Magnetic Dipole Excitation of a Long Conductor in a Lossy Medium

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Abstract—Formulations for the excitation of currents on an infinitely long conductor by electric or magnetic dipoles of arbitrary orientation are presented. The conductor can be either insulated or bare to model ungrounded or grounded conductors. Specific calculations are presented for a vertical magnetic dipole source because this source produces the appropriate horizontal polarization and could be used in a borehole-to-borehole configuration. Numerical results for the induced current and secondary magnetic field indicate that long conductors produce a strong anomaly over a broad frequency range. The secondary magnetic field decays slowly in the direction of the conductor and eventually becomes larger than the dipole source field.

I. INTRODUCTION

The excitation of currents on long, underground conductors is important in many applications. For example, power lines and rails in tunnels can enhance transmission for mine communication [1]. Electromagnetic probing of the earth can be influenced by the presence of cables or pipes [2].

In this paper, we examine the feasibility of electromagnetic detection of long conductors in tunnels. This approach differs from the usual tunnel detection techniques that attempt to detect an air-filled void using VHF frequencies and vertical polarization [3]. For strong excitation of currents on long, horizontal conductors, we consider the use of lower frequencies and horizontal polarization.

Our model consists of an infinitely long conductor in a homogeneous, lossy earth excited by a vertical magnetic dipole. The conductor can be bare or insulated, and the formulations are also included for electric and magnetic dipole sources of arbitrary orientation. We assume that the source, receiver, and conductor are at sufficient depths that we do not need to consider air-earth interface effects. The model is idealized, but it is the simplest model that allows us to consider most of the relevant features of the practical problem.

II. FORMULATION

The geometry is shown in Fig. 1. An infinitely long conductor of outer radius \( b \) is centered on the \( z \)-axis. A vertical (\( y \)-directed) magnetic dipole of magnetic moment \( IA \) is located at \((x_d, y_d, z_d)\). We choose a vertical magnetic dipole because it radiates a horizontal electric field that excites axial (\( z \)-directed) currents in the conductor. A horizontal (\( z \)-directed) electric dipole would also be effective in exciting axial currents, but a horizontal electric dipole is less practical for deployment in a vertical borehole [3]. However, the formulations for arbitrary electric or magnetic dipole orientation are given in the Appendix.

The fields due to the magnetic dipole in the absence of the conductor can be derived from a magnetic Hertz vector with only a \( y \)-component \( \mathbf{H}^d \). For exponential time dependence, \( \mathbf{H}^d \) is

\[
\mathbf{H}^d = \left[ \frac{IA e^{-j\beta r_d}}{4\pi r_d} \right] \mathbf{e}_y
\]

where

\[
r_d = \left[ (x - x_d)^2 + (y - y_d)^2 + (z - z_d)^2 \right]^{1/2}
\]

and \( \mu, \epsilon, \) and \( \sigma \) are respectively the earth permeability, permittivity, and conductivity. The electric and magnetic fields \( E^d \) and \( H^d \) due to the magnetic dipole are [4]

\[
E^d = -j\omega \mu \nabla \cdot (\mathbf{e} \Pi^d_y)
\]

and

\[
H^d = (\nabla \times + k^2) (\mathbf{e} \Pi^d_y)
\]

where \( \mathbf{e} \) is a unit vector in the \( y \)-direction.

Our derivation of the conductor current follows the Fourier transform method in [5]. We first write (1) in integral form [6]

\[
\Pi^d_y = \frac{IA}{4\pi} \int_{-\infty}^{\infty} K_0(\nu r_d) e^{-\nu (z - z_d)} d\nu
\]

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where
\[ \nu = (\lambda^2 - k^2)^{1/2}, \quad \rho_d = \left[(x-x_d)^2 + (y-y_d)^2\right]^{1/2} \]
and \( K_0 \) is the zero-order modified Bessel function of the second kind [7]. The z-component of the electric field \( E_z^e \) due to the dipole is obtained from (2) and (3)

\[ E_z^e = -j\omega \mu \frac{\partial \Pi_z^e}{\partial x} = \frac{j\omega \mu I}{4\pi^2} \int_{-\infty}^{\infty} e^{-j\lambda(z-z_d)} d\lambda \]

We assume that the conductor radius \( b \) is small compared to the wavelength in the earth and that only the axial current \( I(z) \) is significant. The fields produced by a line current \( I(z) \) can be derived in terms of an electric Hertz vector with only a z-component \( \Pi_z^e \), which can be written in integral form [5]

\[ \Pi_z^e = \frac{1}{2\pi (\sigma + j\omega \epsilon)} \int_{-\infty}^{\infty} \hat{I}(\lambda) K_0(v\rho) e^{-j\lambda z} d\lambda \]

where \( \rho = (x^2 + y^2)^{1/2} \).

