Investigation of Angular Smoothing of PET Data

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Abstract
Radial filtering of emission and transmission data is routinely performed in PET during reconstruction in order to reduce image noise. Angular smoothing is not typically done, due to the introduction of a non-uniform resolution loss. The goal of this paper was to assess the effects of angular smoothing on noise and resolution. Angular smoothing was incorporated into the reconstruction process on the Scanditronix PC2048-15B brain PET scanner. In-plane spatial resolution and noise reduction were measured for different amounts of radial and angular smoothing. For radial positions away from the center of the scanner, noise reduction and degraded tangential resolution with negligible loss of radial resolution were seen. Near the center, no resolution loss was observed, but there was also no reduction in noise even for large amounts of angular filtering. These results can be understood by recognizing that angular filtering is equivalent to a weighted sum of rotated images. We conclude that angular smoothing is not optimal due to its anisotropic noise reduction and resolution degradation properties.

I. INTRODUCTION
Image data in positron emission tomography (PET) often suffer from poor statistics due to limitations on scan duration and radiation exposure. Typically, emission data are filtered prior to backprojection in order to reduce the statistical noise in the image. In addition, measured transmission data are smoothed prior to generating attenuation correction factors in order to decrease the contribution of transmission noise to the emission image [1]. For both emission and transmission data, smoothing is most often performed in the radial direction (along rows in the sinogram). Sometimes, it is also performed in the angular direction through the application of a two-dimensional filter [2], [3], [4].

While radial filtering will blur the transmission (or emission) data uniformly across the transverse field of view (FOV), angular filtering will result in a spatially dependent blur. This is due to the spatial variation of the distance between lines of response (LORs) across the FOV. This nonuniform resolution degradation has led many PET investigators to avoid angular filtering altogether.

Stearns [4] developed a metric to determine the conditions under which angular smoothing of transmission data could be used without severely impacting the spatial resolution of the resultant attenuation correction factors. However, the magnitude and spatial distribution of resolution loss and noise reduction due to angular smoothing have not been previously quantified. The goal of this study was to investigate the effects on noise and spatial resolution of angular smoothing of both emission and transmission data.

II. METHODS

A. Angular Smoothing of Emission Data
Angular smoothing was incorporated into the 2D filtered backprojection reconstruction algorithm for emission data after the raw data had been interpolated to an even ray spacing and all corrections for physical effects (detector normalization, random coincidences, attenuation, and scatter) had been performed. A spatially-invariant Gaussian filter was applied in the angular direction across projections for all radial locations, taking into account smoothing across the 0°/180° boundary. Radial filtering was then performed on the angularly smoothed projections immediately prior to backprojection.

B. Angular Smoothing of Transmission Data
To perform angular smoothing of transmission data, the unsmoothed attenuation correction factors (ACFs) were generated from the ratio of the blank (B) and transmission (T) data sets for each projection angle and ray. The blank and transmission data were interpolated to an even ray spacing prior to computing their ratio. The ACF values were originally created with no radial or angular smoothing. The inverse of the ACFs was then calculated and smoothed with a Gaussian function in the radial or angular direction.

This process differs somewhat from how transmission smoothing is usually done on this scanner, in that the ratio of T/B (the inverse of ACF) was smoothed, rather than smoothing the transmission and blank data separately prior to taking their ratio. The method as implemented assumes that there is minimal noise in the blank data, a reasonable assumption, since there is typically no limit to the number of counts that can be acquired in the blank scan.

III. MEASUREMENTS

A. Scanner Characteristics
All scans were carried out on the Scanditronix PC2048-15B brain PET scanner (Scanditronix AB, Uppsala, Sweden) [5], [6]. This system has eight rings of 256 detectors for 15 reconstructed slices, separated by 6.5 mm, and operates only in 2D mode. The detector ring diameter is 52.5 cm, with a reconstructed transverse field of view of 25.6 cm. The sinogram spacing after interpolation is 2 mm and 1.4° in the radial and
angular directions, respectively. Blank and transmission scans are acquired using a rotating line source of Ge-68 (163 MBq, 4.4 mCi at the time of these studies).

