Gamma-Ray Transport Through Material Layers

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Abstract

The transmitted photon energy fluence was calculated as a function of layer thickness for 1.25 MeV photons in Al, Ag, and Pb using the CEPXS/ONELD coupled electron-photon code. The energy fluence vs layer thickness was also calculated as a function of beam radius using the CYLTRAN Monte Carlo code. We compare these results with predictions using the photon "energy absorption coefficient attenuation law" equation recommended in ASTM Standard E666-91 and suggest replacing this equation with a more realistic expression.

I. INTRODUCTION

Radiation testing of an electronic device requires that the dose delivered to the device be determined. ASTM Standard E666-91 [1] provides a methodology for estimating the dose at a specified depth in a material due to an x-ray source with a known energy spectrum and the dose measured in another material (dosimeter). This methodology assumes the validity of the "energy absorption coefficient attenuation law" (Eq. (3) in [1]),

\[ \psi_t(E) = \psi_i(E) \exp \left( - \left[ \frac{\mu_{\text{abs}}(E)}{\rho} \right] t \right) \]

where \( \psi_i(E) \) is the energy fluence spectrum at a distance \( t \) (in g/cm\(^2\)) from the front surface of a material, \( \psi_t(E) \) is the incident energy fluence spectrum at the front surface, and \( \mu_{\text{abs}}(E)/\rho \) is the mass energy absorption coefficient at an energy \( E \). In Eq. (1), all changes in the photon energy spectrum due to the presence of Compton scattered photons are ignored.

In this study, we will be concerned only with a monoenergetic incident beam of energy \( E_i \) and with the total energy fluence \( \psi_t \); in other words we will treat \( \psi_i(E) \) as a delta function. If we write \( \psi_i(E) = \psi_i(\delta(E - E_i)) \) \( \psi_t(E) = \psi_t(\delta(E - E_i)) \) (corresponding to \( t = 0 \) and arbitrary \( t \) respectively) and integrate equation (1) with respect to energy, we obtain the so-called "broad beam attenuation law" [2] for the total energy fluence as

\[ \psi_t = \psi_i \exp \left( - \left[ \frac{\mu_{\text{abs}}(E_i)}{\rho} \right] t \right) \]  (2).

We will henceforth refer to equation (2) as the "energy absorption coefficient attenuation law" or the "E666 law".

The corresponding "total attenuation coefficient attenuation law" (also known as the "narrow-beam attenuation law" [2]), is given by

\[ \psi_t = \psi_i \exp \left( - \left[ \frac{\mu_{\text{abs}}(E)}{\rho} \right] t \right) \]  (3).

This law applies when there are no scattered photons present in the transmitted beam. Here, \( \mu_{\text{abs}}(E_i)/\rho \) is the mass attenuation coefficient at the initial energy \( E_i \). Equations (2) and (3) are equivalent at lower x-ray energies where the photoelectric interaction is dominant and Compton scattering is negligible.

How good an approximation is Eq. (2) for calculating photon energy fluence at Cobalt-60 photon energies? In this paper we examine this question for monoenergetic 1.25 MeV gamma rays, which is the average energy of the primary gamma-rays from a Cobalt-60 source. Using the CEPXS/ONELD coupled electron-photon transport code [3,4], we have calculated the transmission of 1.25 MeV gamma rays as a function of thickness in three commonly used materials of widely differing atomic number, Al (Z=13), Ag (Z=47), and Pb (Z=82) as follows:

(a) With Compton photon scattering rigorously taken into account in the CEPXS/ONELD code, we determined the total energy fluence as a function of layer thickness and compared the results with the predictions of equations (2) and (3).

(b) Using the Monte Carlo code CYLTRAN [5], we studied the effects of the size of the incident photon beam, calculating the transmitted energy fluence vs layer thickness as a function of beam radius.

(c) We fitted the CEPXS/ONELD energy fluence vs thickness curves in Figs. 1-3 with a "physically-based" fitting function involving \( \mu_{\text{abs}}(E)/\rho \), \( \mu_{\text{abs}}(E_i)/\rho \), and two adjustable parameters.

