Magnetic Dipole Excitation of an Insulated Conductor of Finite Length

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Abstract—Excitation of currents on an insulated conductor of finite length and with arbitrary terminations is studied for a magnetic dipole source. For matched terminations, the results agree closely with previous results for an infinitely long conductor, but other terminations produce end reflections that cause standing waves. Specific calculations are presented for a vertical magnetic dipole source, because this source produces the appropriate horizontal electric field and could be used in a borehole-to-borehole configuration. Numerical results for the induced current and secondary magnetic field indicate that long conductors produce a strong anomaly over a broad frequency range for any type of termination.

I. INTRODUCTION

The excitation of currents on underground conductors is important in many applications. Power lines and rails in tunnels can enhance transmission for mine communications [1]. Electromagnetic probing of the Earth can be influenced by the presence of cables or pipes [2], [3]. Most work on dipole excitation of conductors has treated infinitely long conductors [4], [5]. This idealization allows a spatial Fourier transform formulation that simplifies the calculation of the current distribution.

In this paper, our model consists of an insulated conductor of finite length in a homogeneous, lossy Earth, excited by a vertical magnetic dipole. The insulated conductor has an arbitrary terminating impedance at each end. The theory of excitation of an insulated conductor by an external field has been given by King [6], and we use it for the particular case of magnetic dipole excitation. One application of this case is the detection of long conductors in tunnels by means of vertical transmitting and receiving magnetic dipoles in vertical boreholes. This application was studied in [5] for infinitely long conductors, and in this paper we study the effects of finite conductor length.

II. MAGNETIC DIPOLE FIELDS

The geometry for the magnetic dipole and conductor is shown in Fig. 1. The conductor of length $2h$ extends from $-h$ to $h$ on the $z$ axis. A vertical (y directed) magnetic dipole of magnetic moment $I\alpha$ is located at $(x_d, y_d, z_d)$. We choose a vertical magnetic dipole because it radiates a horizontal electric field which excites axial ($z$ directed) currents in the conductor.

The fields due to the magnetic dipole in the absence of the conductor can be derived from a magnetic Hertz vector with only a $y$ component $\Pi^d_y$. For $exp(j\omega t)$ time dependence, $\Pi^d_y$ is [5]

$$\Pi^d_y = \frac{I\alpha e^{-jk\sigma}}{4\pi r_d}$$

where $r_d = [(x - x_d)^2 + (y - y_d)^2 + (z - z_d)^2]^{1/2}$, $k = \omega\sqrt{\mu\varepsilon/\omega}$, and $\mu$, $\varepsilon$, and $\sigma$ are the earth permeability, permittivity, and conductivity. The electric and magnetic fields $E^d$ and $H^d$ due to the magnetic dipole are [7]

$$E^d = -j\omega \varepsilon \nabla \times (\varepsilon \Pi^d_y)$$

$$H^d = (\nabla \varepsilon + k^2)(\varepsilon \Pi^d_y)$$

where $\varepsilon$ is a unit vector in the $y$ direction.

For reception with a second vertical magnetic dipole, the $y$ component $H^r_y$ of the magnetic field is required. This is obtained from substituting (1) into (2) and carrying out the differentiations:

$$H^r_y = \frac{I\alpha e^{-jk\sigma}}{4\pi r_d} \left( k^2 \left[ \frac{1}{k^2} - \frac{(y - y_d)^2}{r^2_d} \right] + \frac{jk r_d + 1}{r^2_d} \cdot \left[ \frac{3(y - y_d)^2}{r^2_d} - 1 \right] \right) \cdot (y - y_d)$$

For excitation of the conductor, the $z$ component $E^d_z$ is required, and it is also obtained from (1) and (2):

$$E^d_z = j\omega \mu \frac{I\alpha e^{-jk\sigma}}{4\pi r_d} \left( x - x_d \right) \left[ \frac{jk + 1}{r_d} \right]$$

Manuscript received June 8, 1989; revised January 2, 1990. This work was supported by the U.S. Army Belvoir RD&E Center.

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IEEE Log Number 9034313.

