Millimeter-Wave Multipath Measurements on Snow Cover

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Abstract—Multipath data were obtained at frequencies of 35.1, 98.1, and 140.1 GHz over a pathlength of 179.5 m by measuring height-gain patterns between 0.2 and 4.0 m with a vertically moving receiving antenna. Grazing angles for this geometry range between 0.5 and 2 degrees. Measured interference patterns between direct and snow-reflected rays were generally coherent in appearance and on occasion exhibited cancellation depths greater than 20 dB. A computer program models the reflection as a coherent process, with the underlying snow surface represented by a series of linear sloping segments derived from actual terrain heights. Reflection coefficient near a 2-degree grazing surface represented by a series of linear sloping segments derived from actual terrain heights. Reflection coefficient near a 2-degree grazing angle ranged from 0.53 to 0.20 over matted grass, from 0.66 to 0.34 over freshly fallen snow, and from 0.85 to 0.71 over old snow. The higher numbers correspond to 35.1 GHz, the lower numbers correspond to 140.1 GHz.

I. INTRODUCTION

MILLIMETER-WAVE radar, guidance, and communication systems near the surface have to contend with multipath propagation from a snow layer on the ground. It is well established that snow is an efficient source of backscatter at millimeter waves [1], particularly if it has been partially melted and refrozen several times. Likewise, one might expect snow multipath signals at low grazing angles to be of substantial magnitude. Three distinct processes are conceivable to produce multipath signals over snow-covered terrain: 1) surface scatter from snow, 2) volume scatter from snow, and 3) surface scatter from the underlying terrain. Whether or not the latter two produce significant contributions to the received signal depends on factors like the snow depth, the type of snow, the grazing angle, and the frequency. Based on experimental results, we assume that multipath returns from snow at millimeter wavelengths in our case originated at or near the surface, with the pathlength differences between individual scatterers small enough in terms of the Rayleigh criterion to produce a largely coherent signal.

Several methods lend themselves to the separate measurement of direct and multipath components in the received signal: time-of-arrival difference, angle-of-arrival difference, and phase interference. The millimeter-wave equipment available for this multipath propagation study had been designed for backscatter measurements [2] and lacks both angle and time resolution required to distinguish component signals in a typical multipath geometry. This leaves the phase interference method. If the transmit and receive antenna beams of a bistatic measurement system encompass both direct and multipath components, then the vertical field distribution at the receiving antenna can be probed to determine the multipath structure. The magnitude of the reflection coefficient may be derived in a height range where only one reflected component contributes to the received signal. Our test equipment operates over a propagation path of several hundred meters with sufficient dynamic range for the analysis of constructive and destructive interference. Transmit and receive antenna heights were chosen to cover a 0.5 to 2 degree range of grazing angles.

II. SNOW MULTIPATH SIMULATION

A perfectly flat snow surface would be ideal for the measurement of antenna height-gain patterns and for the separation of the direct and the single, reflected signal component. However, natural snow surfaces are not flat in terms of the reflection geometry considered here, nor is the underlying terrain. This gives rise to multiple reflecting facets along the propagation path and potentially more than one reflected signal component interfering with the direct signal at any given receiving antenna height.

Owing to the vertical motion of the sensing antenna, phase changes observed between component signals derive largely from the surface variability along the propagation path. A two-dimensional model was therefore developed, describing multipath propagation by specular reflection from a surface composed of linear segments with circular transitions between segments [3]. Details of this model are presented in the Appendix. The area selected for multipath propagation measurements is a relatively plane field of 100 m by 200 m on which the grass is cut during the summer. A terrain profile along the propagation path is displayed in Fig. 1. The actual heights measured were approximated by six straight-line segments. Values for \( h_n, l_n \) were determined at the straight-line intersections.

In order to simulate multipath propagation over a snow layer with the profile of Fig. 1, we have to assume a reflection coefficient for snow. Neglecting the small loss tangent of ice, the magnitude of the reflection coefficient of ice at grazing angle \( \theta \) is [4] for vertical polarization

\[
\rho_v = \frac{(\epsilon_r \sin \theta - (\epsilon_r - \cos^2 \theta)^{1/2})}{(\epsilon_r \sin \theta + (\epsilon_r - \cos^2 \theta)^{1/2})}
\]  

(1)

