Conductor Loss in Superconducting Planar Structures: Calculations and Measurements

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Abstract—We present closed-form expressions of the attenuation constant due to conductor loss for superconducting coplanar waveguide and microstrip transmission lines. These expressions, valid for arbitrary conductor thickness, make use of a numerically determined quantity (the stopping distance $\Delta$) that depends on the material properties and edge shape of the superconducting transmission line. Once $\Delta$ is determined, the attenuation constant for any planar geometry can be obtained without further numerical calculation, making this technique attractive for use in the design of circuits incorporating superconducting planar elements. The results of this calculation compare favorably with full numerical calculations and also with experimental data on high-temperature superconducting coplanar transmission lines, illustrating the accuracy and applicability of the calculation for determining the conductor loss of superconducting circuit elements.

Index Terms—Attenuation, coplanar transmission lines, high-temperature superconductors, microstrip, superconducting microwave devices.

I. INTRODUCTION

HIGH-TEMPERATURE superconducting (HTS) materials are being increasingly used in high-performance passive microwave applications [1], such as filters for wireless communications, due in part to the fact that the quality of HTS thin films has greatly improved in recent years. Since these materials are readily available in thin-film form, many superconducting microwave elements are being implemented in planar geometries such as microstrip and coplanar. It is, therefore, increasingly important to be able to accurately model the critical parameters of superconducting coplanar and microstrip structures, in order to facilitate the design and fabrication of high-performance superconducting microwave components.

To calculate the conductor loss in planar microwave circuits, various techniques can be used, from full numerical calculations to quasi-analytical methods such as Wheeler’s rule. While numerical techniques are capable of very high accuracy, they are computationally intensive and often difficult to use and, hence, do not lend themselves to ready use in design. Instead, closed-form expressions for the conductor loss are desirable for this purpose and can be obtained using Wheeler’s rule. However, if the ratio $t/\delta$ (where $\delta$ is the relevant skin depth and $t$ is the thickness of the conductor) is small or comparable to unity, then Wheeler’s rule breaks down and gives unreliable results. The failure of analytical methods for thin films can be a serious limitation for the design and modeling of planar structures, particularly for superconducting elements, where the film thickness is often comparable to the relevant penetration depth.

In this paper, we present closed-form expressions for the attenuation constant due to conductor loss of superconducting coplanar waveguide (CPW) and microstrip transmission lines; expressions which are valid for any conductor thickness. Analogous expressions for the phase constant will be presented in future work. The calculation makes use of a numerically determined quantity (the stopping distance $\Delta$) that depends on the material properties and shape of the superconducting transmission line. Once determined, the stopping distance is used in an analytical expression for the attenuation due to conductor loss for any transmission-line geometry and film thickness. No further numerical calculations are necessary once $\Delta$ has been obtained, making this approach well suited for use in the design of practical superconducting circuit elements.

In what follows, we first describe the numerical determination of the stopping distance for different material parameters and then present the analytical expressions for the attenuation constant for CPW and microstrip geometries. The results of the calculations are then compared with full numerical simulations of the superconducting attenuation constant found in the literature. A detailed comparison of the calculation with experimental data for high-$T_c$ superconducting CPW transmission lines and resonators of different geometries, measured using a cryogenic microwave probe station, is then presented. Comparisons are also made between normal metal conductors and superconductors in the microstrip geometry. We conclude with a discussion of the limitations of this approach for the calculation of the conductor loss in superconducting planar circuits.

II. DETERMINATION OF THE ATTENUATION CONSTANT

To arrive at a closed-form expression for the conductor loss valid for arbitrary thickness conductors, it is tempting to apply standard perturbation methods. An approximate current density on the strips can be obtained by assuming the conductors are infinitely thin, and this current density can then be used in the expression for the attenuation constant

$$\alpha \approx \frac{R_s}{2\pi} \int_{\text{top}} \left( \frac{J}{I} \right)^2 dl. \quad (1)$$

where the integral is carried over both the top and bottom portions of the strip conductor. Here, $J$ is the approximate current.
density on the strip, \( I \) is the total current on the conductors, \( R_s \)
is the surface resistance of the strip conductors, and \( Z_0 \) is the
characteristic impedance of the structure. However, because the
current density on an infinitely thin perfectly conducting strip diverges as \( 1/\sqrt{r} \) (where \( r \) is the distance from the nearest
edge), the integration of \( |J|^2 \) in (1) diverges logarithmically,
giving no useful results for the conductor loss.

To avoid this difficulty, Lewin [2], Vainshtein and Zhurav
[3], and Holloway and Kuester [4], [5] have developed an
alternative approach, in which the conductor loss is approx-
imated by carrying out the integration in (1) not to the strip
edges, but to some finite distance, the stopping distance \( \Delta \),
away from the edges of the strip. The value of \( \Delta \) is determined
numerically and is a function only of the local edge geometry:
the strip thickness, shape of the edge, and material of the strip
conductor. The stopping distance and resulting expressions for
the normal metal conductor loss for CPW and microstrip lines
are presented by Holloway and Kuester [4], [5].