Our conclusion is that Equations (1) and (2) are not applicable at gamma-ray energies and should be replaced by an expression that takes full account of Compton scattering. Following presentation of the results of these calculations, we discuss some practical implications for radiation shielding and dosimetry at gamma-ray energies.
II. ENERGY FLUENCE

A. Background

The gamma-ray transport and radiation shielding literature is voluminous and dates back to the 1940's. Reference 6 covers a wide range of shielding data. For example, it provides energy fluence vs thickness curves for Pb at 1.0, 2.5 and 6.0 MeV (in the form of energy transmission buildup factors). However, it gives no attenuation curves for Al and Ag nor does it give a curve for Pb at 1.25 MeV. In the context of radiation dosimetry for electronics testing, including the E666 standard [11], more data than is provided in [6] is needed for finding the energy fluence as a function of thickness for any material for an arbitrary photon energy in the 100-1250 keV energy range.

Attix [2] discusses photon transport through layers and defines the terms "narrow-beam" and "broad-beam attenuation"; our Eq. (2) is the same as Eq. (3.10) in reference 2. Beams of intermediate diameter are discussed but not treated quantitatively in [2]. We were interested in seeing what the term "broad-beam" really means for 1.25 MeV gamma-rays; this is why we made Monte Carlo studies as a function of thickness for various beam radii for Al and Pb. Our result is that at infinite beam radius the transmission curve is the CEPXS/ONELD result, not equation (2). Therefore equation (2) is the "broad-beam attenuation law" only at x-ray energies.

B. CEPXS/ONELD Calculations of Energy Fluence

Table 1 lists some physical properties and photon transport quantities (evaluated at \( E_o = 1.25 \text{ MeV} \)) for the three materials we studied. Al, Ag, and Pb provide examples of attenuation in materials of low, intermediate Z, and high atomic number Z respectively. In addition to being representative of low and high Z materials, Al and Pb have a special importance as the materials used in the Pb/Al "filter box" used to minimize dose enhancement in Co-60 radiation effects testing [8,9].

To determine the attenuation behavior at both small and large thicknesses, calculations for a wide range of material thicknesses were needed. Using the CEPXS/ONELD code, we found the transmitted energy fluence as a function of thickness. The results for Al, Ag, and Pb are shown in Figs. 1 - 3 where we have plotted thickness in photon mean free path units (mfp), varying the thickness from near zero to large thicknesses (5 mfp). Here 1 mfp (in g/cm\(^2\)) = \( \rho / \mu \mu (E_o) \) for each material. In each figure we plotted the "energy absorption attenuation law" [Eq. (2)], the "total attenuation coefficient attenuation law" [Eq. (3)], and the energy fluence curve calculated using CEPXS/ONELD. In the case of Pb, experimental photon number fluence data points taken from Evans [10] are also included on the graph; we found complete agreement between the CEPXS/ONELD number fluence curve and this data.

Table 1: Photon transport constants at 1.25 MeV for 3 elements.

<table>
<thead>
<tr>
<th></th>
<th>Al (g/cm(^2))</th>
<th>Ag (g/cm(^2))</th>
<th>Pb (g/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>2.699</td>
<td>10.5</td>
<td>11.35</td>
</tr>
<tr>
<td>( \mu_n/\rho ) (cm(^2)/g)</td>
<td>5.496E-2</td>
<td>5.217E-2</td>
<td>5.876E-2</td>
</tr>
<tr>
<td>( \mu_p/\rho ) (cm(^2)/g)</td>
<td>2.565E-2</td>
<td>2.425E-2</td>
<td>2.988E-2</td>
</tr>
<tr>
<td>1 photon mfp (g/cm(^2))</td>
<td>18.2</td>
<td>19.17</td>
<td>17.02</td>
</tr>
<tr>
<td>1 photon mfp (cm)</td>
<td>6.741</td>
<td>1.826</td>
<td>1.499</td>
</tr>
</tbody>
</table>
From these figures we see that the transmitted photon energy fluence curve calculated by CEPXS/ONELD lies between the two attenuation laws given by Eqs. (1) and (2). The calculated energy fluence decreases more rapidly than the energy absorption law, but not as rapidly as the total attenuation law. At large thicknesses the calculated energy fluence becomes parallel to the total attenuation law (related by a multiplicative factor we call "B").