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are due to the finite length of the antenna gain and reflection loss, to \( h_{R_{\text{min}}} \) where the disadvantage relative to the direct signal both in terms of cancellation. This is apparent as the receiving antenna moves from \( h_{R_{\text{max}}} \) to \( h_{R_{\text{min}}} \) where the disadvantage is less pronounced, leading to a cancellation approaching 20 dB. Slight irregularities in the null depths are due to the finite length of the \( \Delta l \) increment. The spacing between minima is practically constant. Changing \( \varepsilon_r \) to 1.92 increases the maxima by about 0.3 dB and decreases the minima by about 2 dB. Vertical polarization affects the null depth by 0.5 dB, a result to be expected in view of the small differences between \( \rho_Y \) and \( \rho_H \) as calculated from (1) and (2). At 98.1 GHz the number of minima increases to 28 and at 140.1 GHz to 40, that is, proportional with frequency.

Multipath signals at 35.1 GHz over the simulated path can be seen in Fig. 3. Here \( \varepsilon_r = 1.12 \), although no major changes occur for \( \varepsilon_r = 1.92 \). We found that the received signal is dependent on both the straight segments and the circular transition regions. A straight terrain segment generally adds to the received signal only over a limited height range. Its inclination is constant and the change in height at which a multipath signal intercepts the receiver ordinate is small per \( \Delta l \) change of the reflection point along the surface. In contrast, as the reflection point moves along a circular region, the change in inclination per \( \Delta l \) along the surface has a stronger effect on where the reflected ray passes through the receiver ordinate than the change along the path itself. A wider range of heights is covered by the reflected ray, when it passes through the circular region. The choice of length of the circular region, which directly affects the change of the inclination, has a considerable effect on the appearance of the received height-gain pattern. At any given height on the receiver ordinate, only one or two signals reflected from straight segments may contribute to the sum signal. This is because the reflection point distance from the transmitter, its height, and local inclination have to agree. In the circular region, this agreement is reached in more cases, because the local inclination covers a large, contiguous range of values, including one where the reflection condition for the particular receiver height is met. Actual measurements over snow suggest that a low number of reflected signals plus the direct signal compose the

\[
\rho_H = \frac{(\sin \theta - (\varepsilon_r - \cos^2 \theta)^{1/2})}{(\sin \theta + (\varepsilon_r - \cos^2 \theta)^{1/2})}.
\]

Cumming [5] gives a theoretical expression for \( \varepsilon_r \), the dielectric constant of an air-ice mixture

\[
(\varepsilon_r - 1)/\varepsilon_r = \frac{3f}{(\varepsilon_r - 1)/(\varepsilon_r + 2\varepsilon_r)}.
\]

Here \( f \) is the fractional volume occupied by solid ice and \( \varepsilon_r \) its dielectric constant. In our model calculations we assume \( \varepsilon_r \) to be constant and lossless across the whole millimeter-wave range [6]. Substituting \( \varepsilon_r = 3.15 \) and solving for \( \varepsilon_r \), we determine \( \varepsilon_r = 1.12 \) to represent light, freshly fallen snow and \( \varepsilon_r = 1.92 \) to represent old, metamorphic snow.

