag coefficients at various ranges of the mean horizontal wind velocity, depending on whether the dominant ocean-wave phase velocities were slower than, equal to, or greater than the mean horizontal wind velocity. DeLeonibus and Simpson [4] used data observed during 1964, 1967, and 1969 at an open-ocean tower (Argus Island) to present this new analysis of the drag coefficient dependence on wave background.

In this report we use some of the same data to analyze the open-ocean behavior of

$$k/\phi_M(Ri) = U^*/Z \frac{dU}{dZ}$$

where $U^*$ is the friction velocity, observed at height $Z$, $dU/dZ$ is the wind shear, $\phi_M(Ri)$ is the nondimensional wind shear, and $Ri$ is the gradient Richardson number. At neutral stability, where the Richardson number $Ri$ is equal to or, for practical purposes, very close to zero, $\phi_M(Ri) = 1$ and $k$ is the von Karman constant. From data observed in 1964, 1967, and 1969, an average value of $k$ for $|Ri| \leq 0.014$ was calculated to be $0.40$ in agreement with the average values observed by Hogstrom [13] and Zhang et al. [27]. Indeed, for the total of 81 observations discussed in this report, an average value of $k = 0.40$ was also observed over a narrow range of Richardson numbers $-0.054 \leq Ri \leq 0.014$, suggesting that “near neutral” conditions may have been associated with this range.

The scatter in Argus Island tower observations, however, was enormous, as expected. Such large scatter over the open ocean, particularly with respect to eddy-correlation data, seems to be characteristic of open-ocean observations. (See, for example, the drag coefficient data shown in [15, figs. 3 and 5] and [4, fig. 1.]) Nevertheless, we proceed on the assumption that documentation of carefully obtained open-ocean data, even with a large scatter, can provide insight on how to parameterize characteristics of wind turbulence over wind-generated waves and swell. DeLeonibus and Simpson [5], for example, used 1967 and 1969 data to provide additional support to the assertion that acceptable values of dissipation-derivated drag coefficients can be observed over the open ocean with horizontal wind-velocity spectra. Their observations suggest that the measurement of height and atmospheric stability must be taken into account in dealing with the anisotropy observed in higher frequency wind data.

Part of the motivation for this study was that open-ocean tower observations of momentum flux, wind profiles, and atmospheric stability obtained simultaneously with observations of ocean-wave spectra are so very few in number. Such observations are still needed in order to understand and interpret open-ocean data derived from aircraft and polar-orbiting satellites.

II. INSTRUMENTATION AND DATA ANALYSIS

Momentum flux, wind profiles, air temperature differences, sea surface temperature, air temperature, and ocean-surface wave height were observed in 1964, 1967, and 1969 at the Argus Island tower (Fig. 1), which is a research platform formerly located on Plantagenet Bank, 45-km southwest of Bermuda where the water depth is 60 m. The base of the tower was 21 m above the mean sea level, and a vertical probe containing wind and temperature sensors was placed 18-m upwind of the north and south faces of the tower. The vertical probes had fast-response cup anemometers mounted at 6, 7.5, and 10 m for measurements of the horizontal wind component. At the 7.5-m level a three-bladed fast-response windmill
The air temperature sensor consisted of fine-wire thermocouples, and an air temperature difference between the 6 and 10-m levels was recorded continuously. The vertical probe was lowered 2 m for some observations during 1964. In this case the cup anemometers were at 4, 5.5, and 8-m levels, and the windmill anemometer was at 5.5 m to measure vertical velocity.

The sensitive-cup-type wind speed anemometers used during these field experiments were matched to one another to within 0.1 percent and were calibrated against a laboratory standard, with the calibration traceable to National Bureau of Standards before and after the completion of the tower program. Internal friction of each anemometer was measured before the anemometers were shipped to the site. The internal friction was rechecked periodically during the field program to ensure that the sensors retained their minimum friction level during usage. The following specifications will serve to describe the basic characteristics:

- Wind speed range: 0–1450 cm/s.
- Starting speed: Less than 8.9 cm/s.
- Distance constant: 83.3 cm.
- Anemometer cup assembly: Conical, plastic, 5 cm in diameter; total cup assembly weight, 7 g.
- Maximum interference dimension viewed by the wind: 1.3 cm.
- Shaft-counting technique: Frictionless light-bulb-shutter photocell combination.
- Response, general: Response of anemometer is essentially flat for winds having angles of attack of plus or minus 40°.

