BEHAVIOR OF A RELATIVISTIC ELECTRON BEAM IN A GAS-FILLED DRIFT TUBE AS A FUNCTION OF GAS PRESSURE

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Abstract
Several issues pertaining to electron beam propagation and techniques for risetime enhancement of the Aurora electron beam and bremsstrahlung pulses are examined. Time-varying models for the double-humped current waveforms produced by propagating relativistic electron beams through gas cells of various pressures and gas species are developed and compared with experimental results. Specifically, there is a pressure region in which the electron beam propagates until a virtual cathode is formed. After later times the virtual cathode is neutralized by the formation of positive ions by collisions between gas molecules and relativistic electrons. The beam can then propagate.

Figure 1. Drift-tube setup. The early portion of the relativistic electron pulse produced across the anode/cathode gap does not propagate down the drift tube.

* The first portion of this work was carried out while Dr. Roberts was at ARL (HDL) and the later portions while he was associated with Mission Research Corporation, Newington, VA.

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I. INTRODUCTION

The passage of a relativistic e-beam through a gas-filled drift tube can be used to modify the temporal pulse shape of the propagated e-beam and the bremsstrahlung pulse shape produced when the e-beam impinges on a thick tungsten target. The work presented here was motivated by a desire to develop methods of altering bremsstrahlung pulse shapes, e.g. risetimes, pulse widths, etc. By varying gas pressure, gas species, drift tube length, etc., bremsstrahlung pulse shape can be varied in a controlled manner.

II. EXPERIMENTAL SETUP

Enhanced risetime x-ray bremsstrahlung pulses have been produced using the AURORA flash x-ray machine[1]. In these experiments, the electron beam from one arm of AURORA was injected into a drift tube filled with low-pressure neutral gas. By varying the gas pressure in the drift cell, it was found that the risetime of the bremsstrahlung x-ray pulse produced by a high-Z converter located at the end of the chamber could be reduced to about 20 ns.

Figure 1 shows the experimental arrangement employed to study[1-6] the behavior of a relativistic electron beam in a drift tube at the Aurora as a function of gas pressure. The drift tube was attached to only one of the four Aurora electron diodes. Relativistic electrons are produced by the diode and then pass through a vacuum window into a 3-meter-long drift tube gas cell. The gas species and pressure in the cell can be varied. Figure 2 shows the general shape of the beam current and voltage at the diode. The passage of a relativistic e-beam through a gas-filled drift tube can be employed to sharpen the risetime of the e-beam by nose erosion[3-6]. Figures 3 and 4 show radiation pulse shapes for various gas pressures in the drift tube for ambient air and for helium gas measured with a photodiode scintillation counter 2-m in front of the beam exit port. At the lower drift tube pressures, excluding the lowest pressure, measurements show a doubly peaked output pulse. The values of RT above each trace correspond to the pulse risetime in nanoseconds. The risetime decrease at higher pressures compared to the pulse shape in figure 2 can be explained in terms of relativistic beam nose erosion [6, p. 415].

III. DOUBLE-HUMPED PULSE WAVESHAPE ANALYSIS

The double-humped data in figures 3 and 4 can be explained in terms of a relatively simple model which describes the electron-beam neutral-gas-cell interaction processes. The essential physics incorporated in the model are time-dependent ionization processes and the possibility of virtual cathode formation [5, pp. 97, 270].

Consider the simplified sketch of Figure 5. When an electron beam in excess of the geometrical space charge limited current is injected into a low pressure neutral gas, three rather distinct pressure regimes can be envisioned [5, p. 415]: (1) a very low pressure range in which the beam current is less than the space charge limit early in the pulse and more than the space charge limit late in the pulse; (2) an intermediate pressure range in which ionization of the gas allows total beam propagation midway during the beam pulse; and (3) a higher pressure range in which gas ionization is sufficiently rapid that the beam current never exceeds the time-dependent space charge limit.
Figure 3. Photodiode measurements of electron-beam pulse shapes. The drift tube was filled with ambient air at pressure indicated. Time-scale is in nanoseconds and the RT values correspond to risetime in nanoseconds. The photodiode current was converted to a voltage across a resistor.