Based on our experimental parameters we performed a series of multipath signal calculations at 35.1, 98.1, and 140.1 GHz. The modeled 3-dB beamwidth at all frequencies is 7 degrees. Better insight into the effect of various system parameters is gained when multiple reflected signals do not complicate the signal structure. This is accomplished by setting \( h_0 \) through \( h_\alpha \) to zero. As an example, Fig. 2 shows the received signal versus receiver height at 35.1 GHz and horizontal polarization for \( \varepsilon_r = 1.12 \). The beams are not tilted in elevation. For a pathlength \( l_{B} \) of 179.5 m, 10 maxima and minima are plotted as the receive antenna descends from 4 to 0.2 m. The transmit antenna is at a fixed height of 2 m. The 0-dB level corresponds to the magnitude of the direct signal. In-phase superposition of a single reflected signal leads to a sum signal almost 6 dB higher. The sum signal level is fairly constant for all peaks, indicating that the multiplication by the antenna gain factors has little effect there. The more obvious influences can be seen in the minima, where a relatively minor change in the direct or reflected signal amplitude can cause a substantial change in the signal cancellation. This is apparent as the receiving antenna moves from \( h_{R_{\text{max}}} \), where the reflected signal is at a disadvantage relative to the direct signal both in terms of antenna gain and reflection loss, to \( h_{R_{\text{min}}} \) where the disadvantage is less pronounced, leading to a cancellation approaching 20 dB. Slight irregularities in the null depths are due to the finite length of the \( \Delta l \) increment. The spacing
received signal at any height. In order to simulate a similar signal structure, the number of circular transition regions and hence linear segments must be kept low. The model is correct in a qualitative way, based on relatively simple terrain features. It cannot be expected to yield one-to-one correspondence with measured height-gain patterns since that would require much greater precision in describing the path profile. Besides, we did not measure the actual snow profile along the path so that footsteps would not destroy it. In Fig. 3 we assume a circular transition from one linear segment to the next to begin 1 m from the end of each segment. Radii of 100 m or more can be expected for the terrain profile of Fig. 1. The 0-dB level in Fig. 3 corresponds to the direct signal of unity amplitude. An in-phase addition of a reflected signal of equal amplitude results in a sum signal of 6 dB. Three reflected signals and the direct signal add up to 12 dB, if they are of unity amplitude and optimally phased, and so on. Optimal phasing will not likely occur with an increasing number of reflected signals, and hence the sum signal will not reach its maximum height. With perfect out-of-phase addition, cancellation between the direct signal and a single reflected signal will not exceed a level given by the magnitude of $\rho$. With one or more reflected signals added, there is a chance for a more complete extinction of the sum signal. The general trend observed in Fig. 3 is typical of modeled and measured 35.1-GHz patterns. At the greater receiver height and the reflection taking place more toward the transmitter, there is only a single reflected signal interacting with the direct signal. This is the case for heights above 2.2 m in Fig. 3, which has an appearance in this range similar to Fig. 2. The maximum amplitude is close to 6 dB. Below 2.2-m height, at least three phasors interact, because the sum vector reaches a level of 10 dB. In principle, yet another reflected signal could be present, with the phasing never reaching an optimum. In the same region, at 1.5-m height, cancellation stronger than in Fig. 2 is observed ($\geq 25$ dB versus 19.4 dB) pointing toward the superposition of more than two components.

III. Propagation Experiment

Multipath propagation measurements were conducted on four occasions. Fig. 4 shows a picture of the general test area with the transmitter in the foreground and the receiving setup in the background. The peculiar mounting of the transmitter is due to the original function of the equipment. It is irrelevant to this experiment except for the fact that it affords a transmit platform at a constant 2-m height. The 4-m aluminum pole in front of the spruce tree carries a small motor-driven elevator platform for the receiver front end. A ten-turn precision potentiometer serves to derive a height-proportional voltage for recording the receiver position.

Fig. 5 presents a simplified diagram of transmitter and receiver building blocks and a schematic view of the signal paths. A Gunn oscillator at 35.1 GHz and klystron oscillators at 98.1 and 140.1 GHz powered by separate supplies are on/off modulated by a common squarewave modulator at a 1-kHz rate. Power output could not be measured accurately and is estimated at several milliwatts. The transmit antennas are standard gain horns at 35.1 and 98.1 GHz and a scalar feed horn at 140.1 GHz. Beamwidths are listed in Table I.