We also note that with increasing Z (going from Figure 1 to Figure 3), the CEPXS/ONELD curve moves toward the narrow-beam attenuation law [Eq. (3)]. This is a result of the much increased photoelectric absorption relative to the Compton interaction in high Z materials at 1.25 MeV. This increases the amount of photon absorption relative to scattering and causes the relative energy fluence curve to behave more like the total attenuation law.

III. BEAM RADIUS

Using the CYLTRAN Monte Carlo code we calculated the energy fluence transmitted through Al and Pb as a function of layer thickness (in mfp) and beam radius (in cm). The CYLTRAN code was slightly modified to count only the photons escaping the layer within a radius equal to the beam radius specified. The results are shown in Figs. 4 - 7. In Figures 4 and 5 we see that as the beam radius is varied over the range 0.05 - 10.0 cm, the fractional transmitted energy fluence approaches (1) the "narrow-beam" attenuation law as the radius gets smaller, and (2) the CEPXS/ONELD curve as the radius gets larger. No curve in Figures 4 and 5 for any beam radius lies close to the energy absorption coefficient attenuation law [Eq. (2)].

Figures 6 and 7 show somewhat more clearly the effect of beam radius on the transmission of photon beams, where curves of constant attenuation are shown. The crossover between narrow and broad beam attenuation behavior appears to occur in the order of about 2 - 3 cm for Al and less than 1 cm for Pb.
1.25 MeV on Pb - Beam Radius Effect

Figure 7: Log fractional transmitted energy fluence vs beam radius for Pb at 1.25 MeV for various layer thicknesses. (0.1, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0 mfp)

cm for Pb. These curves provide the first quantitative study we have seen of the attenuation of photon beams of finite radius lying between infinitely "narrow" and "broad" beams.

IV. ENERGY FLUENCE FITS

The results shown in Figs. 1 - 3 led us to develop a "physically-based" fitting function.

\[ \frac{\psi_r}{\psi_0} = \exp \left( -\frac{\mu_{\text{in}}(E_0)}{\rho} t \right) \exp \left[ \ln B \left( 1 - \exp \left( -\frac{\mu_{\text{in}}(E_0) - f \cdot \mu_{\text{out}}(E_0)}{\rho \ln B} t \right) \right) \right] \quad (4) \]

Using this function, \( \frac{\psi_r}{\psi_0} \) is approximately given by:

\[ \frac{\psi_r}{\psi_0} \approx \exp \left( -\frac{f \cdot \mu_{\text{in}}(E_0)}{\rho} t \right) \quad \text{for small } t \]

and

\[ \frac{\psi_r}{\psi_0} \approx B \cdot \exp \left( -\frac{\mu_{\text{out}}(E_0)}{\rho} t \right) \quad \text{for large } t. \]

The constant \( B \) is a measure of photon "build-up" (\( B \) is shown on the figure in Evans [10]); it is not the "build-up factor" as defined in the radiation shielding literature [3]. \( f \) is a constant to multiply the energy absorption coefficient to fit the slope at \( t=0 \). The TableCurve™ 2D computer program [11] was used to fit Eq. (4) to the CEPXS/ONELD calculated energy fluence. The results are shown in Figs. 8 - 10 and the two fitting constants \( B \) and \( f \) are tabulated in Table 2. The curve fit is excellent for each of the three cases.

Table 2: The fitting constants \( B \) and \( f \) for Al, Ag, and Pb at 1.25 MeV.

<table>
<thead>
<tr>
<th></th>
<th>( B )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>6.479</td>
<td>1.205</td>
</tr>
<tr>
<td>Ag</td>
<td>5.644</td>
<td>1.302</td>
</tr>
<tr>
<td>Pb</td>
<td>3.179</td>
<td>1.372</td>
</tr>
</tbody>
</table>

1.25 MeV on Al - Fit to Eq. (4)

Figure 8: Log fractional transmitted energy fluence vs thickness for Al at 1.25 MeV (curve fit and calculated data points).

1.25 MeV on Ag - Fit to Eq. (4)

Figure 9: Log fractional transmitted energy fluence vs thickness for Ag at 1.25 MeV (curve fit and calculated data points).