Figure 4. Same as figure 3 except the drift tube was filled with helium.
To semi-quantitatively analyze this concept, we model the time dependent behavior of the space charge limited current as

$$I_b(t) = I_o(t)/[1-f(t)]$$  \hspace{1cm} (1)

where $I_o(t) = (mc^2/e)[\gamma^2(t)-1]^{1/2}/[1+2ln(R/r_b(t))]$ is the geometrical space charge limited current for a solid beam, and $f(t) = n_i(t)/n_a$ is the time-dependent charge neutralization fraction[5, p. 193]. In the above equations, \(m\) is the electron mass, \(c\) the speed of light, \(e\) the electron charge, \(\gamma(t)\) the time dependent relativistic energy factor of the beam, \(R\) the drift tube radius, \(r_b(t)\) the time dependent radius of the beam, \(n_i(t)\) the time dependent positive ion background density, and \(n_a\) the electron density in the beam. All quantities are in cgs Gaussian units. If the beam current \(I_b(t)\) exceeds \(I_o(t)\), then the beam propagation behavior depends critically on the gas pressure.

(1) For the low pressure case the beam will propagate during the time the rising portion of the beam current does not exceed the space charge limited current. During this time the background gas is being ionized by electron impact ionization and ions are being created according to [6, p. 415]

$$\frac{\partial n_i}{\partial t} = \nu_i \sigma_i n_g$$  \hspace{1cm} (2)

where \(\nu_i\) is the electron beam current density, \(\sigma_i\) is the ionization cross section, \(n_g\) is the neutral gas density, and \(e\) is the electron charge. The secondary electrons are assumed to instantaneously leave the region of the beam, being radially expelled by the large space charge field of the beam. When the beam current exceeds the time-dependent space charge limited current, efficient beam propagation is disrupted by the formation of a deep potential well, i.e., a virtual cathode[5]. In the low-pressure region the neutral gas density is too low for electron impact ionization to rapidly neutralize the beam space charge and the major portion of the beam will not propagate.

(2) For somewhat higher pressures ionization of the neutral background gas is rapid enough to allow a major portion of the beam to propagate. The deep potential well collapses midway through the beam pulse, and the beam can again propagate. Any ions trapped in the well can be accelerated. This pressure range thus corresponds to that identified with collective ion acceleration using drifting electron beams.

(3) For still higher pressures ionization of the background gas due to electron impact ionization is sufficiently rapid that the electron beam never exceeds the space charge limited current. The beam propagates immediately, the deep potential well never forms, and ion acceleration will not occur.

In order to analyze the Aurora data in terms of this model, it is necessary to know the beam voltage, current, and radius at the injection point as a function of time. Aurora diode voltage and current waveforms derived from measurements with B-dot probes and capacitative E-field probes positioned in the magnetically insulated transmission line (MITL) were digitized to yield the first two sets of input data. These waveforms are reproduced in Figure 6. In addition, we have used estimates of the time-dependent beam radius obtained from simulations performed using the MAGIC electromagnetic particle-in-cell (PIC) plasma simulation code developed by MRC[7]. These runs simulated the standard AURORA diode configuration, which consists of a 15-inch-diameter, doughnut-shaped cathode, and an 11-inch anode-cathode gap spacing. A sample frame from the simulation is shown in Figure 7, while the resulting variation in beam radius vs time is shown in Figure 8. The geometric space charge limited current, \(I_o(t)\), was then evaluated and is plotted in Figure 9.