The conductor current \( I(z) \) and the spatial transform \( \hat{I}(\lambda) \) are related by

\[ I(z) = \int_{-\infty}^{\infty} \hat{I}(\lambda) e^{-j\lambda z} d\lambda \]

The electric and magnetic fields \( E^e \) and \( H^e \) are [4]

\[ E^e = (\nabla \times + k^2) (e_2, \Pi_z^e) \]

and

\[ H^e = (\sigma + j\omega \epsilon) \nabla x (e_2, \Pi_z^e). \]

The z-component of the electric field \( E_z^e \) due to the conductor is obtained by substituting (5) into (7)

\[ E_z^e = \left(\frac{\partial^2}{\partial z^2} + k^2\right) \Pi_z^e = \frac{-1}{2\pi (\sigma + j\omega \epsilon)} \int_{-\infty}^{\infty} \hat{I}(\lambda) Z_0(v\rho) e^{-j\lambda z} d\lambda \]

At this point \( I(z) \) and \( \hat{I}(\lambda) \) are unknown. They are determined from the axial impedance boundary condition at the surface of the conductor [5]

\[ (E_z^e + E_z^c)|_{\rho=b} = \int_{-\infty}^{\infty} \hat{I}(\lambda) Z_0(\lambda) e^{-j\lambda b} d\lambda \]

where \( Z_0 \) is the axial impedance (series impedance per unit length) of the conductor and is generally a function of \( \lambda \). We can obtain a more convenient form of (9) by taking the Fourier transform [8]

\[ [\hat{E}_z^d(\lambda) + \hat{E}_z^c(\lambda)]|_{\rho=b} = \hat{I}(\lambda) Z_0(\lambda) \]

where \( \hat{E}_z^d \) and \( \hat{E}_z^c \) are Fourier transforms of the form in (7). From (4), (8), and (10), we can solve for \( \hat{I}(\lambda) \)

\[ \hat{I}(\lambda) = \frac{I A}{2\pi} \frac{\rho_d}{\rho_d I v D(\lambda)} e^{-j\lambda z_d} \]

where

\[ \rho_d = (x_d^2 + y_d^2)^{1/2} \quad \text{and} \quad D(\lambda) = K_0(vb) + \frac{2\pi (\sigma + j\omega \epsilon)}{v^2} Z_0(\lambda). \]

The corresponding forms of \( I(\lambda) \) for excitation by magnetic or electric dipoles of arbitrary orientation are given in the Appendix.

A convenient model for a buried conductor is shown in Fig. 2. It consists of a metal conductor of radius \( a \) surrounded by an insulating region of outer radius \( b \). The metal has conductivity \( \sigma_m \) and magnetic permeability \( \mu_m \), and the insulator has permittivity \( \epsilon_m \) and free space permeability \( \mu_0 \). The axial impedance of this model is [5], [8]

\[ Z_a(\lambda) = Z_m + \frac{\lambda^2 - k^2}{2\pi j\omega \epsilon_m} \ln \left(\frac{b}{a}\right) \]

where

\[ Z_m = \frac{1}{2\pi a} \left(\frac{j\omega \mu_m}{\sigma_m}\right)^{1/2} \frac{I_0(jk_m a)}{I_1(jk_m a)}, \quad k_m = \omega(\mu_m/\epsilon_m)^{1/2} \]

and \( I_0 \) and \( I_1 \) are modified Bessel functions of the first kind [7]. If we set \( b = a \), then \( Z_a(\lambda) = Z_m \), and we have a grounded metal conductor (such as a rail in a tunnel). If we set \( \epsilon_m \) equal to free space permittivity \( \epsilon_0 \) and \( \mu_m \) equal to free space permeability \( \mu_0 \), then we can model a conductor (such as a power line) suspended in an air-filled tunnel at a distance \( b - a \) from the tunnel wall.