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III. CONDUCTOR CHARACTERISTICS

A model for a buried insulated conductor is shown in Fig. 2. It consists of a metal conductor of radius \( a \) surrounded by an insulating region of outer radius \( b \). The metal has conductivity \( \sigma_m \) and magnetic permeability \( \mu_m \), and the insulation has permittivity \( \epsilon_i \) and free-space permeability \( \mu_0 \).

For sufficiently low frequencies, the insulated conductor can be characterized by transmission line parameters. The series impedance per unit length \( z_L \) is

\[
z_L = z'_L + z'' + z'''
\]

where

\[
z'_L = \frac{\mu_0}{2\pi a} \mu_0 H_0^2(kb),
\]

\[
z'' = j\frac{\omega}{2\pi} \ln \left( \frac{b}{a} \right)
\]

and \( H_0^2 \) and \( H_1^2 \) are Hankel functions of the second kind [9]. In (5), \( z'_L \) represents the effect of finite conductivity of the inner conductor, \( z'' \) represents the contribution to the impedance from the fields in the Earth, and \( z''' \) represents the contribution from the fields in the insulation region. In (5) and throughout this paper, we take the complex conjugate of King’s expressions because he used \( \exp(-i\omega t) \) time dependence. The shunt admittance per unit length \( y_L \) is [6], [8]

\[
y_L = \frac{j\omega a}{2\pi} \ln \left( \frac{b}{a} \right)
\]

The characteristic impedance \( Z_{ca} \) and the wave number \( k_L \) are [6], [8]

\[
Z_{ca} = \left( z_L/y_L \right)^{1/2}
\]

and

\[
k_L = (-z_L y_L)^{1/2}
\]

The main requirements for the validity of (5)-(7) are that the wavenumber of the Earth be large compared to that of the insulation \( (|k^2| >> \omega^2 \mu_0 \epsilon_i) \) and that the insulated conductor be electrically thin \( (|k| b << 1) \).

To obtain a numerical check of the transmission line approximations in (5)-(7), we compared the real and imaginary parts of the wavenumber \( k_L \) with the solution of the more rigorous mode equation described in [5] for the following parameters: \( a = 0.5 \text{ cm} \), \( \sigma_m = 5.7 \times 10^7 \text{ S/m} \), \( \epsilon_i/\epsilon_0 = 1 \), \( \sigma/\epsilon_0 = 10 \), \( \sigma = 5 \times 10^{-3} \text{ S/m} \), and \( \mu_m/\mu_0 = \mu/\mu_0 = 1 \). These parameters also apply to all the following figures. The Earth parameters are typical of those for nonmagnetic rock. Since we have set \( \epsilon_i = \epsilon_0 \), we can think of \( b-a \) as the distance of the conductor from the wall of an air-filled tunnel. The conductivity \( \sigma_m \) is that of copper, and the results are only weakly dependent on the conductor radius \( a \). Numerical results for the attenuation rate \( \beta_L \) are shown in Fig. 3 as a function of frequency for several values of \( b \). As expected, the attenuation rate increases with frequency, and the agreement with the mode equation is best for low frequencies. Numerical results for the phase constant \( \beta_L \) are shown in Fig. 4, and \( \beta_L \) is normalized to the wavenumber of the insulation, \( k_i = \omega (\mu_0 \epsilon_i)^{1/2} \). Again, the agreement is best for low frequencies. The characteristic impedance \( Z_{ca} \) is important in determining the induced current, and numerical results from (7) are shown in Fig. 5.
IV. INDUCED CURRENT