B. Spatial Resolution

The transverse spatial resolution was measured using line sources filled with F-18 and placed at 25 radial locations between 0.5 cm and 11.0 cm. At least 200 kcts per source were acquired. The scan data were reconstructed with (1) angular Gaussian filters with a full-width-at-half-maximum (FWHM) of 0° (no smoothing), 1.4°, 2.8°, 4.2°, 5.6°, and 7.0° (5 projection angles) and a radial ramp filter with a cutoff at the Nyquist frequency (0.25 mm⁻¹) and (2) no angular smoothing and the radial filters shown in Table 1. No corrections for scatter or attenuation were made during reconstruction. Two-dimensional Gaussian fitting was performed on the resulting images to determine the radial and tangential components of spatial resolution as a function of radial location. The resolution values were averaged over all slices.

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<th>Radial filters for emission data</th>
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C. Emission Noise

A 20-cm uniform cylinder was used to assess the impact of angular smoothing of emission data on noise in the reconstructed image. The phantom was filled with 104 MBq (2.8 mCi) of F-18 and centered in the gantry. Replicate scans were acquired with the scan durations adjusted to account for decay. In that way, 100 scans, each with 150 kcts/slice, were acquired. After the activity had decayed, long emission scan (18.5 Mcts) scans were acquired, in order to minimize the contribution of noise in the attenuation correction to noise in the emission image.

The 100 scans were reconstructed with the same radial and angular filters as described above. Mean and standard deviation (SD) images were created for one slice from the replicate data. An image of the coefficient of variation (COV) was then created from the SD image normalized by the average value in a 16-cm diameter circular region of interest (ROI) drawn on the mean image. To measure noise as a function of radial location, 20 5-mm wide, concentric annular ROIs with increasing radial distance (inner radius: 0 - 9.5 cm) were drawn on the COV image, centered on the image. The average COV(r) was determined from these annular ROIs.

D. Transmission Noise with Angular Smoothing

The same 20-cm uniform cylinder was used to study the effects of angular smoothing of transmission data on noise in the emission image. One hundred replicate transmission scans were acquired with 1.3 Mcts/slice. A long emission scan (22 Mcts/slice) was acquired prior to the transmission scan. A long blank scan (18.5 Mcts) was acquired so that its contribution to noise in the attenuation correction would be negligible. The replicate unsmoothed ACFs were smoothed as described in Section II.B, with (1) no radial smoothing and angular filters of FWHM = 0°, 1.4°, 2.8°, 4.2°, 5.6°, and 7.0° and (2) no angular smoothing and radial filters of FWHM = 0, 2, 4, 6, 8, and 10 mm. The single long emission scan was then reconstructed using the sets of replicate smoothed ACFs, and a Hanning radial reconstruction filter (0.25-mm⁻¹ cutoff) was applied to the emission data prior to backprojection. The mean and SD images were generated and COV(r) determined, as above.

IV. RESULTS

A. Emission Resolution and Noise

Figure 1 shows images from the resolution study for a radial ramp filter and four angular filter widths. It can be observed that the tangential resolution worsens with increased angular smoothing at larger radii (top and bottom spots), as expected, while the radial resolution is essentially unchanged at any radial location. The resolution in the center of the scanner (central spot) is unaffected by angular smoothing.

Figure 2 shows the SD images from the emission noise study with the 20-cm cylinder for a radial ramp filter and the same four angular filters as in Figure 1. The SD decreases at larger radii with increasing angular smoothing, but the noise in the center is unaffected by any of the angular filters shown.

Figure 3 shows the effects of angular filtering on (a) tangential and (b) radial resolutions and (c) COV for the 20-cm cylinder emission study. Each curve shows the behavior for a
different radial location. The tangential resolution is noticeably degraded with increasing angular smoothing, with greater degradation occurring for the large radial distances. If one accounts for the intrinsic resolution of the scanner, the tangential resolution increases linearly with radial distance. Angular smoothing has minimal effect on the radial resolution, as expected. The slight dependence observed is due to the fact that angular filtering results in a circular smearing of the image; thus, smoothing occurs primarily in the tangential direction with a small radial component to the blur (see Discussion). As can be seen from Figure 3c, angular smoothing reduces noise. At the center of the FOV (closed circles), however, both noise reduction and resolution loss are negligible for any amount of angular smoothing.

Figure 2. SD images from the emission noise study. All images shown were reconstructed with a radial ramp filter and are displayed on a common scale. Results are shown for angular filter widths of (a) 0°, (b) 1.4°, (c) 4.2°, and (d) 7.0°.