1.25 MeV on Pb - Fit to Eq. (4)

Figure 10: Log fractional transmitted energy fluence vs thickness for Pb at 1.25 MeV (curve fit and calculated data points).
V. APPLICATION TO RADIATION SHIELDING AND DOSIMETRY

A. Shielding

In Figs. 11 - 13 we compare the CEPXS/ONELD results with the "E666 law" at 1.25 MeV for Al, Ag, and Pb respectively on a linear-linear scale as compared to the semilog scales used in Figs. 1 - 3. Thicknesses are given in cm. We see from Figure 11 that the two attenuation laws are very similar for Al (particularly below 1 cm) but the difference between them becomes progressively greater for Ag (Fig. 12) and Pb (Fig. 13).

Table 3 and 4 provide numerical values for the relative energy fluence for Al and Pb for selected thicknesses of practical interest. As an example, the recommended thickness of the Pb layer in the Pb/Al filter box recommended in ASTM Standard E1249 [8,9] is about 1/16" and that of the Al layer is about 1 mm. For both the Al and Pb layers, the relative energy fluence difference between the two attenuation laws is small; this is not surprising, since the principal purpose of these layers is to filter soft x-rays in the 100-200 keV energy range and not to attenuate gamma-rays.

In general, for Al thicknesses ranging from 1 mm to 1 cm, the transmitted energy fluence values between the two attenuation laws differs by 1.5% or less (see the "Ratio" column in Table 3). Thus, for Al, the new attenuation law makes essentially no difference to the energy fluence. (However, as we discuss in the next section, there is a pronounced difference in the predicted dose at the exiting surface of an Al layer.)

For Pb layers, there is a substantial difference between the two attenuation laws. For example, from Table 4, the ratio of the relative energy fluence in Pb for 1 mm is 1.0129 (about 1.3%), but for 1 cm it is 1.1534 (around 15%). In reference 9 a 2" Pb filter is employed for low dose rate studies using Co-60 radiation; for this thickness, the attenuation predicted by the E666 law is 0.1786 while that predicted by CEPXS/ONELD is 0.0666 (a factor 2.7 difference).

Table 3: Relative energy fluence in Al.

<table>
<thead>
<tr>
<th>Thickness (cm)</th>
<th>E666</th>
<th>CEPXS/ONELD</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1 cm (1 mm)</td>
<td>0.9931</td>
<td>0.9917</td>
<td>1.0014</td>
</tr>
<tr>
<td>.159 cm (1/16&quot;)</td>
<td>0.9891</td>
<td>0.9868</td>
<td>1.0023</td>
</tr>
<tr>
<td>1.0 cm</td>
<td>0.9331</td>
<td>0.9189</td>
<td>1.0154</td>
</tr>
<tr>
<td>2.54 cm (1&quot;)</td>
<td>0.8388</td>
<td>0.8034</td>
<td>1.0441</td>
</tr>
<tr>
<td>5.08 cm (2&quot;)</td>
<td>0.7035</td>
<td>0.6368</td>
<td>1.1047</td>
</tr>
<tr>
<td>6.741 cm (1 mfp)</td>
<td>0.6271</td>
<td>0.5434</td>
<td>1.1539</td>
</tr>
<tr>
<td>10.0 cm</td>
<td>0.5004</td>
<td>0.3926</td>
<td>1.2747</td>
</tr>
</tbody>
</table>

Figure 11: Linear-linear graph of relative energy fluence vs thickness (cm) of Al at 1.25 MeV.

Figure 12: Linear-linear graph of relative energy fluence vs thickness (cm) of Ag at 1.25 MeV.

Figure 13: Linear-linear graph of relative energy fluence vs thickness (cm) of Pb at 1.25 MeV.
electron transport effects. If electron transport were included, this would reduce the dose at small layer thicknesses (in the order of an electron range) below the values shown in Figs. 14 - 16.