There is only one common receiver for the three frequencies. The receiver is designed around a single-ended $K_r$-band waveguide mixer using a Schottky-barrier diode. A mechanically tuned Gunn oscillator serves as the local oscillator. Using an intermediate frequency of 100 MHz, the receiver operates with fundamental mixing at 35.1 GHz (L.O. tuned to 35.0 GHz), and with fourth-order harmonic mixing at 140.1 GHz (L.O. tuned to 35.0 GHz). At 98.1 GHz, third-order harmonic mixing is used with the L.O. tuned to 32.67 GHz. An IF bandwidth of 20 MHz was found to be commensurate with the frequency stability of the free running millimeter-wave sources under-outdoor temperature changes. The choice of 98.1 GHz, which is slightly higher than the center frequency of the atmospheric transmission window, was dictated by the L.O. tuning range. Receiving standard gain horn antenna beamwidths are also listed in Table I. The transmitters and the receiver front end can be rotated by 90 degrees for vertical and horizontal polarization. Referring again to Fig. 5, the receiver mixer is followed by a 35-dB IF amplifier, a precision step attenuator, two cascaded balanced mixers connected as electronically controlled variable attenuators for 70 dB of automatic gain control (AGC), and another 35-dB IF amplifier. After detection, the 1-kHz squarewave modulated signal is amplified in a 15-Hz bandwidth amplifier and rectified. The output signal drives the AGC amplifier, whose gain curve was adjusted for a near-logarithmic receiver response over a 50-dB input signal range. The rectified signal is recorded digitally and also drives the Y-axis of an X-Y recorder. The X-signal is taken from the vertical positioner.
We conducted a series of tests over dead grass for comparison with snow. The grass had been pressed to the ground by previous layers of snow, which then melted. A relatively flat surface remained with only a small number of blades still standing. This suggests that the electromagnetic wave before and after reflection penetrates little vegetation, which would cause attenuation and give the appearance of a reduced reflection coefficient. On the day of measurement the temperature was above freezing. The sky was overcast but no precipitation fell. Fig. 6 shows multipath propagation over grass at the three frequencies.
The highest sum signal received was labeled "0 dB" in the experimental curves. This is in contrast to the modeled curves. A predominant feature in Fig. 6(a) is the low minimum at about 1-m height. Following earlier reasoning we conclude that this is the superposition of the direct signal and at least two reflected components. If we assume that only a single reflected signal interacts with the direct signal in the height range from 2.30 to 3.30 m, then $\Delta V = 10.3$ dB. From (A7) we obtain $\rho = 0.53$. Fig. 6(b) shows results for 98.1 GHz under the same conditions. In the height range for two-signal interaction, $\Delta V = 4.7$ dB is estimated, which converts to $\rho = 0.26$. In Fig. 6(c), the corresponding 140.1-GHz graph, it becomes increasingly more difficult to establish a region of two-signal interaction. This demonstrates the limits of the phase interference technique in determining the reflection coefficient. We estimate from Fig. 6(c) that $\Delta V = 3.6$ dB and $\rho = 0.20$. Wallace [7] has measured the multipath performance of a 140-GHz monopulse antenna over weeds, grass, and asphalt. His equipment to determine height-gain patterns is similar to ours. Antenna heights are comparable but the range is only 100 m. He concludes for vegetative ground cover that $\rho \leq 0.1$. We attribute his low value to substantial attenuation from growing grass and weeds, where the grass was mowed and the weeds reached a height of 1 m.

All propagation data that follow were measured over snow in the frozen or dry state. Multipath propagation curves over freshly fallen snow obtained on a single day are shown in Fig. 7. The snow was light and fluffy with a density of 0.08 g/cm$^3$ and a crystal size of approximately 2 to 3 mm. During the test it was calm and sunny with air and snow temperature in the shade of $-4^\circ$C. Following the same procedure as before, we find values $\Delta V = 13.9$ dB, $\rho = 0.66$ at 35.1 GHz, $\Delta V = 8.9$ dB, $\rho = 0.47$ at 98.1 GHz, and $\Delta V = 6.1$ dB, $\rho = 0.34$ at 140.1 GHz. The snow depth on the ground ranged from 5 to 8 cm. At grazing angles of 0.5 to 2 degrees and over flat terrain this would result in ground-reflected signals traveling between 2.9 and 18.3 m through snow. The similarity of Figs. 6(a) and 7(a) makes one suspect that the underlying grass covered ground and not the snow surface produced most of the multipath signal or signals. It is possible that the path attenuation through snow is moderate enough at 35.1 GHz to achieve this. Snow penetration of ground-reflected signals is much less likely at the other frequencies in Fig. 7(b) and (c), where the snow attenuation is substantially higher. For snow slabs that had aged without melting and refreezing we found attenuation coefficients of 29 dB/km at 35.1 GHz and 495 dB/km at 140.1 GHz.

In Fig. 8(a), the snow cover was so thin, it was thought that multipath signals were mostly coming from the terrain with negligible influence from the snow. In reality this snow, composed of a thin layer of granular structure, resulted in the strongest multipath interferences seen at 35.1 GHz. The effect of the thin snow layer is substantial when comparing Figs. 8(a) and 6(a). The depth was no more than 3 cm with a limited number of blades from the underlying dry grass penetrating the snow surface. Light snowfall had occurred on the previous two days, which appeared to be wet on the first day. During the measurements it was sunny and windy. The wind was strong enough to cover footprints with loose snow in a few hours. The maximum temperature for the day was $-3^\circ$C. The height region from 2.3 to 3.3 m in the graph is evaluated for $\Delta V = 21.9$ dB and $\rho = 0.85$. It is interesting to note...
the similarities of this figure and Fig. 3, a simulation based on comparable parameters. No data exist at the other two frequencies on this particular day. Fig. 8(b) and (c) were obtained under different snow conditions. Commonality in Fig. 8 is achieved through the fact that in all cases the snow was granular in nature. In Fig. 8(b) and (c) we call the ground cover sleet, following the definition by Peterssen [8]. It was clearly different from wet snow, which has a much softer consistency, and it was not freezing rain, as was experienced during the later part of the day. Temperatures hovered around the freezing point and the material deposited on the ground remained essentially as it had fallen two days before the measurements. On the next day temperatures stayed below $-2^\circ$C and light snow fell, making it possible for the sleet to freeze over. The surface hardness was sufficient to support a person walk-
ing on it. A density of 0.47 g/cm³ was determined. Visual inspection revealed mostly spherical particles of 1- to 2-
mm diameter, with some as large as 3 mm. The day of
measurement was sunny with a snow temperature of 0°C
and an air temperature of -1°C. The increase in the num-
ber of deep cancellations is clearly visible at all three fre-
quencies in this figure. In Fig. 8(b) we estimate \( \Delta V \)
= 18.1 dB, \( \rho = 0.78 \) at 98.1 GHz. Correspondingly, in Fig.
8(c) we find \( \Delta V = 15.3 \) dB, \( \rho = 0.71 \) at 140.1 GHz.