King [6] has presented expressions for the current in an insulated conductor of finite length and with arbitrary terminations when the incident field is either a gap feed (antenna problem) or an incident field (scattering problem). Numerous comparisons of the theoretical and measured current distributions for a gap feed are given in [10], and the agreement is generally good. For incident field excitation, the induced current is written as an integral of the incident field times the Green’s function over the length of the conductor, and King [6] has evaluated this integral for uniform plane-wave incidence. For magnetic dipole excitation, the integral expression for the induced current \( I(z') \) is written in the following form:

\[
I(z') = \int_{-h}^{h} E_z(z') G(z', z) \, dz
\]

where \( E_z(z') \) is obtained from (4) and evaluated at \( z = z' \) on the \( z \) axis. The Green’s function \( G \) for a terminated transmission line is obtained by writing King’s result [6], [11] in terms of circular functions:

\[
G(z', z) = \begin{cases} 
\frac{j}{Z_{zo}} \frac{\sin [k_z(h + z_z) - j\theta_+\sin(2k_z h - j\theta_+ - j\theta_-)]}{\sin(2k_z h - j\theta_- - j\theta_+)} & z' \leq z_z \\
\frac{j}{Z_{zo}} \frac{\sin [k_z(h + z_z') - j\theta_-\sin(2k_z h - j\theta_- - j\theta_+)]}{\sin(2k_z h - j\theta_- - j\theta_+)} & z' \geq z_z
\end{cases}
\]

where

\[
\theta_+ = \coth^{-1} (Z_- / Z_{zo}) \quad \text{and} \quad \theta_- = \coth^{-1} (Z_+ / Z_{zo}).
\]

In (10), \( Z_- \) and \( Z_+ \) are the impedances terminating the insulated conductor at \(-h\) and \(+h\).

For the special cases of open ends (\( Z_- = Z_+ = \infty \)), short-circuited ends (\( Z_- = Z_+ = 0 \)), and matched ends (\( Z_- = Z_+ = Z_{zo} \)), the results in (9) and (10) simplify to the results given in the Appendix. The case of an open end is often encountered in practice where the actual conductor is suspended and no grounding is attempted [12]. The opposite case of a short circuit represents perfect grounding of the conductor to the surrounding rock. Fairly good grounding can sometimes be achieved with roof bolts or some other grounding conductor if the grounding conductor is not corroded and makes good contact with the rock. The case of a matched termination is highly idealized, but this reflectionless case is useful for comparison with the earlier calculations for infinitely long conductors [5].

A computer program to evaluate the current distribution in (8) by numerical integration was written. Fig. 6 shows the effect of the conductor termination for the following parameters: \( I = 1 \, \text{A} \cdot \text{m}^2, x_s = -15 \, \text{m}, y_s = 0, z_d = 250 \, \text{m}, 2h = 2 \, \text{km}, \) and \( f = 100 \, \text{kHz} \). The matched line shows a smooth decay of the current with no end reflections, and the result for an infinitely long line [5] is also shown for comparison. This is a good check of the numerical integration, because the result in [5] was obtained by a totally different Fourier transform method. Additional comparisons between the matched line and the infinitely long line showed good agreement for frequencies of less than about 500 kHz. Another requirement of this transmission line model is that the insulation thickness \( (b-a) \) be sufficiently thick so that \( |k_z^2| < |K|^2 \) and that the propagation in the insulation dominates the propagation in the surrounding medium. Also, both the magnetic dipole source and the observation point must be located at least several radii \( b \) from the conductor so that only the rotationally symmetric current is significant.

In Fig. 6, the end reflections for the open and shorted lines are very prominent and the current for the open goes to zero at the ends. When the source is located near one end (\( z_d = b \)), the open line is difficult to excite, as shown in Fig. 7. Here we set \( z_d = h = 1 \, \text{km} \), and the other parameters remain the same. The matched and shorted lines are strongly excited, but the open line is only weakly excited.

Because the open line is often encountered in practice [12], we study the effect of source location on the excitation of an open line in Fig. 8: The excitation improves as the source is moved away from the end toward the center of the line, and this is consistent with experimental results [12]. From Fig. 4 we can calculate that the effective wavelength of the line \( (2\pi / \beta_\lambda) \) is approximately 940 m at 100 kHz. When the source is approximately a quarter wavelength (235 m) from the end of the line, then the reflected wave is in phase with the direct wave and the excitation is very strong. The situation in Fig. 8 is actually more complicated because of multiple reflections from both ends.
Fig. 6. Current induced on an insulated conductor by a magnetic dipole source. The result (---) for an infinitely long conductor [5] is also shown.