### Table 4: Relative energy fluence in Pb.

<table>
<thead>
<tr>
<th>Thickness</th>
<th>E666</th>
<th>CEPXS/ONELD</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1 cm (1 mm)</td>
<td>0.9667</td>
<td>0.9544</td>
<td>1.0129</td>
</tr>
<tr>
<td>.159 cm (1/16&quot;)</td>
<td>0.9476</td>
<td>0.9284</td>
<td>1.0207</td>
</tr>
<tr>
<td>1.0 cm</td>
<td>0.7124</td>
<td>0.6176</td>
<td>1.1534</td>
</tr>
<tr>
<td>1.499 cm (1 mfp)</td>
<td>0.6014</td>
<td>0.4800</td>
<td>1.2529</td>
</tr>
<tr>
<td>2.54 cm (1&quot;)</td>
<td>0.4226</td>
<td>0.2780</td>
<td>1.5201</td>
</tr>
<tr>
<td>5.08 cm (2&quot;)</td>
<td>0.1786</td>
<td>0.0666</td>
<td>2.6795</td>
</tr>
<tr>
<td>10.0 cm</td>
<td>0.0337</td>
<td>0.0033</td>
<td>10.2128</td>
</tr>
</tbody>
</table>

### B. Dosimetry

1) **Dose formulas for E666 and CEPXS/ONELD**

If the energy fluence at thickness $t$ is given by $\psi_0$, then the rate the energy fluence decreases as the thickness increases is given by $d\psi_0 / dt$. The rate energy is deposited in a thickness $dt$ is $-(d\psi_0 / dt)\cdot dt$ so that the dose must be $-d\psi_0 / dt$. From equation (2), the dose $D$ is given by

$$D = \psi_0 \left[ \mu_{en}(E_0) / \rho \right] \exp \left( -\left[ \mu_{en}(E_0) / \rho \right] t \right)$$

while for the curve fit to the CEPXS/ONELD results [equation (3)]

$$D = 1.25 \cdot \left[ \exp \left( -\mu_{en} \cdot t + \ln(B) \cdot \left[ 1 - \exp \left( \frac{\mu_{en} - f \cdot \mu_{en}}{\ln(B)} \cdot t \right) \right] \right) \right]$$

In Figures 14 - 16 we show plots of the calculated dose vs thickness (where the dose is at the exiting surface) from equations (5) and (6) for Al, Ag, and Pb. Here a 20% - 30% increase of the dose value obtained from the equation (6) (obtained by differentiating the CEPXS/ONELD curve fit) over that from the E666 attenuation law. Thus we see that the different slopes of the two energy fluence curves shown in Figures 11 - 13 lead to a large difference in the dose predicted at the exit surface.

It can be seen in Figures 15 and 16 that the dose curves cross each other (this also occurs for Al at greater than 10 cm). That is, the dose predicted by the CEPXS/ONELD curve fit is greater than that predicted by the E666 law for smaller thicknesses, but the E666 law predicts a greater dose at large thicknesses. This of course reflects the fact that the energy fluence drops off faster for the CEPXS/ONELD curve than for the E666 law (Figs. 1 - 3).

We note that all our CEPXS/ONELD calculations were performed in the "photon transport only" mode; we ignored all...
2) Implications for ASTM Standard E666

This paper demonstrates that the E666 attenuation law, Eq. (1), is not valid for 1.25 MeV gamma-rays, since it doesn't take into account the transport and scattering of Compton photons. It is invalid in two respects:

(1) It overestimates the transmitted energy fluence.
(2) It neglects substantial changes (which we observed using CEPXS/ONELD) in the energy fluence spectrum due to Compton-scattered photons.

Our proposed replacement equation (4) appears promising. However, we only made calculations for 3 materials at a single gamma-ray energy. Many more calculations of energy fluence and energy fluence spectra for materials with a wide range of Z over the full range of x-ray and gamma-ray energies would be needed to obtain a general expression that could be used to replace equation (1).

VI. CONCLUSIONS

Using the "photon-only" mode of the state-of-the-art coupled electron/photon transport code CEPXS/ONELD we have performed transport calculations of the transmission of 1.25 MeV gamma-rays through various thicknesses of Al, Ag, and Pb. Comparing the energy fluence transmitted as a function of thickness with the attenuation law given in ASTM Standard E666, we find that the transmission curve differs considerably from the energy absorption coefficient attenuation law [equation (2)]. (This result has been previously shown for Pb by Evans [10]).

We fitted the energy fluence vs layer thickness curves at 1.25 MeV with the function given by Eq. (3) and find it gives an excellent fit to the CEPXS/ONELD curves for Al, Ag, and Pb. This equation, when extended to more photon energies and materials, shows promise as a possible replacement for the exponential attenuation law given in ASTM E666.

VII. REFERENCES


[7] Ibid, page 223, Fig. 4.3-30.