To determine the conductor loss for superconducting planar
structures, a new stopping distance \( \Delta \) must be calculated.
This is accomplished in a manner analogous to the normal
metal case by noting that the power loss in the region local
to the edge of a superconducting strip can be accurately
determined using an asymptotic solution for the fields near
the isolated edge [4]. The stopping distance \( \Delta \) can then
be found by equating this power loss to the power loss
found from the stopping-distance method using the current
distribution of a zero-thickness perfect conductor. To describe
the electrodynamic response of a superconductor, one can, in
general, define a complex conductivity \( \sigma = \sigma_1 - i\sigma_2 \), where
the real (imaginary) part of the conductivity describes the response
of the normal (super) fluid in a two-fluid picture [6]. These two
components of the conductivity can be written conveniently
in terms of a superconducting magnetic penetration depth \( \lambda = \sqrt{1/\mu_0\omega\delta} \) and a normal fluid skin depth \( \delta_n = \sqrt{2/\mu_0\omega\sigma_1} \).
The resulting stopping distance for a superconductor as a
function of the material properties (the strip thickness \( t \),
normal fluid skin depth \( \delta_n \), and magnetic penetration depth \( \lambda \))
is shown in Fig. 1 (for a 90° edge profile); it can be
subsequently used to obtain the conductor loss for planar
superconducting structures. Note that the stopping distance
\( \Delta \) depends sensitively on the ratio \( t/2\lambda \), but is relatively
insensitive to the ratio \( t/2\delta_n \), provided that \( \lambda \ll \delta_n \) (which
implies that the material responds as a good superconductor).

Once the stopping distance \( \Delta \) has been determined, the
attenuation constant for a superconducting CPW transmission
line of dimensions \( a, b \) (see Fig. 2 inset) can be simply
calculated from the expression [5]

\[
\alpha \approx \frac{R_{sm}b^2}{16Z_0K^2(a/b)(b^2-a^2)} \left\{ \frac{1}{a} \ln \left( \frac{2a(b-a)}{\Delta(b+a)} \right) + \frac{1}{b} \ln \left( \frac{2b(b-a)}{\Delta(b+a)} \right) \right\}
\]

(2)

where \( K(a/b) \) is the elliptic integral of the first kind and

\[
R_{sm} = \mu_0\omega t \ln \left( \frac{\cos(k_c t) + \csc(k_c t)}{k_c t} \right)
\]

(3)

is a modified surface impedance for a superconductor strip
of thickness \( t \). Here, \( k_c \) is the (complex) wavenumber in the
superconductor at a frequency \( \omega \) given by [6]

\[
k_c^2 = \left( \frac{1}{\lambda} \right)^2 + 2\lambda \left( \frac{1}{\delta_n} \right)^2.
\]

(4)

The expression in (2) includes the loss due to the signal line
and both ground planes and also assumes that the ground-plane
width is large compared with the strip width \( (2a) \) or the gap
spacing \( (b-a) \).

For a microstrip line of width \( W \), the corresponding expres-
sion for the attenuation constant is [4]

\[
\alpha \approx \frac{R_{sm}}{2\pi^2Z_0} \ln \left( \frac{W}{\Delta} - 1 \right).
\]

(5)

Equation (5) corresponds to the loss of only the strip portion
of the microstrip line. For some microstrip geometries, the
conductor loss associated with the ground plane may become
comparable to the loss of the strip [4] and must be taken into
account in order to accurately calculate the total conductor
loss of a microstrip line. Expanding on previous work [7], we
approximate the conductor loss of the ground plane by

\[ \alpha_{gr} \approx \frac{R_{em}}{Z_0 n W} \left\{ \frac{\pi_1}{2 h} \ln \left( 1 + \left( \frac{W}{2 h} \right)^2 \right) \right\}. \]

(6)

In this expression, \( h \) is the distance from the center line to the ground plane, and the thickness of the ground plane is used in the calculation of \( R_{em} \) in (3).

With \( \Delta \), determined from Fig. 1 for the appropriate material properties, (2) and (5) can be readily used to calculate the loss for finite-thickness CPW or microstrip structures of any geometry, with no further numerical calculations. However, one must use caution if interpolation between curves of constant \( t/2\lambda \) in Fig. 1 is used in order to obtain a more accurate value for \( t/\Delta \). In earlier work [4], \( t/\Delta \) was plotted as a function of \( t/2\lambda \) for the case of a normal metal conductor (\( t/2\lambda = 0 \) in Fig. 1). Such a plot shows that \( t/\Delta \) is a nonmonotonic function of \( t/2\lambda \) for values of \( t/\delta > 1 \). This implies that interpolation between curves in Fig. 1 may not always provide a correct value for \( t/\Delta \). However, as we show below, most good superconductors are described adequately by the curves presented without the need to interpolate between them.