The sensitive vertical velocity anemometer uses an ultra low-friction shutter bearing assembly similar to that employed in the sensitive-cup anemometer. An empirically developed three-bladed lightweight sensor replaces the cup assembly in this instrument. The air foil of the sensor is constructed so that the sensor and its supporting shaft turn at a rate which is directly proportional to the magnitude of the vertical component of the wind. The physical form of the sensor is such that it will turn in one direction when exposed to updrafts, and in the opposite direction when exposed to downdrafts. The anemometer has two photocells working in conjunction with a multislottered shutter and a light source to generate the velocity signals and to provide coded signals for identification of the direction of shaft rotation. The transmitter offers little interference to the natural wind flow. The following specifications summarize the characteristics of this anemometer:

- Range: ±650 cm/s (vertical component).
- Accuracy: ±4.33 cm of vertical air travel per recording period. Calibrated in wind tunnel against a wind speed anemometer with the calibration traceable to the National Bureau of Standards.

The wind sensors were incorporated into a "flux meter" system described by Field and Superior [7] and DeLeonibus [3] for observation of momentum flux. The flux meter system is believed to adequately respond to and record nearly all the turbulent flux of scale lengths associated with the observation height of 7.5 m. A discussion of the four types of errors involved in the flux meter approach 1) inertial lag; 2) resolution by step function; 3) smoothing of component velocities in the flux-meter summation process; and 4) pulse storage loss) is given in [7].

During the 1964 experiment horizontal and vertical wind velocities were used to calculate momentum flux at 7.5 m with a flux meter approach at contiguous 15-min intervals. Flux meter data were also obtained in 1964 at 5.5 m. In 1967, 1/2-h analog recordings of horizontal and vertical wind velocities at 7.5 m were used to estimate the momentum flux. In this report, 4 15-min contiguous observations were combined to represent a 1-h observation for the 1964 data. During 1969, flux meter data were observed consecutively for 55 min during each hour.

The momentum flux \( \tau \) was calculated from the mean product of horizontal \( u' \) and vertical \( w' \) wind velocity fluctuations \( -u'w' \) according to

\[
\tau = \rho \left( -u'w' \right)
\]

with the air density \( \rho \) assumed to be constant. Momentum flux is conveniently interpreted in terms of a friction velocity,

\[
U^* = \left( -u'w' \right)^{1/2}
\]

Air temperature and mean horizontal wind velocity differences \( T_{10} - T_6 \) and \( U_{10} - U_6 \), respectively, at 6 and 10-m levels were used to calculate the gradient Richardson number \( Ri \) at the 7.5-m level according to the expression:

\[
Ri = \frac{\frac{g}{T} \left( \frac{T_{10} - T_6}{7.5 \ln 10/6} + \Gamma \right)}{\left( \frac{U_{10} - U_6}{7.5 \ln 10/6} \right)^2}
\]

where \( g \) is the acceleration of gravity, \( T \) is the absolute temperature, and \( \Gamma \) is the adiabatic lapse rate. Air temperature and wind velocity differences at 4 and 8-m levels were used to calculate \( Ri \) at 5.5 m.

Atmospheric stability for this report is defined in terms of
gradient Richardson numbers according to
\[ |\text{Ri}| \leq 0.014 \quad \text{Neutral} \]
\[ -0.054 \leq \text{Ri} \leq 0.014 \quad \text{Near-neutral.} \] (5)

Wind shears for 7.5 and 5.5-m levels, respectively, were calculated according to
\[ \frac{dU}{dZ} = \frac{U_{10} - U_6}{7.5 \ln 10/6} \]
\[ \frac{dU}{dZ} = \frac{U_8 - U_4}{5.5 \ln 8/4} \] (6)

where \( U_{10}, U_6, U_8, \) and \( U_{10} \) are the wind speeds at 4, 6, 8, and 10 m, respectively, and \( \ln \) refers to the natural logarithm. (Winds at 43 m and ocean-wave spectra were also observed during the month of February 1964 prior to the start of the momentum-flux observation program. These are discussed in Fig. 4.)