Figure 4 shows the comparable plots to Figure 3 for the four radial filters studied (no angular smoothing). Each curve corresponds to a different radial location, and each point on a curve results from a different radial filter (see Table 1). An appreciable and uniform noise reduction is achieved with uniform resolution degradation. For example, a radial Hanning filter with a 0.14-mm⁻¹ cutoff (filter 3) will lead to a noise reduction of 50-60% at all radial locations, compared to a ramp filter (filter 1). It is not possible to achieve comparable noise reduction by angular smoothing for radial locations inside 8 cm with an angular filter width of less than 7°.

B. Transmission Noise with Angular Smoothing

Emission SD images from the study of angular smoothing of transmission data (not shown) are similar to those from angular smoothing of emission data. As was seen in Figure 2, the SD in the emission image decreases at larger radii with
Figures 1-3 and 5 demonstrate that near the center of the FOV, angular smoothing of emission or transmission data provides no noise reduction and no resolution degradation (i.e., angular filtering has little or no effect at the center of the FOV). Away from the center, however, there is radially-increasing noise reduction with a spatially-varying degradation of tangential resolution. Effectively, angular filtering applies a circular smooth with a FWHM that increases with increasing radial position.

The explanation for these results lies in the backprojection process. A pixel value is generated by summing filtered projections that pass through the pixel. For pixels far from the center, this summation is along a sine-wave shaped curve in the sinogram. Away from the center, angular smoothing thus introduces contributions from other, independent projections.
that do not pass through the pixel, thereby reducing the noise and blurring the image. For central pixels, however, the summation is nearly vertical. Near the center, then, smoothing across rows of the sinogram and then summing along that same direction has little effect on either noise or resolution because no "new" projections contribute to these pixels. Angular filtering is simply a weighted sum of angularly shifted sinograms. Because an angular sinogram shift is equivalent to a rotation of the image, angular filtering can be thought of as a weighted sum of rotated images. Rotating an image has little impact near the center (for small rotation angles); angular filtering, therefore, will also have little effect near the center of the FOV.

Figure 6. Noise as a function of transmission radial filter. COV is plotted as a function of radial filter width. Each curve corresponds to a different radial position. Results are shown for no angular smoothing.

The above studies were performed on an older brain scanner with somewhat coarse angular sampling. The results will carry over to scanners with better angular sampling (e.g., whole-body systems or scanners with smaller detectors), although the magnitude of noise reduction achieved will depend upon the sinogram sampling. For these systems, it would be possible to include more projections for the same degradation in tangential resolution (i.e., one would achieve greater noise reduction for the same resolution price). However, near the center, the noise reduction achieved by angular smoothing of any amount will still be negligible. Standard procedure often involves centering the subject in the FOV in order to achieve uniform resolution. Since the data are often noisiest in the center of the subject due to attenuation, in that situation angular smoothing will have minimal effect on the data most in need of noise reduction. Away from the center, the effects of angular smoothing will remain spatially variant, but with a less marked impact on the tangential resolution for the same number of projections included.

At some institutions, transmission data are smoothed in the axial direction in order to reduce further the impact of transmission noise on the image [7], [8]. Previous studies have demonstrated the utility of axial smoothing of both emission and transmission data on reducing noise in the emission image [8]. We have also studied the impact of axial smoothing of transmission data on noise in the emission image and found that comparable noise reduction could be achieved by combined radial/axial smooths of 8mm/0mm (no axial smoothing), 6mm/4mm, 4mm/6mm, and 0mm/10mm (results not shown). Since less blurring is done in any one direction, the desirable goal of more isotropic resolution [8] can potentially be achieved with a combination of radial and axial filtering. For axially smooth emission or attenuation distributions, then, combined radial and axial filtering may be preferable to radial and angular smoothing and may also reduce the bias that arises due to a transmission and emission resolution mismatch [9], [10], depending on the local attenuation structure.

V. CONCLUSIONS

We have assessed the resolution and noise effects of angular smoothing and conclude that angular smoothing is not an optimal noise-reduction technique for emission or transmission data, due to the anisotropy of noise reduction and resolution degradation and the lack of effect on noise reduction at the center of the FOV. Rather, combined radial and axial smoothing of emission and transmission data should be considered for noise reduction.

VI. REFERENCES