### III. CONDUCTOR CURRENT

By substituting (11) into (6), we can write the conductor current as

\[ I(z) = \frac{IA}{2\pi \rho_d} \frac{x_d K_1(v\rho_d)}{v D(\lambda)} e^{-j\lambda(z-z_d)} d\lambda \]

The integrand in (13) has branch points at \( \lambda = \pm k \) and poles at \( \lambda = \pm \lambda_p \). The pole location is determined from the mode equation

\[ D(\lambda_p) = 0. \]
The pole contribution \( I_p(z) \) to the integral in (13) can be evaluated from the residue theorem and is given by

\[
I_p(z) = \frac{-j\mu_0 k^2 \epsilon}{Z_0} \left[ \frac{K_1(z \mu_0)}{vD(k)} \right] e^{-j\lambda_p |z|}. \tag{15}
\]

In general the mode equation (14) must be solved numerically, but we can obtain an approximate solution of the form

\[
\lambda_p \approx k_i (1 + \Delta)^{1/2} \tag{16}
\]

where

\[
\Delta = \left[ j\omega_0 \mu_0 \ln \left( \frac{b}{a} \right) \right]^{-1} \left[ j\omega_0 k_0 \left( \frac{f}{k} \right) + 2\pi \right].
\]

When \( \sigma \) and \( \sigma_m \) are large, then \( |\Delta| \) is small, and \( \lambda_p \) is close to \( k_i \). In this case most of the power in the mode is traveling in the insulation region, \( a < \rho < b \).

A computer program was written to solve the mode equation (14) by Newton's method using (16) as a starting value. The program was checked by comparing with the curves of Wait [9], and agreement was obtained to graphical accuracy. The complex propagation constant can be written \( \lambda_p = \beta_p - j \alpha_p \), where \( \beta_p \) is the phase constant and \( \alpha_p \) is the attenuation rate in nepers per meter. The attenuation rate in decibels per meter is 8.686 \( \alpha_p \).

In Figs. 3 and 4, we show numerical results as a function of frequency for the following parameters: \( a = 0.5 \text{ cm}, \sigma_c = 5.7 \times 10^7 \text{ S/m}, \epsilon_c/\epsilon_0 = 1, \epsilon/\epsilon_0 = 10, \sigma = 5 \times 10^{-2} \text{ S/m}, \) and \( \mu/\mu_0 = 1 \). (These parameters also apply to all following figures.) The earth parameters are typical of those for nonmagnetic rock, but \( \sigma \) can vary greatly [10]. Since we have set \( \epsilon = \epsilon_0 \), we can think of \( b - a \) as the distance of the conductor from the wall of an air-filled tunnel. The conductivity \( \sigma_c \) is that of copper, and the results are weakly dependent on the conductor radius \( a \). In Fig. 3, the attenuation rate increases with increasing frequency or decreasing \( b \), and those trends are consistent with earlier calculations [1]. In Fig. 4, the normalized phase constant increases as \( b \) decreases, which indicates that more of the mode power is carried in the surrounding earth.

The total conductor current \( I(z) \) can be obtained from the integral form in (13). The simplest method of evaluating the \( \lambda \) integration is fast Fourier transform (FFT), and this method automatically yields an array of \( z \) values. Numerical results for the magnitude of \( I(z) \) are shown in Fig. 5 for the following magnetic dipole source parameters: \( IA = 1 \text{ A} \cdot \text{m}^2, x_d = -15 \text{ m}, y_d = z_d = 0, \) and \( f = 100 \text{ kHz} \). The currents are even in \( z \). The currents on the insulated conductor \( (b > a) \) have a slow \( z \) decay because of the dominance of the transmission-line current as given by (15). These curves actually differ very little from \( I_p(z) \) in (15). The current on the bare (grounded) conductor \( (b = a) \) has a higher peak value, but has rapid \( \exp (-ik |z|) \) decay corresponding to plane-wave attenuation in the earth. This result could be obtained from evaluating the branch cut contribution [11] from the branch point at \( \lambda = \pm k \). The branch cut contribution was evaluated directly in [11], but we used the FFT evaluation of (13) here because it is simpler.
IV. MAGNETIC FIELD

At this point we can calculate the primary fields due to the dipole from (2) and the secondary fields due to the conductor from (7). We are most interested in the y-component of the magnetic field because we anticipate reception with a vertical magnetic dipole in a vertical borehole.

The primary y-component \( H_y \) due to the source is determined by substituting (1) into (2)

\[
H_y = \left( \frac{\partial^2}{\partial y^2} + k^2 \right) \frac{d^2}{dz^2} \sum_{n} \frac{e^{-jkz}}{r_n^2} \\
= A_4 e^{j300} \left( k^2 \left[ 1 - \frac{(y - y_0)^2}{r_d^2} \right] + \frac{j k}{r_d} \left[ \frac{3(y - y_0)^2}{r_d^2} - 1 \right] + \frac{1}{r_d^2} \left[ \frac{3(y - y_0)^2}{r_d^2} - 1 \right] \right) \quad (17)
\]

In general we must retain all terms in (17).