Fig. 7. Current induced on an insulated conductor by a magnetic dipole located near the end (\(z_d = h\)).

Fig. 8. Current induced on an open insulated conductor.

In Fig. 9 we show the effect of moving the source farther away from the line for the case of a shorter line (\(2h = 1 \text{ km}\)). Again, \(z_d = 250 \text{ m}\). The induced current falls off rapidly with increasing \(|x_d|\) because of the exponential attenuation of the incident field in the lossy Earth.

In Fig. 10 we show the effect of frequency on the induced current for an open line of length \(2h = 1 \text{ km}\). Again, \(z_d = 250 \text{ m}\) and \(x_d = -15 \text{ m}\). Rapid oscillations occur at 500 kHz because of the shorter wavelength. The excitation is poor at \(\leq 20 \text{ kHz}\) because the line is electrically short (about one-quarter wavelength). To achieve strong excitation, the frequency should be high enough so that the conductor length \(2h\) is at least a half wavelength. Some experimental results for magnetic dipole excitation at frequencies from 50 to 500 kHz have been obtained for open wires, and the agreement of the current distribution and propagation constant with theoretical results is generally good [12].

V. MAGNETIC FIELD

The primary fields of the magnetic dipole are given by (2). The fields of the currents induced in the conductor can be derived from an electric Hertz vector with only a \(z\) component \(\Pi^z\) [5]:

\[
\Pi^z = \frac{1}{4\pi \alpha} \int_{-h}^{h} \frac{I(z')e^{-jkr}}{r_c} \, dz'
\]

where

\[
r_c = \left[ x^2 + y^2 + (z - z')^2 \right]^{1/2}.
\]
The electric and magnetic fields $E'$ and $H'$ due to the conductor are [5]

$$E' = (\nabla \nabla \cdot + k^2)(e_J \Pi_i')$$

and

$$H' = (\sigma + j\omega \varepsilon) \nabla \times (e_J \Pi_i').$$

For reception with a vertical magnetic dipole, the $y$ component $H_y$ of the magnetic field is required, and this is obtained from (11) and (12):

$$H_y = \int_{-h}^{h} I(z') G_i \, dz'$$

where

$$G_i = \left( jk + \frac{1}{r_i} \right) \frac{xe^{-|z_0|}}{4\pi r_i}.$$
APPENDIX
SPECIAL TERMINATIONS

For open ends ($Z_0 = Z_\infty = \infty$), we have $\theta_\infty = \theta_\infty = 0$. In this case, (9) reduces to

$$G_i(z', z) = \begin{cases} \frac{j}{Z_{\infty}} \sin \left( k_i (h + z) \right) \frac{\sin \left( k_i (h - z') \right)}{\sin (2k_i h)}, & z' \geq z_i \\ \frac{j}{Z_{\infty}} \frac{\sin \left( k_i (h + z') \right) \sin \left( k_i (h - z) \right)}{\sin (2k_i h)}, & z' \leq z_i \end{cases} \quad (A1)$$

For short-circuited ends ($Z_\infty = Z_\infty = 0$), we have $\theta_\infty = \theta_\infty = j\pi/2$. In this case, (9) reduces to

$$G_i(z', z) = \begin{cases} \frac{-j}{Z_{\infty}} \frac{\cos \left( k_i (h + z') \right) \cos \left( k_i (h - z) \right)}{\sin (2k_i h)}, & z' \geq z_i \\ \frac{-j}{Z_{\infty}} \frac{\cos \left( k_i (h + z') \right) \cos \left( k_i (h - z) \right)}{\sin (2k_i h)}, & z' \leq z_i \end{cases} \quad (A2)$$

For matched ends ($Z_\infty = Z_\infty = Z_{\infty}$), we have $\theta_\infty = \theta_\infty = \infty$. In this case, (9) reduces to

$$G_i(z', z) = e^{-jz_\infty |z' - z|} \quad (A3)$$

ACKNOWLEDGMENT

The author would like to thank Dr. L. G. Stolarczyk and Dr. F. Ruskey for helpful discussions.

REFERENCES


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