Ocean-surface wave height recordings were obtained with a 15-m-long resistance-wire wave staff. A significant wave height \( (H_{1/3}) \), the average height of 1/3 of the highest waves, was calculated from wave-height spectral density \( S(f) \) according to
\[ H_{1/3} = 4 \left( \int_0^\infty S(f) \, df \right)^{1/2} \] (7)

where \( f \) is the radian frequency. Typical open-ocean gravity wave spectra are relatively narrow band (compared to the wind velocity spectra), ranging in frequency from about 0.05 to 1 Hz. Spectra are usually characterized by a well-defined peak whose frequency \( f_{SM} \) is typically located between 0.08 to 0.14 Hz. In this report we assume that the dominant open-ocean waves are associated with such peak frequencies and that these waves travel with a phase speed \( C_M \), where
\[ C_M = g/2\pi f_{SM} \] (8)
in which \( g \) is the acceleration of gravity. Occasionally, double-peaked spectra occur with the lower frequency peak associated with swell and the higher frequency peak associated with a wind-generated sea. Dominant waves were associated with the lower frequency peaks (double peaks were observed in 15 percent of the data). A dimensionless wave phase speed \( C_M/ U \), sometimes referred to as wave age, is often used as an oceanic parameter.

III. WIND AND WAVE OBSERVATIONS

Out of the 81 observations of momentum flux, wind profile, and atmospheric stability, only 46 observations occurred for \( |\text{Ri}| \leq 0.014 \), which is a strict criterion for neutral atmospheric stratification. Also, we define “near neutral” in the report to include the range \(-0.054 \leq \text{Ri} \leq 0.014 \). Hence (6) is used to calculate the wind shear instead of the formulation for diabatic conditions. Most of the observations for momentum flux were at 7.5 m. The instrument bar was lowered 2 m for 3 runs during 1964, and all corresponding wind and temperature observations were 2 m closer to the mean sea level during these 3 runs. Very few wave spectra were observed in 1964. However, when wave spectra were observed in 1967 and 1969 the phase velocity \( C_M \) of the ocean-wave spectral peak was faster than the mean wind velocity in almost all the observations. This appears to be a common feature of open-ocean experimental data, and the ocean-wave influence on surface-layer wind observations will be discussed below.

A strict criterion was adopted on the allowable range of wind-direction fluctuations under which valid flux measurements were considered to be acceptable at either the north or south outrigged probes. Wind direction fluctuations were generally less than 20° during data runs. Hence, the error involved in the use of cup and windmill anemometers in evaluating (2) was small [3].

IV. VALUES OF \( k \) FOR NEUTRAL AND NEAR-NEUTRAL STABILITY

Flux-profile relationships are usually interpreted as being a universal relationship for wind shear \( dU/dZ \) of the form,
\[ \frac{dU}{dZ} = \left( U^*/kZ \right) \cdot \phi_M(Z/L) \] (9)

where the nondimensional wind shear \( \phi_M \) is a universal function describing the specific form of the flux-profile relationship in terms of \( Z/L \), the well-known Monin-Obukhov stability parameter [1], [6], [26], [13]. In this report we shall focus on a narrow range of the gradient Richardson number \( R_i \) instead of \( Z/L \), since the wind velocity and air temperature differences were simultaneously observed at the Argus Island tower above and below the height at which the friction velocity was observed. At the Argus Island tower independent measurements were made of the friction velocity, wind shear, and Richardson number, and these were used to calculate \( k = U^*/Z (dU/dZ) \) for \(-0.054 \leq \text{Ri} \leq 0.014 \) under the assumption that \( \phi_M = 1 \) within this range (near-neutral).

Fig. 1 illustrates 81 observations of \( k \), of which 64 observations of \( U^* \) were 7.5 m and 17 observations of \( U^* \) were at 5.5 m above the mean sea level. The average of the 46 observations for \( k \) at \( R_i = 0 \), represented numerically by \( |R_i| \leq 0.014 \), equaled 0.40 ± 0.18. The average value of \( k \) over the near-neutral range \(-0.054 \leq \text{Ri} \leq 0.014 \) was also 0.40 ± 0.20.