The secondary y-component \( H'_y \) due to the conductor is determined by substituting (5) into (7)

\[
H'_y = - (\sigma + j\omega\epsilon) \frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} I_p'(z) e^{-j\omega z} dz \\
= \frac{x}{2\pi\rho} \sum_{n} I_n K_1(v_r) e^{-j\omega z} \quad (18)
\]

In general we choose to evaluate the integral in (18) by FFT. However, the pole contribution \( H'_p(z) \) for the case of the insulated conductor can be evaluated from the residue theorem and is given by

\[
H'_p(z) = \left[ \frac{x\pi K_1(v_r)}{2\pi\rho} \right] I_p(z) \quad (19)
\]

where \( I_p(z) \) is given by (15).

In Fig. 6 we show the primary and secondary magnetic fields observed at \( x = 15 \text{ m} \) and \( y = 0 \) for a frequency of 100 kHz. As with the conductor current in Fig. 5, the magnetic field due to an insulated conductor decays slowly in \( z \). The secondary magnetic field was computed from (18) using FFT, but the results for the insulated conductor are closely approximated by the pole contribution in (19). For large \( z \), the secondary magnetic field is larger than the primary field, and the insulated conductor should be easy to detect. The secondary field for a grounded conductor has a more rapid decay, but it is still nearly as large as the primary field.
In Fig. 7 we show magnetic field results for three values of $y$, and these results are relevant to reception at different heights in a borehole. The other parameters are the same as in Fig. 6. Both the primary and secondary fields decrease as $y$ is increased. The same results would be obtained if we varied the source height $y_d$.

In Fig. 8 we show magnetic field results for different frequencies from 20 kHz to 2 MHz. The other parameters are the same as in Fig. 6. In all cases we observe the crossover effect where the secondary magnetic field is larger than the primary field for large $z$. Thus this entire frequency range should be useful in detection of conductors.

**V. CONCLUSIONS**

The results presented here indicate that a vertical magnetic dipole can excite significant axial currents on long horizontal conductors. These currents produce secondary magnetic fields that are smaller than the primary magnetic field near the source but are larger than the primary field at large axial distances. An insulated conductor can support a transmission line current that has slow decay in the axial direction. A grounded conductor has a larger current excited near the source, but that current decays more rapidly in the axial direction. In either case, the secondary magnetic field is large enough that detection of long conductors appears promising. Numerical results have been presented for frequencies from 20 kHz to 2 MHz, and the entire frequency range appears to be useful.

A number of extensions to this work would be useful. Excitation (or reception) by electric or magnetic dipoles of other orientations or by long electric line sources could be examined. The presence of the air-earth interface [12] could be included for cases of shallow conductor depth. Our infinitely long, single-conductor model is fairly simple, and multiple conductors or conductors of finite length could be studied. Also, earth layering or inhomogeneities could be studied to obtain an estimate of the competing geologic noise [13].

**APPENDIX—OTHER DIPOLE SOURCES**

The transformed current $\hat{I}(\lambda)$ induced on the conductor for a $y$-directed magnetic dipole source is given in (11). We can derive the corresponding expression for other dipole sources by substituting the appropriate expression for $E_y^0(\lambda)$ into (10). For an $x$-directed magnetic dipole source, $\hat{I}(\lambda)$ is given by

$$\hat{I}(\lambda) = -\frac{IA}{2\pi} \frac{y_g k^2 K_1(v_p d_0)}{\rho_0 v D(\lambda)} e^{j\lambda z_0}. \quad (A1)$$

For a $z$-directed magnetic dipole, there is no coupling to the conductor, and $\hat{I}(\lambda) = 0$.

For an $x$-directed electric dipole source of moment $I d_s$, $\hat{I}(\lambda)$ is

$$\hat{I}(\lambda) = \frac{I d_s}{2\pi} \frac{j x g K_1(v_p d_0)}{\rho_0 v D(\lambda)} e^{j\lambda z_0}. \quad (A2)$$

For a $y$-directed electric dipole source, $\hat{I}(\lambda)$ is

$$\hat{I}(\lambda) = \frac{I d_s}{2\pi} \frac{j y g K_1(v_p d_0)}{\rho_0 v D(\lambda)} e^{j\lambda z_0}. \quad (A3)$$

For a $z$-directed electric dipole source, $\hat{I}(\lambda)$ is

$$\hat{I}(\lambda) = \frac{I d_s K_1(v_p d_0)}{2\pi D(\lambda)} e^{j\lambda z_0}. \quad (A4)$$

All of these results for $\hat{I}(\lambda)$ are consistent with the results in [5].

**REFERENCES**

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