For comparison, data at the three frequencies over all
types of ground cover have been compiled in Table II.
Reflection coefficients calculated from (1) for fresh snow
(\( \varepsilon_r = 1.12 \)) and for old snow (\( \varepsilon_r = 1.92 \)) at 2 degrees
and vertical polarization have been added. The calculated
values are independent of frequency. An inspection of Ta-
ble II shows good agreement between measurement and
model at 35.1 GHz over old snow, with the measured val-
ues decreasing somewhat below the calculated one at the
higher frequencies. Over fresh snow, there is a larger dis-
crepancy at 35.1 GHz. The discrepancy increases toward
higher frequency with the measured value being less than
half the calculated value at 140.1 GHz. Three mecha-
nisms may account for this: 1) The snow surface is rough
in terms of the Rayleigh criterion, rather than a smooth
dielectric interface as considered by the model. At 2-m
receiver height, a \( \lambda/4 \) pathlength difference [9] on our
path corresponds to a surface height difference of 1.2 cm
at 140.1 GHz, 1.9 cm at 98.1 GHz, and 4.8 cm at 35.1
GHz. Surface roughness was not measured in situ. It is
possible that diffuse surface scatter reduced the specularly
reflected signal, an effect increasing with frequency. 2)
The snow surface is largely specular, but has protrusions,
which the grazing signal has to penetrate. In the process
it is being attenuated, and progressively so with fre-
quency. An additional path loss of 6 dB for the reflected
ray will halve the measured reflection coefficient. Such
loss is conceivable in view of the high attenuation in snow
quoted earlier. 3) Multipath propagation over snow is not
simply a surface reflection phenomenon but depends to
some extent on near-surface scattering. The scattering is
cohesive and constructive at least to a depth of 1.2 cm at
140.1 GHz and deeper at the lower frequencies (see (1)).
That is, the composite phasing of the received signal is
practically the same as for surface reflection in the model.

If surface reflection is reduced and volume scattering is
frequency dependent due to attenuation suffered in snow,
then the shortcomings of the model can be explained. Note
that penetration into the snow is more important in fresh
than old snow. The latter acts much more like a surface
reflector at all wavelengths with a correspondingly lower
difference between measured and calculated values.

V. CONCLUSIONS

Measurements over a range of grazing angles from 0.5
to 2 degrees and over a relatively flat grass or snow sur-
face have yielded multipath signals that appear to be made
up of a limited number of phasors, typical of specular re-
fection from one or a few facets along the propagation
path. A model composed on the basis of relevant param-
ters of the experimental setup including major features
of the terrain showed good qualitative agreement with
measurements at 35.1 GHz. This includes features such
as number of lobes, cancellation depths, and number of
phasors involved. The surface model was based on terrain
heights and not snow heights that were not measured along
the path. At 98.1 and 140.1 GHz, surface roughness and
snow attenuation become more of a factor over the same
terrain and the same range of grazing angles. Despite this
fact, the number of measured lobes does not increase sig-
ificantly over that expected for two-signal interaction for
the given path geometry. As stated previously, model cal-
culations yield 10, 28, and 40 lobes at 35.1, 98.1, and
140.1 GHz, respectively. A count in Fig. 8 leads to cor-
responding numbers of 12, 33, and 50. This count is
somewhat subjective, depending on what size lobe con-
stitutes a major one. It must, however, be emphasized that
the relatively close resemblance between counts (theoret-
ical specular model versus experimental data) was the pri-
mary reason for treating our multipath propagation phe-
nomenon as specular. The specific value derived for \( \rho \)
must be used with some caution. Neither have sufficient
data been accumulated to firmly establish these values,
nor does the measurement technique lend itself to a simple
derivation of \( \rho \). The measurements do give an indication
of the range of constructive and destructive phase inter-
ference that one can expect at a particular millimeter-
waveband and grazing angle range over various types of
ground cover, if the antenna beams cannot discriminate
against surface reflections.