The experimentally determined value of \( k \) (over land sites or wind tunnel) has ranged from 0.2 to 0.8 [18], although most of the data cited in the review of 46 experimental investigations [18] ranged from 0.38 to 0.43. Tennekes [22], [23] developed a relationship between \( k \) and the Reynolds number which predicted an asymptotic approach to 0.33 and discussed the uncertainty of \( k \) in terms of the Rossby-number similarity. Telford [20] has recently derived a theoretical value of 0.37. Table I summarizes past and more recent experimental determinations of \( k \). The reanalysis of the data of Businger et al. [2] by Wieringa [25] resulted in a value of 0.41. In two recent experiments [14], a value of 0.39 was obtained based on 72 field observations at \( |R_i| \leq 0.02 \), and Hogstrom [11] observed \( k = 0.40 \pm 0.01 \), revised to \( k = 0.39 \pm 0.01 \) after reanalysis of his 16 field observations. (See the comments by
investigator: Goddard (1970)  
Average value for $k$: 0.41  
Businger et al. (1971)  
Pruitt et al. (1973)  
Hicks (1976)  
Kondo and Sano (1982)  
Hogstrom (1985)  
Hogstrom (1986)  
Hogstrom (1988)  
Zhang et al. (1988)  
Argus Island  

Part of the scatter in Fig. 1 may be associated with statistical variability of the covariance $-\langle u'w' \rangle = U^2$. Whereas $u'$ and $w'$ are approximately normally distributed, the product $u'w'$ is not. Stewart [19] has remarked on the difficulty associated with observing stable averages of this statistic before the overall conditions change during a data run in the field. Haugen et al. [9] observed large negative "bursts" of $u'w'$ at a Kansas field site during unstable conditions with moderate-to-high wind velocities which they associated with submesoscale phenomena such as longitudinal roll vortices. Drag coefficients ($U^2' / U^2$) observed over the open ocean and plotted as a function of the mean horizontal wind velocity scatter widely about a constant or slowly varying mean value [15], [4]. Some scatter is associated with the vertical outrigged probe which holds the wind sensors. The nature of such a disturbance of the mean wind and the influence of this disturbance on observations of momentum flux has been the subject of many papers. At Argus Island a special study was made of the disturbance of the natural airflow caused by the tower [24]. Results of this study specified locations around the tower where accelerated or retarded flow existed. One important result indicated the existence of a region of undisturbed flow about 15-m upwind of the tower where measurement of turbulent fluxes could be attempted. Subsequent wind-tunnel studies [16] on a 1/4-scale model of the Buzzards Bay Electric Light Tower indicated regions about 15-m upwind of that tower where accelerated or decelerated flows can occur. Since Argus Island wind measurements were made at a distance of 18-m upwind of the tower, there is ample reason to conclude that flux measurements were made in undisturbed natural air flow with due allowance, of course, for the outrigged probe itself.

V. Discussion

Part of the scatter in Fig. 1 may be associated with statistical variability of the covariance $-\langle u'w' \rangle = U^2$. Whereas $u'$ and $w'$ are approximately normally distributed, the product $u'w'$ is not. Stewart [19] has remarked on the difficulty associated with observing stable averages of this statistic before the overall conditions change during a data run in the field. Haugen et al. [9] observed large negative "bursts" of $u'w'$ at a Kansas field site during unstable conditions with moderate-to-high wind velocities which they associated with submesoscale phenomena such as longitudinal roll vortices. Drag coefficients ($U^2' / U^2$) observed over the open ocean and plotted as a function of the mean horizontal wind velocity scatter widely about a constant or slowly varying mean value [15], [4]. Some scatter is associated with the vertical outrigged probe which holds the wind sensors. The nature of such a disturbance of the mean wind and the influence of this disturbance on observations of momentum flux has been the subject of many papers. At Argus Island a special study was made of the disturbance of the natural airflow caused by the tower [24]. Results of this study specified locations around the tower where accelerated or retarded flow existed. One important result indicated the existence of a region of undisturbed flow about 15-m upwind of the tower where measurement of turbulent fluxes could be attempted. Subsequent wind-tunnel studies [16] on a 1/4-scale model of the Buzzards Bay Electric Light Tower indicated regions about 15-m upwind of that tower where accelerated or decelerated flows can occur. Since Argus Island wind measurements were made at a distance of 18-m upwind of the tower, there is ample reason to conclude that flux measurements were made in undisturbed natural air flow with due allowance, of course, for the outrigged probe itself.