Assuming a time-varying beam radius, \(r_b(t)\), the time-dependent space charge neutralization fraction, \(f(t)\), can be determined by integration of Equation (2)

$$f(t) = \sigma_i n_a \int_0^t \frac{\nu(t) r_b^2(t)}{I(t)} \frac{I(t')}{r_b^2(t')} \, dt'$$  \hspace{1cm} (3)

where \(\nu(t)\) is the electron beam velocity. A comprehensive summary of the measured electron impact ionization cross sections of neutral gases is available in the literature[8]. For each gas the cross section is described accurately by a relation of the form

$$\sigma_i = (1.874 \times 10^{-20}) (M_x x_1 + C_x x_2) \, (cm^2)$$  \hspace{1cm} (4)

in which

$$x_1 = (2\beta^2) \ln \gamma - 1, \quad x_2 = \beta^2$$  \hspace{1cm} (5)

where \(\gamma\) is the electron energy factor and

$$\beta = (1 - 1/\gamma^2)^{1/2}.$$
Figure 6. Diode voltage (a) and current (b) as a function of time for 90 kV Marx charge voltage.

Figure 7. Sample frames from a MAGIC (PIC) simulation of the Aurora diode.

Figure 8. Time-dependent average radius for the Aurora beam. (These data are estimated from PIC simulations such as those shown in Fig. 7.)

Figure 9. The geometric, time-dependent, space-charge-limited current $I_s(t)$, as calculated from the digitized voltages and beam radius data.
and $M^2$ and $C$ are tabulated empirical constants characteristic of the gas. These constants are summarized in Table 1 below for several common gases.

Table 1. Ionization Cross Section Constants for Several Common Gases[8].

<table>
<thead>
<tr>
<th>GAS</th>
<th>$M^2$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>0.77</td>
<td>7.65</td>
</tr>
<tr>
<td>Ne</td>
<td>2.02</td>
<td>18.2</td>
</tr>
<tr>
<td>Ar</td>
<td>4.22</td>
<td>37.9</td>
</tr>
<tr>
<td>H2</td>
<td>0.69</td>
<td>8.12</td>
</tr>
<tr>
<td>N2</td>
<td>3.74</td>
<td>34.8</td>
</tr>
<tr>
<td>O2</td>
<td>4.20</td>
<td>38.8</td>
</tr>
<tr>
<td>CO2</td>
<td>5.75</td>
<td>55.9</td>
</tr>
</tbody>
</table>

If we assume an average electron energy of 4 MeV, then $x_1 = 3.36$ and $x_2 = 1$. For He we obtain $\sigma = 1.92 \times 10^{-19}$ cm$^2$; also we have $n_g = (3.53 \times 10^{16}$ atoms/cm$^3$) x P(Torr), where P(Torr) is the pressure in Torr. Thus,

$$f(t)_{he} = 2.03 \times 10^8 P(Torr) \left( f_\alpha(t)/f(t) \right) \left( (I(t')/I_\alpha(t'))^2 \right) dt' \quad (6)$$

For air, we assume an 80/20 mixture of nitrogen and oxygen, and obtain

$$f_{we} = 9.65 \times 10^8 P(Torr) \left( f_\alpha(t)/f(t) \right) \left( (I(t')/I_\alpha(t'))^2 \right) dt' \quad (7)$$

Equations (6) and (7) can be numerically evaluated using the digitized beam current and beam radius data. Writing $f(t) = f_\alpha(t) P(Torr)$, we can then compute the variation of the space charge limited current as a function of time for various fill pressures of air and helium. These results are presented in Figures 10 and 11.

During the virtual cathode stage, propagation of the total injected beam current is not possible; however, a significant fraction of the space-charge-limited current can still propagate. Effectively, the virtual cathode can emit. A simple one-dimensional model which attempts to estimate the magnitude of the current which propagates is given by[5, p.98]

$$(2I - I_p)^{1/2} + I_p^{1/2} = 2I_\alpha^{1/2} \quad (8)$$

where $I$ is the injected current, $I_\alpha$ is the propagating current, and $I_\alpha$ is the space charge limited current. The ratio of $I_p/I_\alpha$ is plotted in Figure 12 as a function of $I/I_\alpha$.