III. COMPARISON WITH NUMERICAL CALCULATIONS

To verify the validity of our approach, we show, in Fig. 2, a comparison of the attenuation constant calculated from (2) with numerical results from the mode-matching calculation of Heinrich [8] for a superconducting CPW line of thickness \( t = 0.3 \) \( \mu \)m and with \( a = 7.5 \) \( \mu \)m and \( b = 17.5 \) \( \mu \)m over the frequency range 1–100 GHz. The material properties of the superconductor are given by a London penetration depth of \( \lambda = 200 \) nm and a normal fluid conductivity of \( \sigma_1 = 8.2 \times 10^{6} \) S/m. These material parameters are typical for HTS materials at 77 K. With this value for \( \sigma_1 \), the quantity \( t/2\theta_{nf} \) ranges from \( 8.5 \times 10^{-3} \) to \( 8.5 \times 10^{-2} \) over the frequency range 1–100 GHz. Fig. 1 shows that \( \Delta \) is effectively independent of the value of \( \delta_{nf} \) over this entire frequency range, implying that a single value of \( \Delta \) is sufficient for a complete description of the superconductor in this example. The excellent agreement between the full numerical simulation of Heinrich and the present calculation illustrates the accuracy of our calculation, specifically for a range of materials parameters representative of high-\( T_c \) superconductors.

IV. COMPARISON WITH EXPERIMENTAL DATA

In order to further demonstrate the validity of our calculation and to illustrate its application, we compare the results of (2) to experimental data on actual superconducting CPW transmission lines and resonators fabricated from YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) (YBCO) thin films (with critical temperatures of \( T_c \approx 91 \) K) grown by pulsed-laser deposition on LaAlO\(_3\) substrates. Before the thin films are patterned into coplanar structures, the surface impedance of the superconducting samples is measured as a function of temperature using a sapphire dielectric resonator [9], giving quantitative values for the material properties of the superconducting thin films. Our CPW structures are subsequently patterned from these same films by optical photolithography and Ar ion milling. Gold contact pads are evaporated onto the CPW devices to facilitate reproducible electrical contact.

The patterned CPW transmission lines and resonators are measured at temperatures below \( T_c \) using a low-temperature microwave probe station [10]. The probe station incorporates movable air coplanar probes that are cooled to cryogenic temperatures under vacuum along with the superconducting samples. This allows many different superconducting devices, which are often patterned onto a single thin film, to be characterized rapidly during a single cooldown. Two different measurement techniques are used: transmission-line measurements and resonator measurements. For characterization of the transmission lines, a low-temperature wafer calibration scheme [11] is used to obtain the attenuation constant as a function of frequency from 50 MHz to 40 GHz. The errors associated with the attenuation-constant determination are primarily due to the precision with which the transmission coefficient can be reproducibly measured, which is probably limited in the current measurements by irreproducible contact and temperature drift during the measurements. For structures which have very low losses, resonator measurements become necessary as the attenuation constant becomes smaller than the resolution limit of the transmission measurements. For these measurements, the movable probes are decoupled from the transmission-line structures, creating a resonant structure. The quality factor of the fundamental resonance and several overtone resonances are then measured. Although the resonant measurements are much more sensitive than the transmission measurements described above, data is obtained only at discrete frequency points. In addition, as the conductor losses decrease, contributions to the measured quality factor due to dielectric loss in the substrate and radiation losses become increasingly important. For the measurements presented here, the quality factor appears to be dominated by conductor losses. The results of these different measurements can then be directly compared to the response calculated from (2) for a number of different CPW geometries.

Fig. 3 shows the measured attenuation constant at 76 K for three CPW transmission lines of different geometries fabricated from the same YBCO film of 50-nm thickness. The center conductor linewidths of the different transmission lines are: 1) 21 \( \mu \)m; 2) 53 \( \mu \)m; and 3) 105 \( \mu \)m, and the gap spacing between the center conductor and ground planes is varied to maintain 50-\( \Omega \) characteristic impedance. Also shown in Fig. 3 are the results of (2), using values for \( \sigma_1 = 1.72 \times 10^6 \) S/m, and \( \lambda = 316 \) nm. Both \( \sigma_1 \) and \( \lambda \) are extracted from the dielectric-resonator measurements of the surface impedance of the thin films at \( T = 76 \) K. In order to determine the absolute value for the penetration depth \( \lambda \), we combine our dielectric resonator measurements, which yield \( \Delta \lambda(T) \) from the resonant frequency shift, with results for the absolute value of the penetration depth \( \lambda(T = T_{k_BT}) \) obtained from mutual inductance measurements of similar samples [12], [13]. The calculation results obtained in Fig. 3, therefore, make use of no adjustable material parameters and yield results that agree...
very well with the measured experimental data for all three different transmission-line geometries.