Table II illustrates the distribution of the mean horizontal wind velocity observed at 7.5 m. Fifty-six of the 81 observations were for $U_{7.5} = 7$ ms$^{-1}$, suggesting that the scatter was not particularly associated with the instrument response for light or weak winds.

Finally, some scatter is associated with the influence of the open-ocean waves. For example, Fig. 3 illustrates that $C_M / U_{3.3} > 1$ for most of the 2- to 3-m significant wave heights observed in 1969. Under such conditions the open-ocean surface roughness cannot be a function only of the local wind stress regime. Fig. 4 illustrates this ubiquitous open-ocean condition in which $C_M / U_{3.3} > 1$ during most of the entire month of February 1964, and where significant wave heights ranged mostly from 2 to 4 m. Figs. 3 and 4 are somewhat
TABLE II

DISTRIBUTION OF OBSERVED MEAN HORIZONTAL WIND VELOCITIES, $U_7$, FOR THE RICHARDSON NUMBER, $Ri$, IN THE RANGE: $0.054 \leq Ri \leq 0.014$

<table>
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<th>$U_7$ (m.s.$^{-1}$)</th>
<th>Number of Observations</th>
<th>$U_7$ (m.s.$^{-1}$)</th>
<th>Number of Observations</th>
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<tr>
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<tr>
<td>9</td>
<td>16</td>
<td>15</td>
<td>7</td>
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</table>

Fig. 2. One-hour observations of the mean and standard deviation of $k = U^4/2dU/dZ$ as a function of the Richardson number ($Ri$). The number of observations are indicated by each point. The standard deviation is indicated by a vertical bar for 8 or more observations.

Fig. 3. Significant wave height ($H_{1/3}$) and ratio ($C_m/U_{1/3}$) of wave phase velocity of spectral ($C_m$) to mean horizontal wind velocity ($U_{1/3}$) as a function of time for the 1969 observations at the Argus Island tower. The gaps in time represent periods when no data were taken.

The wind velocities in Fig. 4 are hourly averages. Wave directions were determined from visual observations at the tower, from ship reports, and were also inferred from 6-h surface weather maps.

Thus, just as near-neutral open-ocean drag coefficients $C_{10}$ = $U^4/2dU/dZ$ scatter widely around average values of 1.0 to 1.5 x 10$^{-3}$ [15], [4], so it appears that open-ocean observations of near-neutral values of the von Karman constant $k$ scatter widely around the traditional land-site value of 0.40. Hence, in addition to serving as a modulating carrier of capillary and short gravity waves, longer waves may influence the momentum flux from atmosphere to ocean surface by imposing an ambiguous roughness signature on the wind-generated roughness field associated with the shorter waves. Since these mobile roughness elements, the shorter waves, are superimposed on the much longer dominant waves, whose phase velocities are greater than the mean horizontal wind velocities more than half of the time [4], it would appear that additional open-ocean experiments, which include wave spectra observations, are needed to clarify scaling laws and flux-gradient relationships over the ambiguously defined roughness regimes normally encountered over the open ocean.

VI. CONCLUSIONS

A mean value of the von Karman constant $k = 0.40 \pm 0.18$, calculated from sets of earlier data observed at an open-ocean tower, agrees with recent observations [13], [27]. The larger scatter in the observations was associated partly with the influence of open-ocean wind-generated gravity waves and swell, partly with the statistical variability of observed covariances of the horizontal and vertical wind velocity fluctuations, and partly with the observing system itself, which includes the disturbance associated with support structures as well as the wind sensor response.

The scatter in the Argus Island tower data in evaluating the von Karman constant and the scatter in evaluating the open-ocean drag coefficients [15], [4] strongly suggest that future experiments with which to validate transfer functions for polar-orbiting satellites include observations of wave spectra as well as wind velocity and atmospheric stability. The large
scatter also suggests that new field experiments with modern instrumentation be conducted from ocean towers to evaluate the von Karman constant.

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REFERENCES


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