APPENDIX

Consider Fig. 9, in which a six-segment linear profile
is indicated, defined by segment heights \( h_0 \) through \( h_6 \)
and segment length \( l_1 \) through \( l_6 \). The actual test path has
a comparable profile. The transmit antenna at \( T \) and the re-
ceive antenna at \( R \) have Gaussian-shaped beams with tilt
angles \( \delta \) relative to the horizontal. Assume that a ray from
the transmitter with takeoff angle \( \beta_{TR} \) is incident at point
\( S \), which is at height \( h_s \), above the reference level. The
slope from which reflection takes place forms the angle
\( \gamma = \tan^{-1}[(h_s - h_{s-1})/l_s] \) (A1)
with the horizontal direction. With $\theta$ the grazing angle on the local slope we can write

$$\beta_{RR} = 2\gamma + \beta_{TR}. \quad (A2)$$

The reflected ray reaches the receiver ordinate at height

$$h = h_i + (l_{TR} - l) \tan (\beta_{TR} + 2\gamma) \quad (A3)$$

where $l_{TR}$ is the sum of the segment lengths and $l$ is the horizontal distance between $T$ and $S$.

The length of the direct path $l_D$ and the reflected path $l_T + l_R$ follow from trigonometry. At the shallow grazing angles of interest here the phase shift upon reflection is accounted for by an additional pathlength of half the radio wavelength $\lambda$.

To compute the height-gain pattern, the reflection point on the surface is moved in increments $\Delta l$ from $l = 0$ to $l_{TR}$. The height $h$ at which the reflected ray from each reflection point crosses the receiver ordinate is determined according to the local height and slope of the reflection point. If height $h$ falls within the range $h$, over which the probing receiver travels, a phasor addition is performed between the direct signal calculated for the same height and the reflected signal. Depending on the distance of the reflection point from the transmitter and the local height and slope, multiple reflected signals may superimpose with the direct signal to form the sum signal at any particular height.

It is obvious that the simple terrain model of Fig. 9 results in discontinuities in the local slope, when the reflection point transitions from one segment to the next. In this case the reflected ray breaks off at one height $h$ and jumps to another one. For a more realistic profile, circular transition regions between segments were introduced. These transition regions prevent the reflected signal from varying discontinuously with height. The circular segment is tangent to the present and the next-segment straight slopes.

The circle diameter is determined by selecting a distance $q$ between the present-segment tangent point and the end of that segment. In Fig. 10 one such circular transition region is drawn between segments $n$ and $n + 1$. With the segment slopes $\gamma_1$ and $\gamma_2$ given by (A1), the tangent points $x_1, y_1$, and $x_2, y_2$ as well as the center point $x_0, y_0$ follow from trigonometry. The height of the reflection point in the circular region of radius $r$ becomes

$$h_i = \left( r^2 - (x - x_0)^2 \right)^{1/2} + y_0 \quad (A4)$$

and the surface slope

$$\gamma = \tan^{-1} \left( -1/\left[ r/(x - x_0) \right]^2 - 1 \right)^{1/2}. \quad (A5)$$
For all receiver heights the model assumes range independent amplitudes on both the direct and indirect paths. The magnitude of the sum signal thus becomes

\[ V = G_T G_R e^{-2\pi h/\lambda} + \sum_k G_T G_R e^{-2\pi (l_k + h_k + \lambda/2)/\lambda} \]  

(A6)

Antenna gains \( G_T \) and \( G_R \) have to be taken along the direction of the particular rays, assuming there are \( k \) reflected rays. If only a single reflected ray exists, and the path geometry makes it possible to neglect the effect of antenna gain patterns on direct and reflected signals, then the sum signal varies between \( 1 + \rho \) and \( 1 - \rho \). From the voltage ratio \( \Delta V \) between adjacent maxima and minima of a measured height-gain pattern

\[ \Delta V = 20 \log \left[ \frac{(1 + \rho)/(1 - \rho)} \right] \text{ (in decibels)} \]  

(A7)

the reflection coefficient \( \rho \) can be determined.

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