Figure 10. The space-charge-limited current $I_\alpha$ versus time for air at various pressures. (Numbers correspond to pressure in milliTorr.)

Figure 11. The space-charge-limited current $I_\alpha$ versus time for helium at various pressures. (Numbers correspond to pressure in milliTorr.)

Using the digitized beam current data, the space-charge-limited current data of Figures 10 and 11 and the ratio data of Figure 12 then gives the predicted time-dependent propagating beam current in the drift tube as a function of gas pressure. These results are presented in Figures 13 and 14 for helium and air.
Figure 12. Ratio of propagating current to space-charge-limited current as a function of the ratio of injected beam current to space-charge-limited current.

![Figure 12](image)

The ratio of the propagating current to the space-charge-limited current is given by the equation $I/I_s$. The figure shows the relationship between these two quantities as a function of the ratio of the injected beam current to the space-charge-limited current.

As the final step, we try to correct in an approximate fashion for the non-linear photodiode response. We assume that the photodiode signals depend primarily on the electron beam dose rate, rather than on the bremsstrahlung dose rate produced by the relativistic electron beam. The bremsstrahlung dose rate in the axial direction of the diode can be approximated by $V^2I$ where $V$ is the voltage across the relativistic diode and $I$ the beam current [9]. As a result, we calculate the quantity $V^2I$ at every timestep. These data have been normalized to the maximum value of the peak propagating current and voltage, and the results are presented in Figures 15 and 16 for various pressures of helium and air. These waveshapes are to be compared with the experimental data presented in Figures 3 and 4.

Figure 13. Propagating beam current versus time in air at various pressures. (Numbers correspond to pressure in milliTorr.)

![Figure 13](image)

Figure 14. Propagating beam current versus time in helium at various pressures. (Numbers correspond to pressure in milliTorr.)

![Figure 14](image)

Figure 15. Normalized radiation intensity versus time in air at various pressures. (Numbers correspond to pressure in milliTorr.)

![Figure 15](image)
Figure 16. Normalized radiation intensity versus time in helium at various pressures. (Numbers correspond to pressure in milliTorr).

IV. CONCLUSIONS

The original purpose of these e-beam propagation experiments was to manipulate the relativistic e-beam pulse shape impinging on a thick tantallum bremsstrahlung target. The passage of the e-beam through a gas cell can be used to control the risetime, pulse width, and general pulse shape of the final bremsstrahlung temporal pulse shape.

At the higher pressures (200 mTorr-He) nose erosion sharpens the 70 ns Aurora pulse risetime to approximately 20 ns. There is, however, a double-humped phenomenon at lower pressures.

Although the detailed agreement is somewhat sensitive to exactly when the current begins to rise in comparison with the voltage pulse, for example, and several assumptions are inherent (erosion processes are not modeled, for example), the qualitative picture presented here agrees surprisingly well with the experimental data. To summarize, at air pressures of 2-4 mTorr only the leading edge of the beam current trace is expected to propagate well, and the remainder of the beam pulse will be strongly attenuated because of virtual cathode formation. From about 6-10 mTorr (air) we expect to see the "double-humped" propagation characteristic, while for pressures in excess of roughly 15-20 mTorr the entire pulse should propagate. An examination of the available data indicates just this behavior. (Although at the higher pressures beam front erosion is evidently significant.)

In addition, if other gases are used, the essential scaling parameter is the ionization cross section. For example, the ratio of the air ionization cross section to that of helium is about 4.75. Thus, we would expect to observe the same characteristic beam propagation behavior in helium, but at pressures a factor of 4.75 times higher than for air. This prediction also seems to be verified by the experimental data.

The decrease in risetime due to e-beam nose erosion at the higher pressures (see figures 3 and 4) can be combined with other risetime enhancement techniques. See reference 10.

V. REFERENCES

[9] T. Sanford, Sandia National Laboratory, Private communication (Recent work yields a much more accurate relationship incorporating angular distributions).