The experimental data in Fig. 3 are obtained in a region where $\Delta$ is nearly independent of material parameters $(t/\lambda \ll 1)$. We can obtain further confirmation of our calculations by using material parameters such that $t/\lambda \approx 1$. Such data are obtained using CPW resonators fabricated from thicker films and measured at low temperatures where the penetration depth $\lambda$ is smallest. Fig. 4 shows the quality factors of CPW resonators of center linewidth $21 \mu m$ measured at $30 \text{ K}$ for film thicknesses: 1) $50 \text{ nm}$; 2) $320 \text{ nm}$; and 3) $500 \text{ nm}$. Resonator measurements for these samples are necessary because the attenuation constant has become smaller than the sensitivity limit of our calibrated transmission-line measurements over a large portion of the measurable frequency range. Fig. 4 also shows results of our calculation for the resonator quality factor based on material parameters obtained from our dielectric-resonator measurements, as described previously. For this example, the quantity $t/\lambda$ varies in the $0.24-2.4$ range. Since the quality factor $Q$ is given by $Q = \beta / 2\alpha$, where $\beta$ and $\alpha$ are the phase and attenuation constants, respectively, we need an expression for $\beta$ in order to obtain the calculated $Q$ from our calculated attenuation constant. For the data in Fig. 4, we use the approximation $\beta \approx \omega / c \sqrt{\varepsilon_{\text{eff}}}$ (where $c$ is the speed of light, and $\varepsilon_{\text{eff}}$ is the effective dielectric constant, related to the substrate dielectric constant $\varepsilon_{\text{sub}}$ for the coplanar geometry by $\varepsilon_{\text{eff}} = (\varepsilon_{\text{sub}} + 1)/2$. Using $\varepsilon_{\text{sub}} = 24$ for the dielectric constant of LaAlO$_3$, this gives $Q = (3.216)f/\alpha$, where $f$ is the frequency in gigahertz and $\alpha$ is in decibels/centimeter.

The above approximation ignores the kinetic inductance of the superconducting thin film. However, calculations of the kinetic inductance [14] show that this contribution is less than 8% for the 50-nm films, which have the largest kinetic-inductance contribution of the three measured samples. Therefore, the calculated $Q$ in Fig. 4 slightly underestimates the actual value for the quality factor and is more accurate for the thicker films. Nonetheless, this comparison shows that the calculation also agrees well with experimental data in the regime where $t/\lambda \approx 1$. These comparisons to experimental data on actual HTS circuits demonstrate the applicability of our calculation for a wide range of materials parameters appropriate to the HTS materials.

In order to demonstrate the application of these results to geometries other than coplanar, we present in Fig. 5 calculations of the attenuation constant of a microstrip transmission
normal metal conductors approaches the dimensions of the transmission line, violating the assumption of isolated edges. Also, the results of this calculation are easily extended to superconductors with arbitrarily shaped edges, although only the 90° edge profile has been considered here.

One complication of applying this calculation to superconducting materials is the fact that two material parameters need to be specified in order to determine the stopping distance $\Delta$ from Fig. 1 (in contrast to the case of normal metal conductors, which require only one material parameter). This means that instead of simply looking up the value for $\Delta$ in a table, it may be necessary to search a two-dimensional parameter space for the value of $\Delta$ given values for $\lambda$ and $\delta_{nf}$. This is complicated by the nonmonotonic nature of $t/\Delta$ as a function of $t/2\delta_{nf}$ or $t/2\lambda$. However, for the practical examples given above, the values of $t/2\delta_{nf}$ range from $4.6 \times 10^{-4}$ to $6.3 \times 10^{-3}$, showing that, for most superconducting applications, we can in practice make use of the $t/2\delta_{nf} \to 0$ limit and use just one of the curves shown in Fig. 1.

VI. CONCLUSIONS

In summary, we have presented closed-form expressions for the attenuation constant due to conductor loss for superconducting CPW and microstrip lines. These expressions are based on a calculation of the stopping distance $\Delta$, which is a function of the strip thickness $t$, normal-fluid skin depth $\delta_{nf}$, and London penetration depth $\lambda$. Universal curves for $\Delta$ have been presented and are used in (2) and (5) to determine the attenuation constant for arbitrary CPW or microstrip geometries. Unlike other quasi-analytical results found in the literature, the expressions presented here are valid for arbitrary conductor thickness. Comparisons to both experimental results and to full numerical techniques found in the literature show very good agreement.

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REFERENCES


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